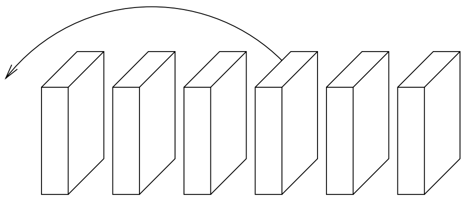
A self-organising system

N books are placed on a shelf. They have unknown probabilities of being selected $(p_1, \dots, p_N)$. If book j is requested at it is the ith book from the left hand end of the shelf then the access cost is i.

The move-to-the-front rule

Suppose that whenever a book is selected it is replaced at the left hand end. This has the advantage that a book for which p_j is large will tend to reside near the left hand end.



For $N = 6$, $p = (.850, .146, .001, .001, .001, .001)$ the average access cost per unit time is 1.271130.

The transposition rule

An alternative is that whenever a book is selected it is replaced just one place closer to the left hand end than where it was extracted. I.e., it is transposed with the book on its left. Again, books that are frequently requested will percolate to the left hand end. The average access cost is now 1.263704.

Theorem. *For all values of $p_1, \dots, p_N$, the transposition rule is less costly than the move-to-front rule.*

Rivest (1976) conjectured that the transposition rule is optimal, for all $p_1, \dots, p_N$. However, this is not true (Anderson, Nash and Weber (1982)).

Consider the following policy. When the book that is ith counting from the left is selected,

$i = 1$: the book remains in position 1.

$i = 2$: the book remains in position 2.

$i = 3$: the book is moved to position 1.

$i = 4$: the book is moved to position 3.

$i = 5$: the book is moved to position 4.

$i = 6$: the book is moved to position 5, the book in position 1 is moved to position 3, the book in position 2 is moved to position 4, the book in position 3 is moved to position 1, and the book in position 4 is moved to position 2.

move-to-front-rule:	1.271130
transposition rule:	1.263704
weird rule above:	1.216094