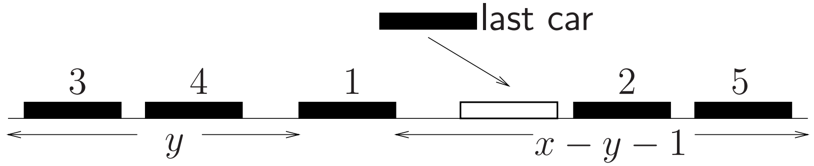
The Palásti conjecture

Cars of unit length are to parked randomly in a street of length x. Cars are parked one at a time, each car being positioned uniformly over all places it would fit, until there is no gap of size 1 remaining.



What is the expected number of cars, $m(x)$, that are packed?

There is an iterative equation that for $x \geq 1$,

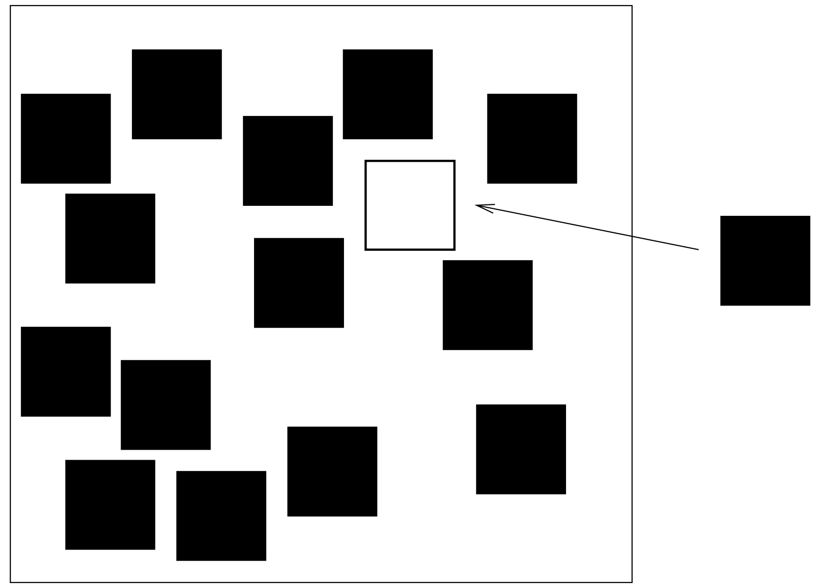
$$
\begin{aligned}
m(x) &= 1 + \frac{1}{x-1} \int_{y=0}^{x-1} [m(y) + m(x-y-1)]dy \\
&= 1 + \frac{2}{x-1} \int_{y=0}^{x-1} m(y)dy
\end{aligned}
$$

where y is the distance that the left side of the first car lies from the left hand of the street and we have the boundary condition $m(x) = 0$, $x < 1$.

The asymptotic packing density is

$$\mu_1 = \lim_{x \to \infty} \frac{m(x)}{x} = 0.74759\ldots$$

What about a similar problem in the square?



As the side of the square $x \to \infty$, simulation experiments give

$$\mu_2 = \lim_{x \to \infty} \frac{m(x)}{x^2} \approx 0.7501^2 \approx \mu_1^2.$$

This is the Palásti conjecture (1960), that in d dimensional space,

$$\mu_d = \lim_{x \to \infty} \frac{m(x)}{x^d} = \mu_1^d.$$

It has never been resolved. However, refined simulations indicate that it justs fails to be true and that the packing density in $\mathbb{R}^2$ is slightly more than the square of the packing density in $\mathbb{R}$.