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Percolation Theory for Mathematicians

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PREFACE

Quite apart from the fact that percolation theory had its origin in an honest applied problem (see Hammersley and Welsh (1980)), it is a source of fascinating problems of the best kind a mathematician can wish for: problems which are easy to state with a minimum of preparation, but whose solutions are (apparently) difficult and require new methods. At the same time many of the problems are of interest to or proposed by statistical physicists and not dreamt up merely to demonstrate ingenuity.

Progress in the field has been slow. Relatively few results have been established rigorously, despite the rapidly growing literature with variations and extensions of the basic model, conjectures, plausibility arguments and results of simulations. It is my aim to treat here some basic results with rigorous proofs. This is in the first place a research monograph, but there are few prerequisites; one term of any standard graduate course in probability should be more than enough. Much of the material is quite recent or new, and many of the proofs are still clumsy. Especially the attempt to give proofs valid for as many graphs as possible led to more complications than expected. I hope that the Applications and Examples provide justification for going to this level of generality. I taught a graduate course on this material at Cornell University in Spring 1981, but the beginning of the monograph was a set of notes for a series of lectures at Kyoto University, Japan, which I visited in summer 1981 on a Fellowship of the Japan Society for the Promotion of Science.

I am indebted to a large number of people for helpful discussions. I especially value various suggestions made by J.T. Cox, R. Durrett, G.R. Grimmett and S. Kotani. I also wish to thank members of the Department of Mathematics at Kyoto University, and in particular my host, Professor S. Watanabe, for their hospitality and for giving me the opportunity to try out a first version of these notes in their seminar. Last but not least, I am grateful to the National Science

Foundation and the Japan Society for the Promotion of Science for their financial support during my work on this monograph.

The reader should be aware of the fact that some standard notation is defined only in the Index of Symbols at the end.

Ithaca, New York, July 1982

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