Mixing times of Markov chains (M16)

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An ergodic Markov chain converges to its equilibrium distribution as time goes to infinity. But how long should one wait until the distribution is "close" to the invariant one? How many times should one shuffle a deck of cards until the order becomes uniform? This question lies at the heart of the modern theory of mixing times for Markov chains. The classical theory of Markov chains studied fixed chains and the focus was on large time asymptotics of their distribution. Recently the need to analyse large spaces has increased and the focus has shifted on studying asymptotics of the mixing time as the size of the state space tends to infinity. The area of mixing times is at the interface of mathematics, statistical physics and theoretical computer science.

In this course we will develop the basic theory and some of the main techniques and tools from probability and spectral theory used to estimate mixing times. We will apply them to study the mixing time of several chains of interest. We shall also discuss the *cutoff* phenomenon which was first discovered by Diaconis in the context of card shuffling and it says that a Markov chain converges to equilibrium abruptly. This phenomenon seems to be widespread but it remains a challenging question to obtain criteria for cutoff for general classes of chains.

Pre-requisites

This course assumes almost no background, except for prior exposure to Markov chains at an elementary level.

Literature

1. D. Levin and Y. Peres and E. Wilmer *Markov chains and Mixing Times.* American Mathematical Society, 2008.

2. D. Aldous and J. Fill, *Reversible Markov Chains and Random Walks on Graphs.* book in preparation available online at https://www.stat.berkeley.edu/∼aldous/RWG/book.html

3. R. Montenegro and P. Tetali, *Mathematical aspects of mixing times in Markov chains.* Foundations and Trends in Theoretical Computer Science: Vol. 1: No. 3, pp 237-354, 2006.