

TIME SERIES

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Syllabus

Time series analysis refers to problems in which observations are collected at regular time intervals and there are correlations among successive observations. Applications cover virtually all areas of Statistics but some of the most important include economic and financial time series, and many areas of environmental or ecological data.

In this course, I shall cover some of the most important methods for dealing with these problems. In the case of time series, these include the basic definitions of autocorrelations etc., then time-domain model fitting including autoregressive and moving average processes, spectral methods, and some discussion of the effect of time series correlations on other kinds of statistical inference, such as the estimation of means and regression coefficients.

Books

1. P.J. Brockwell and R.A. Davis, *Time Series: Theory and Methods*, Springer Series in Statistics (1986).
2. C. Chatfield, *The Analysis of Time Series: Theory and Practice*, Chapman and Hall (1975). Good general introduction, especially for those completely new to time series.
3. P.J. Diggle, *Time Series: A Biostatistical Introduction*, Oxford University Press (1990).
4. M. Kendall, *Time Series*, Charles Griffin (1976).

Keywords

ACF, 2
Akaike's AIC, 26
AR(p), 2
ARIMA(p, d, q), 6
ARMA(p, q), 5
autocorrelation function, 2
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