A repeated game with hidden knowledge

Consider the usual two person, zero-sum game, but where the payoff matrix is equally likely to be either of

$$\begin{pmatrix} 4 & 0 & 2 \\ 4 & 0 & -2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 4 & -2 \\ 0 & 4 & 2 \end{pmatrix}$$

- The row player knows which matrix it is.
- The column player does not.

  The only thing the column player can observe is the choice of row made by the row player each time the game is played.
- The row player's advantage is that he can choose whether to reveal information and how much to reveal.
- The value of the game in the repeated version of this game is the average payment per game to the row player.
- Payments are deferred to the end so that the column player only knows the sequence of plays, not whether he won or lost.

$$\begin{pmatrix} 4 & 0 & 2 \\ 4 & 0 & -2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 4 & -2 \\ 0 & 4 & 2 \end{pmatrix}$$

- If the row player tells the column player which matrix it is, the value of the game is clearly 0.
- If the row player gives away nothing (which he can do by choosing his successive plays independently and equally likely over the two rows, or by choosing one row at random and sticking with it), then the value of the game is also 0 (and the column player should play column 3).
- But suppose the row player chooses one row and then plays that row in every game, but he chooses the row with the 2 in column 3 with probability $3/4$ and the row with the -2 in column 3 with probability $1/4$. This gives the matrix:

$$\begin{pmatrix} 4 & 0 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 4 & 1 \end{pmatrix}$$

The column player is not sure which of these it is, and places equal prior probability on each.

So the column player now thinks the game is

$$\begin{pmatrix} 4 & 0 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 4 & 1 \end{pmatrix}$$

If the column player knows the strategy the row player is following and sees him choosing row 1, then the column player will deduce that it is the first matrix with probability 3/4 (by Bayes's theorem). Hence the column player sees the game as having matrix

$$(3/4)\begin{pmatrix} 4 & 0 & 1 \end{pmatrix} + (1/4)\begin{pmatrix} 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \end{pmatrix}$$

and the value of the game is 1.

Similarly, if the column player sees the row player choosing row 2 then the game appears to have matrix

$$(1/4)\begin{pmatrix} 4 & 0 & 1 \end{pmatrix} + (3/4)\begin{pmatrix} 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \end{pmatrix}$$

and the value of the game is 1.

This is perhaps paradoxical! If the row player reveals everything or nothing the value of the game 0, but if the row player reveals partial information the game is now in his favour.