Instabilty of a Random Access Communication Channel

Suppose the number of messages arriving during each of time periods $n = 1, 2, \ldots$ are i.i.d., with $P(i \text{ arrivals}) = a_i$, and $a_0 + a_1 < 1$.

Each arriving message will transmit at the end of the period in which it arrives. If exactly one message is transmitted then the transmission is successful and the message leaves the system. However, if at any time 2 or more messages simultaneously transmit, then a collision is deemed to occur and these messages remain in the system. Once a message is involved in a collision it will, independently of all else, transmit at the end of each additional period with probability p, $p < 1$. This is the Aloha protocol.

Let "state k" correspond to there being k messages are in the system.

$$I_k = \begin{cases} 1 & \text{if the first time state } k \text{ is departed} \\ & \text{the new state is } k-1, \\ 0 & \text{otherwise.} \end{cases}$$

For example, if the states are 0,1,3,3,2 then $I_3 = 1$, but if they are 0,1,3,3,4, then $I_3 = 0$. Now

$$\begin{aligned} E\left[\sum_{k\geq 1} I_k\right] &= \sum_{k\geq 1} P(I_k = 1) \\ &= \sum_{k\geq 1} \frac{P_{k,k-1}}{1 - P_{kk}} \\ &= \sum_{k\geq 1} \frac{a_0 kp(1-p)^{k-1}}{1 - a_0(1 - kp(1-p)^{k-1}) - a_1(1-p)^k} \\ &< \infty. \end{aligned}$$

This implies $\sum_k I_k < \infty$ with probability 1. Hence, with probability 1, there are only a finite number of states which are departed via a successful transmission. Eventually all we see is collisions!

A more general protocol is that a message that has collided i times will transmit with probability $p_i = 2^{-i}$. (This is called an 'exponential backoff scheme'). But even Aloha with exponential backoff is unstable.

In practice, however, the instability is not significant. The Aloha protocol is common in ethernet and internet communications.