Selection bias in 2020

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Selection bias: An umbrella term

- The Cambridge Dictionary of Statistics: "The bias that may be introduced into all types of scientific investigations whenever **a treatment is chosen by the individual involved or is subject to constraints that go unobserved by the researcher**".
- Wikipedia: "the bias introduced by the **selection of individuals, groups or data for analysis in such a way that proper randomization is not achieved**, thereby ensuring that the sample obtained is not representative of the population intended to be analyzed."
- Wikipedia collects many types of selection bias: (non-random) sampling bias; time interval (censoring/truncation); susceptibility bias; indication bias; data dredging; attrition/survivorship bias; observer selection bias; volunteer bias; Berkson's paradox (collider bias).

My experiences: Before 2020

- Don't remember taking a course that involved > 1 lectures about selection bias.
- In consulting sessions, clients often want to know how to interpret their results after an array of data processing and model selection. What should I say?
- Have seen many anecdotes about selection bias.
- Always thought they are far away from me. The current practice cannot be that bad, right?

My experiences: 2020

A statistician's existential crisis

Rest of the talk: Two topical examples

1. Initial estimates of COVID-19's infectiousness and incubation period.
   - Reference: Z, Ju, Bacallado, Shah. (2020). BETS: The dangers of selection bias in early analyses of the coronavirus disease (COVID-19) pandemic. *The Annals of Applied Statistics* (in press). arXiv: 2004.07743.
2. Racial bias in policing.
   - Reference: Z, Keele, Small, Joffe. (2020). A note on post-treatment selection in studying racial discrimination in policing. arXiv: 2009.04832.

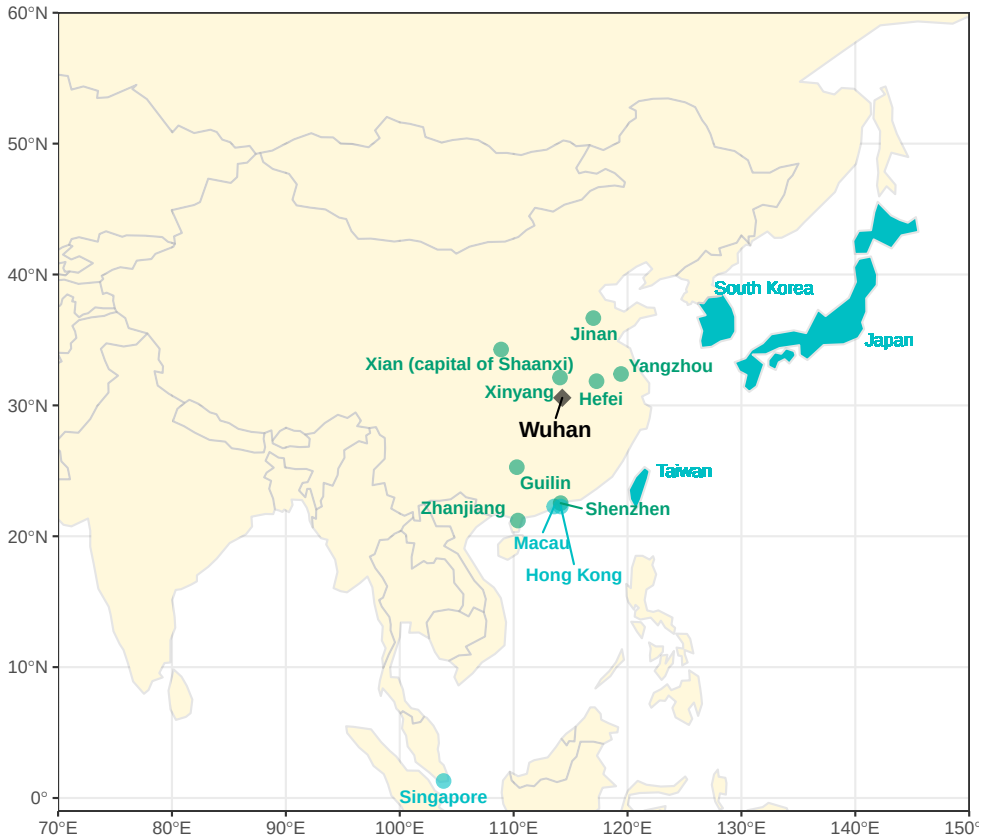
Acknowledgement

Many people who have offered helpful suggestions: Cindy Chen, Yang Chen, Yunjin Choi, Hera He, Michael Levy, Marc Lipsitch, James Robins, Andrew Rosenfeld, Dylan Small, Yachong Yang, Zilu Zhou; Dean Knox, Joshua Loftus, and Jonathan Mummolo.

Initial COVID-19 studies

- Many were based on "exported" cases from Wuhan.
- Extremely influential.
- Many types of selection bias incurred: Under-ascertainment; Non-random sample selection; right-truncation; ignoring travel restrictions and fast epidemic growth on unobserved data.
- Common mistake: New data + Existing model = New results.

Data collection



- 14 locations where the local health agencies published full case reports.
- 1,460 COVID-19 cases that were confirmed by February 29 for locations in mainland China (February 15 for international locations).
- 378 exported cases from Wuhan.

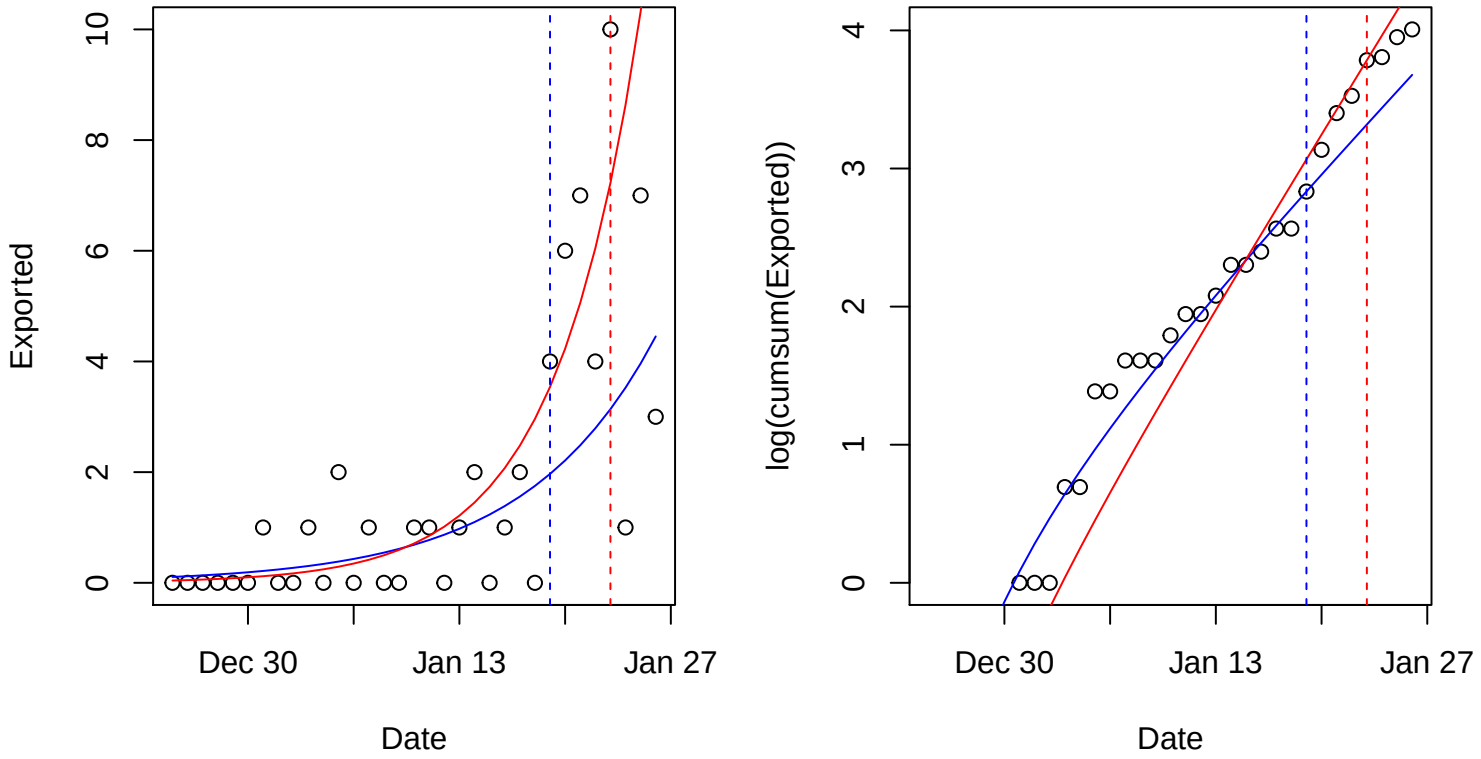
Overview of the dataset

Column name	Description	Example	Summary statistics
`Case`	Unique identifier for each case	HongKong-05	1460 in total
`Residence`	Nationality or residence of the case	Wuhan	21.5% reside in Wuhan
`Gender`	Gender	Male/Female	52.1%/47.7% (0.2% NA)
`Age`	Age	63	Mean=45.6, IQR=[34, 57]
`Known Contact`	Known epidemiological contact?	Yes/No	84.7%/15.3%
`Cluster`	Relationship with other cases	Husband of HongKong-04	32.1% known
`Outside`	Transmitted outside Wuhan?	Yes/Likely/No	58.5%/7.7%/33.8%
`Begin Wuhan`	**Begin of stay in Wuhan (B)**	30-Nov[4]	
`End Wuhan`	**End of stay in Wuhan (E)**	22-Jan	
`Exposure`	Period of exposure	1-Dec to 22-Jan	58.9% known period/date 8.2% known date
`Arrived`	Final arrival date at the location where confirmed a COVID-19 case	22-Jan	40.6% did not travel
`Symptom`	**Date of symptom onset (S)**	23-Jan	9.0% NA
`Initial`	Date of first medical visit	23-Jan	6.5% NA
`Confirmed`	Date confirmed	24-Jan	

Naive method

- Wu, J. T. et al. (2020). Nowcasting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: A modelling study. *The Lancet*, 395(10225).
- They used a SEIR (Susceptible-Exposed-Infectious-Recovered) model for the epidemic in Wuhan and a Poisson process to model case exportation.
- They fitted the model using 17 (!) international cases who showed symptoms before January 20, 2020.
- To replicate their analysis, I fitted some simple Poisson log-linear models.

Initial doubling time



- **Blue** (using symptom onsets before January 20): 5.9 days (95% CI 3.4–15.7).
- **Red** (before January 24): 3.9 days (2.9–5.5).
- Original study: 6.4 days (5.8–7.1).

Problems

These models

- Do **NOT** take into account Wuhan's travel ban on January 23.
- Ignore the rich information available for the individual cases.

Let's start from the first principles

Four crucial epidemiological events

- B: Beginning of stay in Wuhan;
- E: End of stay in Wuhan;
- T: Time of transmission (unobserved);
- S: Time of symptom onset.

Below we will:

- Define the support $\mathcal{P}$ of (B, E, T, S) for the **Wuhan-exposed** population;
- Construct a generative model for (B, E, T, S);
- Define the sample selection set $\mathcal{D}$ corresponds to **Wuhan-exported** cases;
- Derive likelihood functions to adjust for sample selection.

Wuhan-exposed population $\mathcal{P}$

Intuitively, $\mathcal{P}$ = All people who stayed in Wuhan between 12am December 1, 2019 (time 0) and 12am January 24, 2020 (time L, the lockdown).

Conventions

- $B = 0$: **Started their stay in Wuhan before time 0.**
- $E = \infty$: **Did not arrive in the 14 locations we are considering before time L.** (We do not differentiate between people who stayed in Wuhan or went to a different location).
- $T = \infty$: **Were not infected during their stay in Wuhan.** (We do not differentiate between infection outside Wuhan and never infected.)
- $S = \infty$: **Did not show symptoms of COVID-19** (never infected or asymptomatic).

Under these conventions.

$$\mathcal{P} = \Big\{(b, e, t, s) \mid b \in [0, L], e \in [b, L] \cup \{\infty\}, t \in [b, e] \cup \{\infty\}, s \in [t, \infty]\Big\}.$$

A generative BETS model

$$f(b, e, t, s) = \underbrace{f_B(b) \cdot f_E(e \mid b)}_{\text{travel}} \cdot \underbrace{f_T(t \mid b, e)}_{\text{disease transmission}} \cdot \underbrace{f_S(s \mid b, e, t)}_{\text{disease progression}}.$$

To allow extrapolation from Wuhan-exported sample to Wuhan-exposed population, the BETS model makes two basic assumptions

Assumption 1: Disease transmission independent of travel

$$f_T(t \mid b, e) = \begin{cases} g(t), & \text{if } b < t < e, \\ 1 - \int_b^e g(x)\, dx, & \text{if } t = \infty. \end{cases}$$

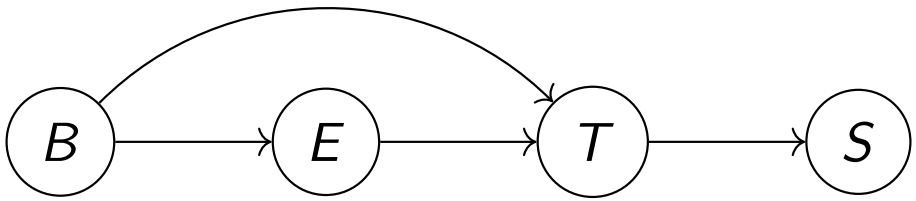
Here $g(\cdot)$ models the **epidemic growth** in Wuhan before the lockdown.

Assumption 2: Disease progression independent of travel

$$f_S(s \mid b, e, t) = \begin{cases} \nu \cdot h(s - t), & \text{if } t < s < \infty, \\ 1 - \nu, & \text{if } s = \infty. \end{cases}$$

Here $h(\cdot)$ is the density of the **incubation period** $S - T$ (for symptomatic cases).

Graphical model representation



- This is in temporal/causal order if we view E as the **planned** traveling date.
- Assumption 1 restricts the density of T given B, E.
- Assumption 2 says that $S \perp\!\!\!\perp B, E \mid T$.

Parametric assumptions

To ease the interpretation and simply the likelihood functions, we assume

Assumption 3: Exponential growth

$$g(t) = g_{\kappa,r}(t) \stackrel{\Delta}{=} \kappa \cdot \exp(rt), \ t \leq L,$$

Assumption 4: Gamma-distributed incubation period

$$h(s-t) = h_{\alpha,\beta}(s-t) \stackrel{\Delta}{=} \frac{\beta^\alpha}{\Gamma(\alpha)}(s-t)^{\alpha-1}\exp\{-\beta(s-t)\}.$$

- Assumptions 3 & 4 are relaxed in a Bayesian nonparametric analysis (see the paper).

Wuhan-exported cases

The event of observing Wuhan-exported cases can be written as

$$\mathcal{D} = \{(b, e, t, s) \in \mathcal{P} \mid b \leq t \leq e \leq L, t \leq s < \infty\}.$$

This makes three further restrictions on $\mathcal{P}$:

1. $B \leq T \leq E$, because we only use cases who contracted the virus during their stay in Wuhan;
2. $E \leq L$, because the case can only be observed if they left Wuhan before the travel ban;
3. $S < \infty$, because we only consider COVID-19 cases who showed symptoms.

Which likelihood function?

For a moment, let's pretend the time of transmission T is observed.

✗ Sample from $\mathcal{P}$

$$\prod_{i=1}^{n} f(B_i, E_i, T_i, S_i)$$

✓ Sample from $\mathcal{D}$ (Unconditional likelihood)

$$\prod_{i=1}^{n} f(B_i, E_i, T_i, S_i \mid \mathcal{D}), \text{ where } f(b, e, t, s \mid \mathcal{D}) \triangleq \frac{f(b, e, t, s) \cdot 1_{\{(b,e,t,s)\in\mathcal{D}\}}}{\mathbb{P}\big((B, E, T, S) \in \mathcal{D}\big)}.$$

✓ Sample from $\mathcal{D}$ (Conditional likelihood)

$$\prod_{i=1}^{n} f(T_i, S_i \mid B_i, E_i, \mathcal{D}), \text{ where } f(t, s \mid b, e, \mathcal{D}) \triangleq \frac{f(t, s \mid B = b, E = e) \cdot 1_{\{(b,e,t,s)\in\mathcal{D}\}}}{\mathbb{P}\big((B, E, T, S) \in \mathcal{D} \mid B = b, E = e\big)}.$$

Unobserved T

In reality, the time of transmission T is unobserved. We can either treat T as a latent variable and use e.g. an EM algorithm, or use the **integrated likelihood**:

Unconditional likelihood

$$L_{\text{uncond}}(\theta) = \prod_{i=1}^{n} \int f\left(B_i, E_i, t, S_i \mid \mathcal{D}\right) dt,$$

where $\theta = (f_B(\cdot), f_E(\cdot \mid \cdot), g(\cdot), h(\cdot))$.

Conditional likelihood

$$L_{\text{cond}}(\theta) = \prod_{i=1}^{n} \int f\left(t, S_i \mid B_i, E_i, \mathcal{D}\right) dt,$$

where $\theta = (g(\cdot), h(\cdot))$.

The conditional likelihood is less efficient because it does not use information in $f(b, e \mid \mathcal{D})$; but it is robust to misspecifying the travel models $f_B(\cdot), f_E(\cdot \mid \cdot)$.

Conditional likelihood function

Proposition

Under Assumptions 1–4,

$$L_{\text{cond}}(r,\alpha,\beta) = \begin{cases} r^n\left(\dfrac{\beta}{\beta+r}\right)^{n\alpha} \cdot \displaystyle\prod_{i=1}^{n} \dfrac{\exp(rS_i)\left[H_{\alpha,\beta+r}(S_i-B_i) - H_{\alpha,\beta+r}((S_i-E_i)_+)\right]}{\exp(rE_i)-\exp(rB_i)}, & \text{for } r>0, \\ \displaystyle\prod_{i=1}^{n} \dfrac{H_{\alpha,\beta}(S_i-B_i) - H_{\alpha,\beta}((S_i-E_i)_+)}{E_i-B_i}, & \text{for } r=0, \end{cases}$$

where $H_{\alpha,\beta}(\cdot)$ is the CDF of Gamma(α,β) and $(\cdot)_+ = \max(\cdot, 0)$.

- This does not depend on ν (proportion of symptomatic cases) and κ (baseline transmission).
- When $r = 0$, this reduces to the likelihood function in Reich et al. (2009) *Statistics in Medicine*, 28.
- The unconditional likelihood function assuming "stable travel" can be found in the paper.

Results

Location	Sample size	Doubling time (in days)	Incubation period Median	Incubation period 95% quantile
		Conditional likelihood		
China - Hefei	34	2.1 (1.2–3.7)	4.3 (2.9–6.0)	12.0 (9.1–17.3)
China - Shaanxi	53	1.7 (1.0–2.8)	4.5 (3.1–6.2)	14.6 (11.5–19.8)
China - Shenzhen	129	2.2 (1.7–3.0)	3.5 (2.8–4.3)	11.2 (9.5–13.6)
China - Xinyang	74	2.3 (1.5–3.5)	6.8 (5.4–8.2)	16.4 (13.8–20.1)
China - Other	42	2.0 (1.1–3.4)	5.1 (3.6–6.7)	12.3 (9.8–16.4)
International	46	2.1 (1.4–3.4)	3.8 (2.5–5.3)	10.9 (8.4–15.1)
All locations	378	2.1 (1.8–2.5)	4.5 (4.0–5.0)	13.4 (12.2–14.8)
		Unconditional likelihood		
China - Hefei	34	1.8 (1.4–2.4)	4.1 (2.8–5.5)	11.9 (9.0–17.2)
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(Point estimates obtained by MLE. Confidence intervals obtained by inverting LRT.)

What's wrong with simple exponential growth?

✗ Density of S in $\mathcal{P}$

It is reasonable to assume incidence of symptom onset is growing exponentially in **Wuhan-exposed population** $\mathcal{P}$:

$$f(s \mid \mathcal{P}) \overset{\propto}{\sim} \exp(rs), \text{ for } s \leq L.$$

But the observations are from the **Wuhan-exported cases** $\mathcal{D}$.

✓ Density of S in $\mathcal{D}$

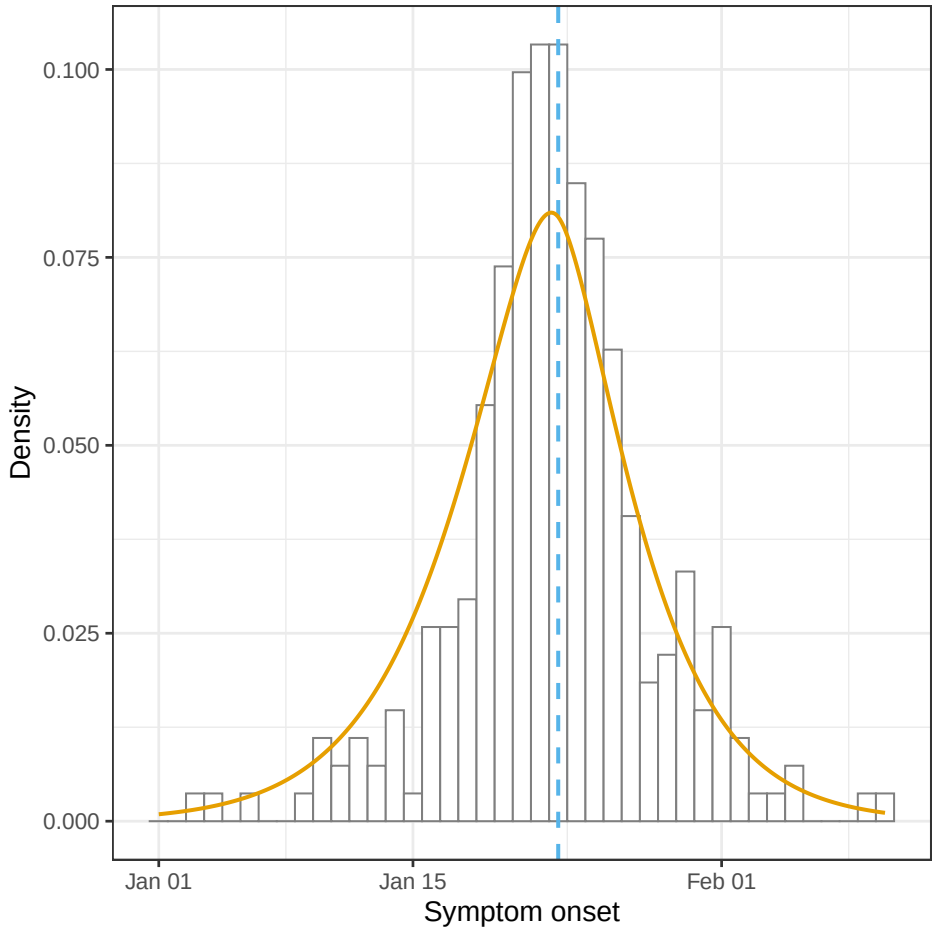
Under Assumptions 1–4 and reasonable approximations,

$$f(t \mid \mathcal{D}) \overset{\propto}{\sim} \exp(rt)\,(L - t)\,1_{\{t \leq L\}},$$

We can further derive the theoretical $f_S(s \mid \mathcal{D})$; in particular,

$$f_S(s \mid \mathcal{D}) \overset{\propto}{\sim} \exp(rs)\left(L + \frac{\alpha}{\beta + r} - s\right), \text{ for } s \leq L.$$

Illustration of the selection bias

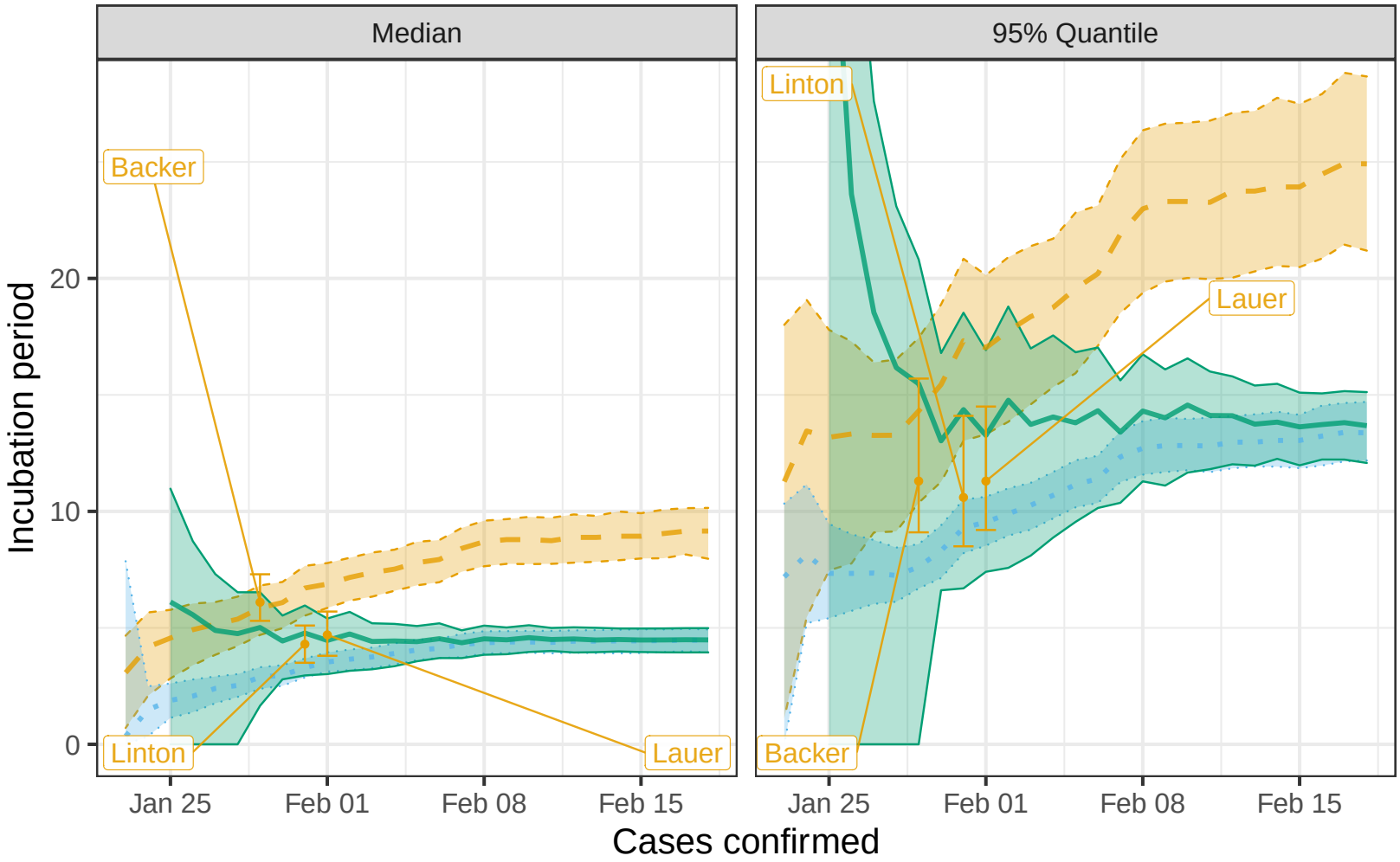


- Histogram: Symptom onsets of Wuhan-exported cases;
- Orange curve: Theoretical fit $f_S(s \mid \mathcal{D})$ using the MLE of (r, α, β).
- Blue dashed line: January 23, 2020 (time L).

Incubation period estimates

An experiment

- For each day between January 23 and February 18, obtain the subset of cases confirmed by that day.
- Fit the parametric BETS model by using one of the following likelihoods:
  1. **Adjusted for nothing:** $L_{\text{cond}}(0, \alpha, \beta)$ (likelihood function in Reich et al. (2009) used in other studies).
  2. **Adjusted for growth:** $L_{\text{cond}}(r, \alpha, \beta)$.
  3. **Adjusted for growth and right-truncation:** $L_{\text{cond,trunc}}(r, \alpha, \beta; M)$ (conditional on $S \leq M$).
- Obtain point estimates by MLE and CIs by nonparametric Bootstrap.
- Compare with previous studies:
  1. Backer, J. A. et al. *Eurosurveillance*, 25(5), 2020. PubMed: 32046819.
  2. Lauer, S. A. et al. *Annals of Internal Medicine*, 2020. PubMed: 32150748.
  3. Linton, N. M. et al. *Journal of Clinical Medicine*, 9(2), 2020. PubMed: 32079150.



Ignore epidemic growth $\Longrightarrow$ Overestimate incubation period.
Ignore right-truncation $\Longrightarrow$ Underestimate incubation period.

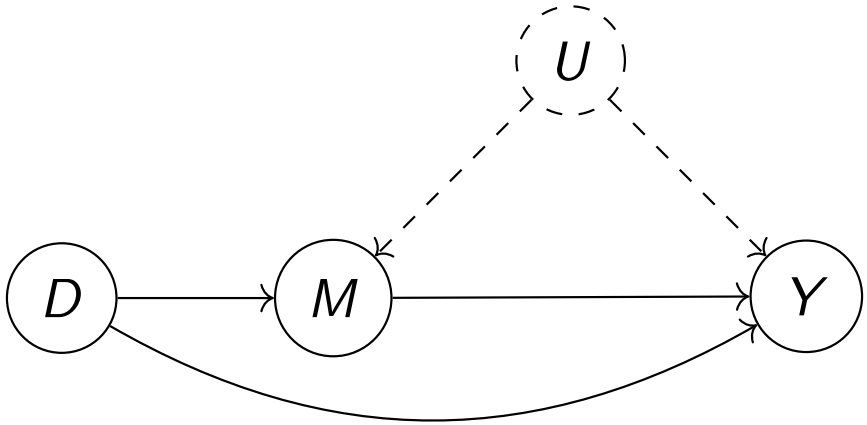
Questions about the first study?

Second example: Racial bias in policing

This work is motivated by

- Knox, D., et al. (2020) Administrative records mask racially biased policing. *American Political Science Review* 114(3).
- Gaebler, J., et al. (2020) A causal framework for observational studies of discrimination. arXiv:2006.12460.

Setup in Knox et al.



- D: binary, 1 means minority.
- M: binary, 1 means police detainment.
- Y: binary, 1 means use of force.

Key challenges

1. Only observe data with $M = 1$ in police admin data.
2. There can be unmeasured M-Y confounders.

$\Longrightarrow$ Collider bias (when conditioning on $M = 1$) in influential studies.

What can be learned from police admin data?

Let $Y(d)$ be the potential outcome for race $D = d$.

Two methods in Knox et al.

1. Partial identification of

$$\text{ATE}_{M=1} = \mathbb{E}[Y(1) - Y(0) \mid M = 1],$$
$$\text{ATT}_{M=1} = \mathbb{E}[Y(1) - Y(0) \mid M = 1, D = 1].$$

2. Identification of $\text{ATE} = \mathbb{E}[Y(1) - Y(0)]$:

Key assumptions in Knox et al.

1. **Mandatory reporting:** $Y(M = 0) = 0$ and all police stops are recorded.
2. **Treatment ignorability:** $D \perp\!\!\!\perp M(d), Y(d, m)$.
3. **Mediator monotonicity:** $M(1) \geq M(0)$. (Not needed for ATE.)

Our results

1. $\text{ATE}_{M=1}$ and $\text{ATT}_{M=1}$ can be difficult to interpret: They may have a different sign even if the natural direct and indirect effects have the same sign.
2. ATE estimation requires estimating the magnitude of $\mathbb{P}(M = 1)$:

$$\begin{aligned}\text{ATE} =& \mathbb{E}[Y \mid D = 1, M = 1]\mathbb{P}(M = 1 \mid D = 1) \\ & - \mathbb{E}[Y \mid D = 0, M = 1]\mathbb{P}(M = 1 \mid D = 0).\end{aligned}$$

This can be circumvented by considering the risk ratio:

$$\text{RR} = \frac{\mathbb{E}[Y(1)]}{\mathbb{E}[Y(0)]} = \underbrace{\frac{\mathbb{E}[Y \mid D = 1, M = 1]}{\mathbb{E}[Y \mid D = 0, M = 1]}}_{\text{naive estimator}} \cdot \underbrace{\left\{\frac{\mathbb{P}(D = 1 \mid M = 1)}{\mathbb{P}(D = 0 \mid M = 1)}\right\} \Big/ \left\{\frac{\mathbb{P}(D = 1)}{\mathbb{P}(D = 0)}\right\}}_{\text{selection bias factor}}.$$

How large is the selection bias?

$$\mathrm{RR} = \frac{\mathbb{E}[Y(1)]}{\mathbb{E}[Y(0)]} = \underbrace{\frac{\mathbb{E}[Y \mid D=1, M=1]}{\mathbb{E}[Y \mid D=0, M=1]}}_{\text{naive estimator}} \cdot \underbrace{\left\{\frac{\mathbb{P}(D=1 \mid M=1)}{\mathbb{P}(D=0 \mid M=1)}\right\} \Big/ \left\{\frac{\mathbb{P}(D=1)}{\mathbb{P}(D=0)}\right\}}_{\text{selection bias factor}}.$$

- Police admin data: NYPD stop-and-frisk.
- We estimated $\mathbb{P}(D=1)$ using two external surveys.

External dataset	Estimated risk ratio	95% Confidence interval
	Naive estimator	
None	1.29	1.28–1.30
	Adjusted for selection bias	
CPS	13.6	12.8–14.3
PPCS	32.3	31.3–33.3
PPCS (Large Metro)	16.7	15.4–18.4

- The selection bias could be $>$ **10-fold!!**

Summary

- **Ridiculously large selection bias** in naive analyses of two topical problems.
- These examples bring discredit on our professions—statistics, epidemiology, social science, data science, ...
- Things are much better in well established research topics, but we cannot be complacent.
- The only solution (I think): **Start from the first principles**.
- Causal inference researchers are uniquely well positioned.
- Act now!!