Percolation and related topics I

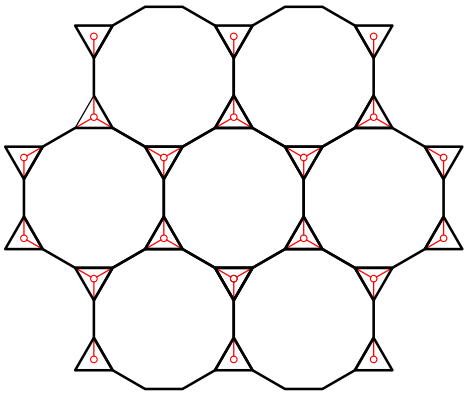
Exercises will be taken mostly from my book *Probability on Graphs*, published by Cambridge University Press in 2010, and available online at

`http://www.statslab.cam.ac.uk/~grg/books/pgs.html`

This set of exercises is extracted mostly from Chapters 3 and 4.

Handout for 19 February. Scripts to be handed in to lecturer (or via pigeon-hole) by **12 noon on 26 February**. Class to be arranged.

1. *Subadditive inequalities.* Exercises 3.1 and 3.2.
2. *Bond and site critical probabilities.* Exercise 3.3. Do you have an idea for the last part?
3. *Covering graphs.* Exercise 3.4.
4. *The problem of runs.* Exercises 3.7 and 3.8. Do not worry overmuch about the fact that you need to work with integer-values. This is 'just noise'.
5. Exercise 3.9. You will need to know that $\theta(\frac{1}{2}) = 0$ for bond percolation on the square lattice.
6. Exercise 3.10, suggested by a French colleague.
7. *Positive-correlation for product measures.* Exercises 4.2 and 4.3.
8. *Equivalence of FKG condition and monotonicity.* Exercise 4.5.
9. The *Archimedean lattice* denoted $(3, 12^2)$ is obtained from the hexagonal lattice by replacing each vertex by a triangle:



Show that the connective constant κ of $(3, 12^2)$ satisfies

$$\kappa^{-2} + \kappa^{-3} = \frac{1}{\sqrt{2+\sqrt{2}}}.$$

Hint: use generating functions.