## $\mathbf{R}$ as a calculator

R can be used as a calculator:
$>\left(9.1^{\wedge} 3\right) * \operatorname{sqrt}(14) * \exp (-5) / \log (4)$
Help on any R function can be found by typing a question mark followed by the function, e.g.

## > ? exp

You will need to use this help facility extensively (and get used to skim-reading to find the relevant bit!). Note that R is case-sensitive.
The <- symbol is the usual assignment operator in R, though = can also be used. For instance, we can assign the value 3 to the variable x , and then perform operations on x . Anything which appears after the hash symbol \# is a comment and need not be typed.

```
> x <- 3
> round(x^2 + log10(x), 3) # try ?round to see what it does
[1] 9.477
> 37 %/% 3 # try ?'%/%'
[1] 12
> 37 %% 3
[1] 1
```


## Creating vectors

The c function (for 'concatenate') combines values into a vector.

```
> x <- c(3, 6, 4, 2)
> x
[1] 3 6 4 2
> length(x)
[1] 4
```

There is no such thing as a scalar in $R$; what one might think of as a scalar is treated as a vector of length 1. Note that R does not distinguish between row and column vectors unlike Matlab.
You can create a vector $y$ with the same entries using y <- scan(). Enter one component per line and leave a blank line after the last.
A sequence of equally spaced numbers can be created using the seq function. The rep function provides different ways of repeating vectors.

## Operations on vectors

Operations on vectors in R are performed component by component. For example

```
> x + x
[1] 6 12 8 4
> x*x
[1] 9 36 16 4
> exp(x)
[1] 20.085537 403.428793 54.598150 7.389056
```

When operations are performed on vectors of different lengths, the shorter vector is cycled until it is the same length as the longer vector.

```
> x <- c(3, 6, 4, 2)
> y<- c(1, 2)
x + y
```

```
[1] 4 8 5 4
> x*y
[1] 3 12 4 4
> x^y
[1] }\begin{array}{l}{3}\\{36}\end{array}
> y <- 1:3 # same as y <- c(1, 2, 3)
> x + y
[1] 4 8 7 3
Warning message:
In x + y : longer object length is not a multiple of shorter object length
```

What are the values of $x+2,3 * x$ and $(2+x)^{\wedge} 3$ ?

## Indexing vectors

```
>x <- c(3, 6, 4, 2)
> x[2] # 2nd component of x
[1] 6
> x[c(1, 3)] # 1st and 3rd components of x
[1] 3 4
> x[-1] # All of x except the 1st component
[1] 6 4 2
> x[-(1:2)] # All of x except the 1st two components
[1] 4 2
> x[1:2] <- c(7.1, 3.4) # We can assign values to components
> x
[1] 7.1 3.4 4.0 2.0
```

Note that after the final command, x has automatically transformed from a vector of integers to a vector of floating point numbers (these are a way of representing real numbers on computers, though of course only to a certain degree of accuracy).
We can also index components of a vector using a TRUE / FALSE (logical) vector.

```
> index_vec <- c(TRUE, TRUE, FALSE, TRUE)
> x[index_vec]
[1] 7.1 3.4 2.0
```

Logical vectors can also be created using the binary operator < which performs componentwise comparisons.
$>\mathrm{x}$
[1] 7.133 .44 .02 .0
$>x>3.6$
[1] TRUE FALSE TRUE FALSE
$>\mathrm{x}[\mathrm{x}>3.6]$
[1] 7.14 .0

## Matrices

We can create a matrix using the matrix function.

```
> A <- matrix(1:8, 2, 4)
> A
\begin{tabular}{lrrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} & {\([, 4]\)} \\
{\([1]\),} & 1 & 3 & 5 & 7 \\
{\([2]\),} & 2 & 4 & 6 & 8
\end{tabular}
```

Can you enter the terms by row instead? Rows and columns of matrices can be extracted in the following way:

```
> A[1, ]
[1] 1 3 5 7
> A[, 3]
[1] 5 6
```

Note that the rows and columns thus formed are now vectors. We can check this using the very helpful str function.

```
> str(A[1, ])
    int [1:4] 1 3 5 7
```

Here we see that $\mathrm{A}[1$,$] is an integer vector of length 4$. To create a 2 by 1 matrix, we use

```
> A[, 2, drop = FALSE]
    [,1]
[1,] 3
[2,] 4
```

Submatrices can be formed by e.g. A [, 1:3]. The diagonal can be extracted using diag. We can perform many standard operations on matrices.

```
> A %*% x # matrix vector multiplication
    [,1]
[1,] 51.3
[2,] 67.8
> A*A # componentwise multiplication
        [,1] [,2] [,3] [,4]
[1,] 1
[2,] 
> t(A)
        [,1] [,2]
[1,] 1 2
[2,] 3
[3,] 5 6
[4,] 7 8
> A %*% t(A) # matrix matrix multiplication
        [,1] [,2]
[1,] 84 100
[2,] }10
```

The solve function can be used to invert matrices and solve linear systems.

```
> solve(A %*% t(A))
        [,1] [,2]
[1,] 1.50 -1.25
[2,] -1.25 1.05
```


## A few important functions

```
> x <- c(3, 6, 4, 2)
>sum(x)
[1] 15
> sum(x > 3) # TRUE is treated as 1 and FALSE, 0
[1] 2
>mean(x)
[1] 3.75
> sort(x)
[1] 2 3 4 6
> sd(x) # standard deviation
```

```
[1] 1.707825
> mean(A) # mean treats A as a vector
[1] 4.5
> colMeans(A)
[1] 1.5 3.5 5.5 7.5
> rowSums(A)
[1] 16 20
```

How is the standard deviation being calculated? The function cbind 'glues' columns of matrices together.

| > cbind(1, A) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | [,2] | [,3] | [,4] | [,5] |
| [1,] | 1 | 1 | 3 | 5 | 7 |
| [2,] | 1 | 2 | 4 | 6 | 8 |

## Generating random numbers

Independent and identically distributed sequences of random numbers are generated with commands like rnorm, runif, rchisq etc. (normal, uniform, $\chi^{2}$ ). The corresponding density, cumulative distribution and quantile functions are, e.g. dnorm, pnorm, qnorm.

```
> x <- rnorm(1000)
> hist(x, freq = FALSE)
> x_vec <- seq(-3, 3, by = 0.1)
> lines(x_vec, dnorm(x_vec), col = "red") # adds lines to an existing plot
```

What does the following code do?

```
> X <- matrix(runif(50*1000, min=-1, max=1), 50, 1000)
> hist(sqrt(50) * colMeans(X) / sd(X), freq = FALSE) # sd treats X as a vector
> lines(x_vec, dnorm(x_vec), col = "red")
```

Experiment with other distributions and other sample sizes.

## Exercises

1. Let $Z \sim N(0,1)$. Estimate $\mathbb{E}(Z \mid\{Z \geq 1\})$ and $\mathbb{E}\left(Z^{6}\right)$.
2. What is the upper $5 \%$ point of a $\chi_{6}^{2}$ distribution?
3. Use R to solve

$$
\begin{aligned}
3 a+4 b-2 c+d & =9 \\
2 a-b+7 c-2 d & =13 \\
6 a+2 b-c+d & =11 \\
a+6 b-2 c+5 d & =27 .
\end{aligned}
$$

