Statistical Theory (M24)
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The first three chapters of this course concern parametric statistical theory. We begin with a discussion of concepts and principles of statistical inference and use these in the second chapter to develop inferential methods based on the likelihood function. These basic first-order techniques rely on large-sample asymptotic approximations and may be inappropriate in some situations. The third chapter, therefore, is devoted to modern, sophisticated refinements of the procedures which yield more reliable inference for small sample sizes.

In the second half of the course, on nonparametric statistical theory, we do not assume that the underlying population is of a pre-specified parametric form, and consider such problems as estimating a distribution function, a quantile, or a density function from a sample. I also intend to cover some extreme value theory, including analogues of the Central Limit Theorem for maxima and minima of a sample. There are many beautiful mathematical results in these areas, as well as several open problems.

Concepts and Principles: Likelihood and related quantities including pseudo-likelihoods, sufficiency, exponential families, transformation models, maximal invariants and equivariance. Discussion of approaches to inference, ancillarity, parameter orthogonality. [5]

First-order theory: Review of basic probability, modes of convergence, Slutsky’s theorem, stochastic order notation, moments and cumulants, the delta method. Review of Wald, score, likelihood ratio statistics and signed root versions, distribution theory in no nuisance parameter case. Discussion of nuisance parameters, profile likelihood. [3]

Higher-order theory: Asymptotic expansions, Edgeworth expansions, saddlepoint approximations, Laplace’s method, the $p^*$-formula, Bartlett correction, modified profile likelihood, Bayesian asymptotics. [6]

Kernel density estimation: Optimality criteria, asymptotic approximations, asymptotically optimal bandwidth, canonical kernels, higher order kernels, bandwidth selection, multivariate density estimation. [6]


Pre-requisite Mathematics

Basic familiarity with statistical inference, including point estimation and hypothesis testing, will be assumed. Part IID Principles of Statistics is recommended as background. Measure theory is certainly not necessary, but would be a small bonus.

Literature

