Summary of Nilanjana Datta's Research

Since 2002, my research has primarily been in the field of Quantum Information Theory. I have been working on various aspects of this field, e.g., data compression for sources with memory, perfect transfer of quantum states and entanglement over spin networks, proof of the additivity conjectures of the Holevo capacity and the minimum output entropy for various models of quantum channels, complementary channels, entanglement manipulation, and the evaluation of the optimal rates of various quantum information protocols using the quantum information spectrum method.

Before moving into the field of Quantum Information Theory, my research was primarily in the field of Quantum Statistical Mechanics. Part of my PhD thesis was also on the Quantum Hall Effect. Below, I give brief summaries of certain selected topics of my research.

Brief summary of research on memory effects in Quantum Information Theory

Optimal rates of quantum information protocols, such as compression and transmission of information, were initially obtained under the assumption that the information source, or the channel used in the protocol, was memoryless. A memoryless quantum information source is one for which successive signals emitted by it are independent of each other. A quantum channel is said to be memoryless if the noise acting on successive inputs to the channel is uncorrelated.

In real world communication systems, the assumption of sources and channels being memoryless is not always justified. Hence memory effects need to be taken into account. In 2002, with Yuri Suhov of the University of Cambridge, I proved that the optimal data compression limit for a quantum information source with memory, modelled by a system of interacting qubits governed by a suitable Hamiltonian, is given by its von Neumann entropy rate.

Classical and quantum capacities in the presence of memory, have been established only for a sub-class of channels, the so-called forgetful channels. A forgetful channel is one for which the noise acting on successive inputs dies out after a finite number of uses. Capacities of channels with long-term memory, however, had remained an open problem.

Alongwith T.C.Dorlas of the Dublin Institute of Advanced Studies, I studied the transmission of classical information through a class of channels with long term memory. A channel in this class is given by a convex combination of memoryless channels. Hence, the memory of such a channel can be considered to be given by a Markov chain which is aperiodic but not irreducible. We obtained an expression for both the classical capacity and the entanglement-assisted capacity of such a channel.

With Garry Bowen of the University of Cambridge, I employed the quantum information spectrum method, which is a powerful mathematical tool in understanding information theory beyond the memoryless scenario, to compute the classical capacity of any arbitrary quantum channel. The latter work was an important step in the direction of establishing a unifying mathematical framework for integrating many different quantum information protocols together, without making specific assumptions about the nature of the sources or channels.

We have recently used this method further, to obtain optimal rates of entanglement manipulation protocols of bipartite pure states, such as entanglement concentration and entanglement dilution. We have also been able to go beyond pure states and evaluate the entanglement cost of sequences of *arbitrary* quantum states.

Brief summary of research on perfect transfer and mirror inversion of quantum states

Quantum information is encoded in quantum mechanical states of a physical system. Hence, reliable transmission of quantum information from one location to another entails the perfect transfer of quantum mechanical states between these locations. We considered the situation in which the underlying system used for this information transmission is a system of N interacting spins and addressed the problem of arranging the spins in a network in a manner which would allow perfect state transfer over the largest possible distance. The network is described by a graph G, with the vertices representing the locations of the spins and the edges connecting spins which interact with each other. State transfer is achieved by the time evolution of the spin system under a suitable Hamiltonian. This can be equivalently viewed as a continuous time quantum walk on the graph G. We obtained the maximal distance of perfect state transfer and proved that the corresponding quantum walk exhibited an exponential speed-up over its classical counterpart.

We could also go beyond state transfer and achieve the more general operation of mirror inversion of quantum states of a linear register, in each excitation subspace with respect to the centre of the register. The appealing feature of our construction was that it required no dynamical control over individual inter-qubit interactions. This work was done with a host of collaborators from Cambridge, Imperial College London and MIT, namely, C.Albanese, M.Christandl, T.Dorlas, A.Ekert, A.Kay and A.Landahl.

Brief summary of research on the additivity and multiplicativity conjectures

The capacity of a memoryless quantum channel for transmitting classical information, under the constraint that successive inputs to the channel our independent of each other, is given by the so-called *Holevo capacity*. An important open question is whether the capacity of the quantum channel can be increased by lifting this constraint, i.e., by using entangled inputs. This question is related to the conjecture that the Holevo capacity is additive. Along with Alexander Holevo of Moscow and Yuri Suhov of Cambridge, I proved this conjecture for different classes of quantum channels.

We also obtained a sufficient condition for additivity of the minimum output entropy for a pair of given channels and an analytic verification of this condition for specific quantum channels breaking a closely related multiplicativity property. This validated the additivity conjecture for these channels.

Along with Mary Beth Ruskai, I studied the additivity conjecture, as well as the associated multiplicativity conjecture for the maximal output p-norm, for quantum channels which are generalizations of the depolarizing channel.

Brief summary of research on Quantum Statistical Mechanics

As a graduate student, in 1994, along with C.Albanese, I studied a family of Hamiltonians describing a system of spin polarized, itinerant lattice fermions with short range repulsive interactions in the strong coupling limit. We proved rigorously that this family of Hamiltonians exhibit a Mott transition at zero temperature. We developed a convergent cluster expansion algorithm to study the system at low temperatures. By means of this expansion, we could avoid the difficulties related to the *sign problem* of Fermi systems.

Along with R.Fernandez and J.Fröhlich, I developed contour expansion methods to study the effect of thermal fluctuations and quantum perturbations of classical lattice interactions of a system in $d \geq 2$ dimensions. The classical Hamiltonian was required to have a finite number of periodic ground states and yield a regular zero-temperature phase diagram. Our analysis was based on an extension of the well-known Pirogov Sinai theory of classical phase transitions to quantum lattice models. We proved that the only effect of the quantum perturbation was to cause small deformations of the classical phase diagram and does *not* lead to any drastic new effects. Hence it provides a "no–go" theorem which narrows the search for new quantum effects.

Subsequently, we developed a convergent perturbation scheme, with the aim of extending the applicablity of the (above mentioned) low temperature analysis to systems in which quantum perturbations have the more drastic effect of breaking the degeneracies of the classical ground states. A classical Hamiltonian with an infinitely degenerate ground state was considered. It was proved that if the quantum perturbation reduced the degeneracy to a finite number in the nth order of perturbation theory then there existed a unitary operator which block diagonalized the Hamiltonian to order n in the perturbation parameter. The transformed Hamiltonian has a new classical part with finitely many degenerate ground states and a new quantum perturbation which is "small" with respect to the classical part in such a way that the low temperature expansion methods can be used.

We also made a rigorous analysis of the low temperature phases of two variants of the Hubbard model, namely, the Falicov Kimball model and the asymmetric Hubbard model. We established a mathematically controlled perturbation expansion which enabled us to show how – as a consequence of the Pauli principle – *effective exchange interactions* are generated, even though the interactions of the intial Hamiltonian are *spin–independent*.

Along with Bruno Nachtergaele of UC Davis, USA, and Alain Messager of C.N.R.S. Marseilles, I studied the problem of rigidity of interfaces in the Falicov-Kimball model in a cubic lattice, under mixed boundary conditions. We developed a technique to evaluate the low temperature properties of such a system by means of a low temperature contour expansion, while treating the long range tail of the potential by a high temperature expansion. This transformed the problem from that of an interacting contour model to a model of non-interacting "decorated" contours, the decorations resulting from the long range of the potential. The latter model could then be analysed by the method of cluster expansions. We proved that both the 100– and the 111 interfaces of the Falicov Kimball model are rigid at low temperatures. Moreover, the rigidity of the 111 interface results from a phenomenon of "ground state selection" and is a consequence of the Fermi statistics of the electrons in the model.

I further investigated properties of interfaces in quantum lattice models with Tom Kennedy of Tucson, Arizona. We proved that at zero temperature, interfaces in the highly anisotropic XZ and XXZ quantum spin chains are not stable. This is in contrast to the ferromagnetic XXZ chain, for which the existence of localized interface ground states has been proved, for any amount of anisotropy in the Ising-like regime.