CORRECTIONS and IMPROVEMENTS
to the book by Y.Suhov and M.Kelbert
'Probability and Statistics by Example. Volume 1'

Introduction

1) Page x, Line 10 from below. Printed: [Wil]. Should be: [Will].

Chapter 1

1) Page 5, Problem 1.4. A shorter solution was proposed by John Haigh. Consider what happens after John and Mary each had n tosses. That is, let x denote the probability that Mary and John have an equal number of heads before her last toss. Then, by the symmetry, the probability that Mary has more heads than John before her last toss equals $(1-x)/2$. Then the probability that Mary finally has more heads than John equals $(1-x)/2 + x/2 = 1/2$.

Another nice solution was proposed by Rob Eastaway. Since Mary has one more toss, she either has in the end more heads than John or more tails. Furthermore, these two possibilities are mutually exclusive. By symmetry, the probability of each of them is 1/2.

2) Page 12, Problem 1.12. The last line of the solution does not make sense. The probability to have two balls of different colours is $\frac{1}{2}$ if you not just put back the removed ball but also start the whole selection procedure all over again, i.e. select an urn at random before choosing a ball. The solution to the last part of Problem 1.12, as it is stated in the book, is as follows:

$$\begin{aligned} P(1st \text{ red}, 2nd \text{ blue}) &= \frac{1}{n(n-1)^2}\left[\frac{n^2(n-1)}{2} - \frac{(n-1)n(n+1)}{3}\right] \\ &= \frac{n-2}{6(n-1)}. \end{aligned}$$

Hence, the answer to Problem 1.12, as it is stated in the book, is $\dfrac{n-2}{3(n-1)}$.

3) Page 26, Line 12 from above: a displayed equation, the last factor. Printed: $(1-p_2)^{n-\sum_i I_{A_i}(\omega)}$. Should be: $(1-p_2)^{n-1-\sum_i I_{A_i}(\omega)}$.

4) Page 29, Line 4 from below: Problem 1.26 Printed: "Given that $\mathbb{P}(A_1)=p_1$ for". Should be: "Given that $\mathbb{P}(A_i)=p_1$ for".

5) Page 36, Line 14 from below. Printed: We see that formula (1.23) is merely a paraphrase of (2.6). Should be: We see that formula (1.23) is merely a paraphrase of (1.7).

6) Page 36, Line 3 from below. Printed: $i+2,3,7,11,12,$. Should be: $i=2,3,7,11,12,$.

7) Page 42, Line 7 from below. Printed: z^{S_n}. Should be: z^{S_N}.

8) Page 43, Line 16 from above. Printed: $N=I_{A_1}+\ldots+I_{A_n}$. Should be: $N=I_{A_1}+\ldots+I_{A_{10}}$.

9) Page 44, Line 11 from below (a displayed equation). Printed: $\left(\sum_{i=1}^{n} x_i^2 - n\overline{X}^2\right)$. Should be: $\left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right)$.

10) Page 44, Line 7 from below (a displayed equation). Printed: $\sum_{i=1}^{n} X_k$.

Should be: $\sum_{i=1}^{n} X_i$.

11) Page 45, Line 2 from above (the displayed equation). in the first sum (after the symbol $\mathbb{E}$) the limits of summation are printed as $j = m + 1$ (the lower limit) and n (the upper limit). Should be: $j = n + 1$ and m: $\mathbb{E} \sum_{j=n+1}^{m}$.

12) Page 50, Line 15 from below: (the displayed equation). Printed: $\mathbb{E}_{H+2,R-1,G-1}$. Should be: $E_{H+2,R-1,G-1}$.

13) P51, Page 51, Line 3 from above. Printed: $m + n = k$ and n. Should be: $m + n$ and $m - n$.

14) Page 51, Lines 2 and 7 from above. Printed: $E(m+1, n-1)$. Should be: $\mathbb{E}(m + 1, n - 1)$.

15) Page 52, Line 1 from below: the end of the remark, before the box. Insert the following new paragraph:

A similar reasoning is helpful to produce a short solution to Problem 1.40 above. Let H_n, R_n and G_n denote the number of Hamlet's, Rosencrantz's and Guildenstern's coins, respectively, after n flips. Consider the variable

$$Y_n = H_n G_n R_n + \frac{3}{4}(H_n + G_n + R_n - 2)n.$$

Here, $\frac{3}{4}(H_n + G_n + R_n - 2)$ is in fact constant in n, equal to 18, as $H_n + G_n + R_n = 26$. It is instructive to note that the conditional expectation

$$\mathbb{E}\left(H_{n+1}G_{n+1}R_{n+1} \middle| H_n G_n R_n\right) = H_n G_n R_n - 18,$$

which is left as an exercise. Then, by using (1.23),

$$\begin{aligned}\mathbb{E}Y_{n+1} &= \mathbb{E}H_{n+1}G_{n+1}R_{n+1} + 18(n+1)\\ &= \mathbb{E}\left[\mathbb{E}\left(H_{n+1}G_{n+1}R_{n+1}\middle|H_nG_nR_n\right)\right] + 18(n+1)\\ &= \mathbb{E}H_nG_nR_n + 18n = \mathbb{E}Y_n = 14\times 6\times 6 = 504.\end{aligned}$$

The fact (again requiring use of the martingale theory) is that the same is true for the random time τ when the game ends (i.e., one of the players is left without coins): $\mathbb{E}Y_\tau = 504$. On the other hand,

$$\mathbb{E}Y_\tau = 18\mathbb{E}\tau,$$

as the product $H_\tau G_\tau R_\tau = 0$. Hence, $\mathbb{E}\tau = \dfrac{504}{18} = 28$. $\square$

16) Page 53, Line 6 from below, a displayed equation. Printed:

$$\text{Cov}\,(X,Y) = \mathbb{E}XY - \mu_X\mu_Y - \mu_X\mu_Y + \mu_X\mu_Y = \mathbb{E}XY - \mu_X\mu_Y.$$

Should be:

$$\text{Cov}\,(X,Y) = \mathbb{E}XY - \mu_X\mu_Y.$$

17) Page 65, Line 8 from above. Printed: Var $X_n = np/q^2$. Should be: Var $X_n = nq/p^2$.

18) Page 85, in the end of the solutiuon to Problem 1.56, add the following text. Observe that using the normal approximation (the De Moivre–Laplace Theorem) yields a better lower bound: $n \geq (1.96\times 10)^2 35/12 \approx 794$.

19) Page 91, Line 1 from below (a displayed equation). Printed: $\ln \sum_{i=1}^{n} x_i$.

Should be: $\ln \left(\dfrac{1}{n}\sum_{i=1}^{n} x_i\right)$.

20) Page 95, Line 6 from above: Problem 1.71. The Hint about the sum of squares is incorrect. The correct version is

$$\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6},$$

as used in the solution.

21) Page 99, Line 3 from above and Page 100, Line 5 from above. Printed: $F_n(sF_{n-m}(0))$. Should be: $F_n(sF_{m-n}(0))$.

22) Page 99, Line 5 from below: Problem 1.72 (ii). The first indicator should be $I(X_m = 0)$.

Chapter 2

1) Page 110–111, Problems 2.2 and 2.3. The printed solution to Problem 2.2 is wrong (we thank J. Haigh for this remark). Also, the text after Problem 2.3 should be changed. The correct solution to Problem 2.2 is as follows.

Solution Let the stick length be ℓ. If x is the place of the first break, then $0 \le x \le \ell$ and x is uniform on $(0, \ell)$. If $x \ge \ell/2$, then the second break point y is uniformly chosen on the interval $(0, x)$. See Figure 2.2. Otherwise y is uniformly chosen on (x, ℓ).

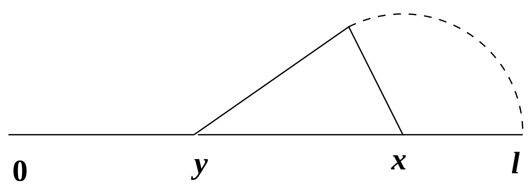

Figure 2.2

Thus

$$\Omega = \{(x, y) : \ 0 \le x, y \le \ell; \ y \le x \ \text{ for } x \ge \ell/2 \text{ and } x \le y \le \ell \text{ for } x \le \ell/2\}\,.$$

See Figure 2.3.

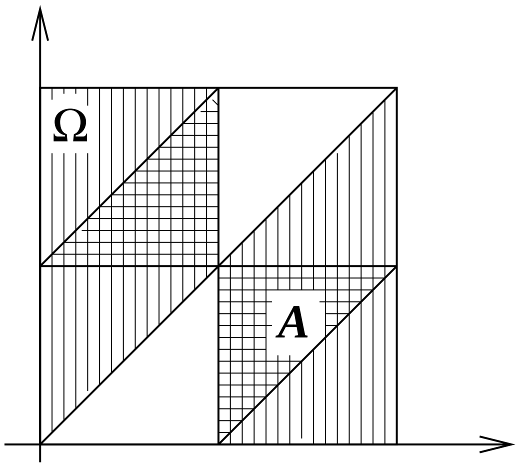

Figure 2.3

To make a triangle (x, y) must lie in A, where

$$A = \left\{ \max\, x, y] > \frac{\ell}{2},\ \min\,[x, y] < \frac{\ell}{2},\ |x - y| < \frac{\ell}{2} \right\}.$$

Owing to the symmetry:

$$\mathbb{P}(\text{triangle}) = \frac{2}{\ell} \int_0^{\ell/2} \frac{1}{l - x} \int_{\ell/2}^{\ell/2+x} \mathrm{d}y \mathrm{d}x = \frac{2}{\ell} \int_0^{\ell/2} \frac{x}{l - x} \mathrm{d}x,$$

which leads to the answer $2 \ln\, 2 - 1 = 0.3862944$.

Problem 2.3 A stick is broken in two places chosen beforehand, completely at random along its length. What is the probability that the three pieces will make a triangle?

Answer: 1/4. This value is less than the previous one. One difference between the two is that the whole sample space Ω in the last problem is the square $(0, \ell) \times (0, \ell)$, i.e., is larger than in the former (as the shorter of the two halves can be broken, which is impossible in the first problem). However, the event of interest is given by the same set A. Intuitively, this should lead to a smaller probability. This is indeed the case, despite the fact that the probability mass in the two problems is spread differently.

3) Page 126, Line 8 from above: displayed equation (2.29). Printed: "$X_1 < y_1, \dots, X_n < y_1$". Should be: $X_1 < y_1, \dots, X_n < y_n$

4) Page 126, Line 4 from below: displayed equation (2.31). Printed: $\frac{\mathrm{d}^2}{\mathrm{d}x\mathrm{d}y}$. Should be: $\frac{\partial^2}{\partial x \partial y}$.

5) Page 152, Line 2 from above: Example 2.15. Include a constant in the last expression, say

$$\psi_X(t) = e^c e^{-t^2/2}.$$

6) Page 155, Line 8 from above, after the answer to Problem 2.20. Insert the following text:

Remark. This problem is of interest in physics, for the design of an atomic clock based on photon emission. There is a complication here, as n itself has to be random (e.g., $\sim$Po(λ)). However, the main idea of the solution still works. See M. Kelbert, I. Sazonov and A.G. Wright. Exact expression for the variance of the photon emission process in scintillation counters. *Nucl. Instruments & Methods in Phys. Res.*, Sect. A, **563** (2006).

7) Page 163, Problem 2.30. Printed: A pharmaceutical company produces a drug based on a chemical Amethanol. The strength of a unit of a drug is taken to be $-\ln(1-x)$, where $0 < x < 1$ is the portion of Amethanol in the unit and $1-x$ that of an added placebo substance. You test a sample of three units taken from a large container filled with the Amethanol powder and the added substance in an unknown proportion. The container is thoroughly shaken up before each sampling.

Find the CDF of the strength of each unit and the CDF of the minimal strength.

Should be: A pharmaceutical company produces drugs based on a chemical Amethanol. The strength of a unit of a drug is taken to be $-\ln(1-x)$, where $0 < x < 1$ is the portion of Amethanol in the unit and $1-x$ that of an added placebo substance. You test a sample of three units taken from three large containers filled with the Amethanol powder and the added substance in unknown proportions. The containers have been thoroughly shaken up before sampling.

Find the joint CDF of the strengths of the units and the CDF of the minimal strength.

8) Page 172, Lines 9, 11 from above. Printed: 0.1. Should be: 0.01.

Chapter 3

1) Page 212, Line 1 from below. Printed: $T(\mathbf{x})) = \left(\sum_i x_i, \sum_j x_i^2\right)$. Should be: $T(\mathbf{X})) = \left(\sum_i X_i, \sum_j X_i^2\right)$.

2) Page 216, Line 16 from below: Printed: S_{XX}/n. Should be: $\mathbb{E}S_{XX}/n$.

3) Page 220, Line 7 from below (a displayed equation). Printed: $-\frac{\mathrm{d}^2}{\mathrm{d}x\mathrm{d}y}$. Should be: $-\frac{\partial^2}{\partial x \partial y}$.

4) Page 222, Line 12 from above. Printed: Var $T \geq$. Should be: Var $\theta^* \geq$.

5) Page 224, Line 8 from above. Printed: $D(x_{i_1}, \theta)D(x_{i_1}, \theta)$. Should be: $D(x_{i_1}, \theta)D(x_{i_2}, \theta)$.

6) Page 230, Line 5 from above: Printed: $\Phi(a) = 1 - 0,005$. Should be: $\Phi(z) = 1 - 0,005$.

7) Page 233, Line 12 from above. Reference [P] should be changed to [PH].

8) Page 240, Line 10 from above, after "unpunished'.)". Insert the following text. It was Robbins who first proved rigorously the bounds $\frac{1}{12n+1} < \theta(n) < \frac{1}{12n}$ for the remainder term in Stirling's formula; see Eqn (1.81). He did it in his article A remark on Stirling's formula. *Amer. Math. Monthly*, **62** (1950), 26-29.

Chapter 4

1) Page 266, Line 8 from below. Printed: $\ln(1+z)$. Should be: $\ln(1+u)$.

2) Page 280, Lines 6,7 from above. Should be: $z = 1.1974$. Since $\Phi(z) = 0.8847$, which is less than 0.975, we still have no reason to reject H_0. $\square$

3) Page 290, Line 4 from above. Printed: To solve this problem, one uses linear regression, as data available are scarce; related measurements on galaxies are long and determined.

Should be: To solve this problem, one uses linear regression, as data available are scarce (related measurements on galaxies are long and often not precise).

Chapter 5

1) Page 299, Lines 10 from above, 1 from below. Change T to M.

2) Page 335, Line 14 from below. Printed: Replacing 0.2275 by -1 we obtain a shorter interval. Should be: Replacing 0.2275 by -1 we obtain a shorter confidence interval, of the form $[-1.6912, -1]$.

Appendices

1) Page 346: the range of the geometric distribution. Should be given as $0, 1, 2, \ldots$.

2) Page 347: the PDF of the Cauchy distribution. Should be given as

$$\frac{\tau}{\pi(\tau^2 + (x - \alpha)^2)}.$$