ANALYSIS OF FUNCTIONS (D) **24 lectures, Lent Term**

Part II Linear Analysis and Part II Probability and Measure are essential.

Lebesgue integration theory
Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of L^p spaces and their completeness. The Lebesgue differentiation theorem. Egorov's theorem, Lusin's theorem. Mollification by convolution, continuity of translation and separability of L^p when $p \neq \infty$. [5]

Banach and Hilbert space analysis
Strong, weak and weak-* topologies; reflexive spaces. Review of the Riesz representation theorem for Hilbert spaces; the Radon–Nikodym theorem; the dual of L^p. Compactness: review of the Ascoli–Arzelà theorem; weak-* compactnesss of the unit ball for separable Banach spaces. The Riesz representation theorem for spaces of continuous functions. The Hahn–Banach theorem and its consequences: separation theorems; Mazur's theorem. [7]

Fourier analysis
Definition of Fourier transform in L^1; the Riemann–Lebesgue lemma. Fourier inversion theorem. Extension to L^2 by density and Plancherel's isometry. Duality between regularity in real variable and decay in Fourier variable. [3]

Generalized derivatives and function spaces
Definition of generalized derivatives and of the basic spaces in the theory of distributions: $\mathcal{D}/\mathcal{D}'$ and $\mathcal{S}/\mathcal{S}'$. The Fourier transform on $\mathcal{S}'$. Periodic distributions; Fourier series; the Poisson summation formula. Definition of the Sobolev spaces H^s in $\mathbb{R}^d$. Sobolev embedding. The Rellich–Kondrashov theorem. The trace theorem. [5]

Applications
Construction and regularity of solutions for elliptic PDEs with constant coefficients on $\mathbb{R}^n$. Construction and regularity of solutions for the Dirichlet problem of Laplace's equation. The spectral theorem for the Laplacian on a bounded domain. *The direct method of the Calculus of Variations.* [4]

Appropriate books

H. Brézis *Functional Analysis, Sobolev Spaces and Partial Differential Equations.* Universitext, Springer 2011

A.N. Kolmogorov, S.V. Fomin *Elements of the Theory of Functions and Functional Analysis.* Dover Books on Mathematics 1999

E.H. Lieb and M. Loss *Analysis.* Second edition, AMS 2001