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Khalifa University

Michael's major contributions to stochastic optimization and finance

Quantitative Finance Conference 2023
A tribute to Michael
University of Cambridge

Giorgio Consigli

Cambridge, April 13, 2023

Outline

A short summary of Michael's CV

Stochastic optimization, Financial Economics and Computations

Research paths
- Research path 1: from SLP to quantitative ALM
- Research path 2: from Complementarity problems to Derivatives pricing
- Research path 3: Financial economics

SP-based decision support

Concluding remarks

Summary CV

Michael completed his BA in Mathematics at University of Toronto in 1961 before moving to Carnegie Institute of Technology (to become Carnegie Mellon in 1967) where he completed an MSc program in Mathematics and then a PhD (1965) with a dissertation *On Stochastic Programming* under the supervision of Professor Morris H. DeGroot and of Prof. Malenpati M. Rao.

In 1965 Michael moved to Oxford (U.K.) initially as IBM Research fellow and then from 1967 to 1981 as University lecturer in Industrial mathematics affiliated with Balliol College.
Between 1981 and 1987 Michael was Lecturer in Mathematics at Balliol College and took a position as Research Professor of Management and Information sciences at Dalhousie University in Halifax (CA) held until 1990.

In 1990 he moved back to the U.K. as Professor of Mathematics at the University of Essex (1990-1995) and then from 1995 as Professor at the University of Cambridge and director of Research and the PhD program at the Judge Institute of Management Studies. He is currently Professor Emeritus of Mathematics and Statistics with the Statistical Laboratory (StatsLab) and of Finance and Management Science with the JBS of the University.

Worth recalling ...

Among very many, I would like to mention just few relevant initiatives and achievements that have characterized Michael's career, some of which maybe unknown to some of you.

- In 1974 Michael organized and chaired the 1^{st} **International Conference on Stochastic Programming**, whose 16^{th} triannual edition will be held next July at UC Davis (CA).
- In 1996 he founded **the Centre of Financial Research** of the University of Cambridge.
- In 1996 he founded with dr. Elena Medova **Cambridge Systems Associates** Ltd, repository of the *StochasticsSuite*^{TM} (2005) for optimal financial planning.
- In 2000 as we know he co-founded **Quantitative Finance**. The same year he was also appointed Honorary Fellow of the prestigious UK Institute of Actuaries.
- In 2004 for his contributions he was recognized as **Pioneer of Stochastic Programming** by the Math.Prog.Society.
- In 2013 he was appointed Foreign member of the Italian **Accademia Nazionale dei Lincei** established in 1603 by Federico Cesi and whose third affiliate was Galileo Galilei.

A consistent evolution of research contributions

Michael's scientific interests are broad as we know and the fields, quite many, to which he has contributed over the years include optimization, stochastic modeling, mathematical statistics, information theory, system analysis, mathematical and computational finance, management science, numerical analysis, intelligent systems.

On the grounds of my experience, I tried to frame his research interests and contributions along some few scientific seasons (not to be taken too strictly), such as:

- The origins, 60s and 70s, following Michael's doctoral studies, with a predominant interest in linear and stochastic linear programming, accompanied by research on public economics.
- then the 80s characterized by studies of dynamic stochastic systems in discrete time with major results on dynamic stochastic programming and information theory.
- 90s increasingly into computations and continuous time finance.
- Late 90s and 00s with relevant successful industry projects and contributions on institutional and individual asset-liability management, and derivative pricing.
- 00s and 10s more and more industry cooperation and a remarkable set of contributions related to commodity markets.

Stochastic optimization

Consider the following general formulation of a stochastic optimization problem:

$$\begin{aligned} & inf_x \mathbb{E} f(x,\omega) \\ s.t. \quad & x \in \mathcal{P}(\omega), \quad g(x,\omega) \in \mathcal{Q} \subseteq \mathbb{R}^m \quad a.s. \end{aligned} \tag{1}$$

Here $\omega := \{\omega_t\}_{t \in \mathcal{T}} \in (\Omega, \Sigma, \mathbb{P})$ is a vector random data process, $\mathbf{x} \in \mathcal{P}(\omega)$ is a *nonanticipative* Σ-measurable vector decision process, $f : \mathcal{P} \times \Omega \to \mathbb{R}$ is a real-valued objective functional, and $g : \mathcal{P} \times \Omega \to \mathbb{R}^m$ is a vector-valued constraint function with $\mathcal{Q}$ convex. $\mathbb{E}$ denotes expectation with respect to $\mathbb{P}$.

The stochastic program (1) has been studied extensively by Michael, whose contributions have been path-breaking in terms of mathematical characterization since [MAHD(1969), MAHD(1980a)] dynamic formulation [MAHD(1988)], the analysis of the underlying information processes in strict connection with the random process properties ω, and computational solution approaches.

Discrete and continuous stochastic systems

Michael's primary interest in the study of the evolution and control of dynamic stochastic systems exposed him and led to relevant contributions in and across discrete versus continuous mathematics.

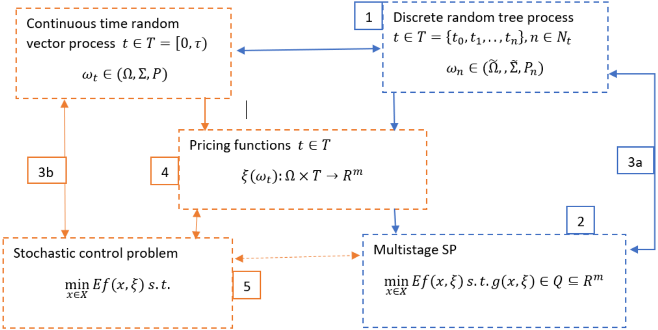


Figure: A glimpse on Michael's stochastic optimization core research domain

Major contributions

Of specific interest in my story-line the following key contributions in:

- **Stochastic Optimization**
  - SLP and the distribution problem [MAHD(1968), MAHD(1969), MAHD(1974)]
  - multistage stochastic programming [MAHD(1980b), MAHD(1988)]
  - SLP-OCP-LCP formulation: [MAHD and Cryer(1980), J.M.Borwein and MAHD(1989)]
  - information theory:
    [MAHD(1980b), Davis et al.(1991)Davis, MAHD and Elliott, MAHD(2006)]
- **Financial Economics**
  - Public economics: [MAHD with Davis and A.Wildavsky(1966)]
  - Asset-Liability management: institutional [Consigli and MAHD(1998), MAHD(2003)] and individual [Medova E.A.(2008)]
  - Option and complex derivatives pricing
    [MAHD and J. Hutton(1997), MAHD and J. Hutton(1999)]
  - Commodity markets [MAHD and S.Hong(2000), MAHD with Medova and Tang(2018)].

Major contributions (ctd)

- **Computations**
  - Cutting plane and decomposition methods, high performance computing [MAHD et al(1983), Consigli and MAHD(1998), MAHD and Thompson(1998), MAHD et al(2018)], and SMPS format [MAHD et al(1987)]
  - sampling - scenario generation in [Chen et al.(1997)Chen, Consigli, Dempster and Hicks-Pedrón] and then in [MAHD(2006), MAHD with Medova and Yong(2011)]
  - decision support systems [MAHD and Ireland(1988), MAHD with Scott and Thompson(2005), MAHD with Medova and Yong(2017)]

From theory to applications and back

I follow next few very comprehensive and original **research paths**:

(1) From the early work on stochastic linear programming [MAHD(1968)] and on the distribution problem [MAHD(1974)] through multistage stochastic programming [MAHD(1980a), MAHD(1988)] to asset-liability management (ALM) applications and ALM post-optimality analysis [MAHD with Medova and Yong(2017)].

(2) From optimization in finite and infinite dimensions [MAHD(1971)] and early work on linear complementarity problems [MAHD and Cryer(1980)] through [J.M.Borwein and MAHD(1989)] to numerical methods for derivatives pricing of complex options [MAHD and Hutton(1997), MAHD and J. Hutton(1999)] and commodity derivatives [MAHD with Medova and Tang(2018), MAHD and K.Tang(2022)].

(3) From mathematical economics for prediction of budget cycles in U.S. [MAHD with Davis and A.Wildavsky(1966), MAHD with Davis and Wildavsky(1974)] through early decision support systems [MAHD and Ireland(1988)], to the development of economic scenario generator for large global institutional investors [MAHD(2003), MAHD et al and Ustinov(2015)] and increasing interest on commodity markets [MAHD with Medova and Tang(2008), MAHD and K.Tang(2015)].

Research path (1): From SLP to quantitative ALM

Consider explicitly ω as a vector stochastic data process $\omega := \{\omega_{\mathbf{t}} : \mathbf{t} \in \mathcal{T}\}$ with $\mathcal{T} := \{1, 2, ..., T\}$ finite and $\mathbf{x} := \{x_t : t \in \mathcal{T}\}$ as a decision process which is measurable with respect to the σ-algebra generated by ω. The decision space is specified in [MAHD(1980a)] as the Banach space $L_p^n := \Pi_{t=1}^T L_p(\Omega, \Sigma, \mathbb{P}; \mathbb{R}^n)$. Let $t = 1, 2, .., T$ and the filtration $\mathbb{F} = \{\Sigma_t\}_{t=1}^T$ generated by the data process describe the current information available to the decision maker. Then a decision process $\{x_t | \pm_t\}$ is *nonanticipative*: this condition can be enforced implicitly or explicitly in the SP formulation.

- A relaxation of the nonanticipativity constraints leads to the formulation of a stochastic program under partial or perfect foresight and the definition of the *expected value of perfect information* (EVPI) [MAHD(1988)].
- An explicit formulation of the constraint leads to the definition of the dual process or marginal EVPI process [MAHD(1982), MAHD(2006)].

The static primal and dual SP problems formulations are already established in Michael's PhD thesis [MAHD(1968)], while [MAHD(1974)] presents a first solution algorithm based on quantile arithmetic to the so-called distribution problem. Michael's key intuition was already in the 70's to see this solution as an upper bound of (1) and the difference between the two as the fundamental information process needed to characterize the true problem solution.

Multistage stochastic programming with recourse

Let $t \in \mathcal{T} = \{1, ..., T\}$, ω a vector tree process, the objective function be separable $f(x, \omega) = \sum_{t \in \mathcal{T}} f_t(x_t, \omega^t)$ and consider a.s. feasibility. The following is a MSP recourse formulation, particularly suitable to address, discrete time strategic Asset-Liability Management (ALM) problems:

$$max_{x_1} f_1(x_1) + \mathbb{E}_{\Sigma_2}\left[max_{x_2}\mathbf{f}_2(x_2, \omega^2) + ... + \mathbb{E}_{\Sigma_{T-1}}\left(max_{x_T}\mathbf{f}_T(x_T, \omega^T)\right)\cdots\right] \quad (2)$$

subject to, a.s.:

$$\begin{aligned}
&A_{11}x_1 && && = b_1\\
&\mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 && && = \mathbf{b}_2\\
&\quad\cdots\cdots && \cdots && \cdots\\
&\mathbf{A}_{T1}\mathbf{x}_1 + \mathbf{A}_{T2}\mathbf{x}_2 && +\mathbf{A}_{T3}\mathbf{x}_3\cdots && +\mathbf{A}_{TT}\mathbf{x}_T = \mathbf{b}_T
\end{aligned}$$

The optimal strategy is a decision process $x(t, \omega^t)$ or optimal contingency plan whose first element is the root node or implementable decision [MAHD(1980b), MAHD(1988)]. Of specific interest the case of a Markovian decision problem with $\mathbf{A}_{t,t-1} = \mathbf{B}_t$ and $\mathbf{A}_{t,t-k} = 0$ for $1 < k < t$.

The value of information

In a multiperiod formulation, let $\phi_t(\omega^t)$ denote the *value function* of a stochastic program under perfect foresight:

$$\phi_t(\omega^t) = \mathbb{E}\{sup_{\mathbf{x_t}} \left[f_t(\omega^t, x^{t-1}, \mathbf{x}_t) + \phi_{t+1}(\omega^{t+1}, x^{t-1}) \right] |\mathcal{F}_t\} \quad (3)$$

subject to the feasibility conditions of the problem. What is to be noted is that under this formulation rather than a stochastic program we are estimating the expected value of a sequence of deterministic programs, one for each random path ω^t. Also denote the SP solutions of the subproblems for each t in (2) by $\pi_t(\omega^t)$, then we recover the definition of the expected value of perfect information (EVPI) process [MAHD(1980a), MAHD(1988), MAHD(2006)] as

$$\eta_t(\omega^t) = \phi_t(\omega^t) - \pi_t(\omega^t) \quad t = 1, ..., T \quad (4)$$

The EVPI was proven by Michael to be a nonnegative supermartingale process with absorbing state at 0.

The value of information

Indeed consider the MSP (2) with a block-diagonal Markovian structure and reformulate the problem in the canonical dynamic programming form $max_{x_t}\{\left[f_t(x_t)+v_{t+1}(x^t,\omega^t)\right]$ s.t. $\mathbf{A}_t x_t + \mathbf{B}_t x_{t-1} = \mathbf{b}_t\}$ a.s.. with $v_{t+1}(x^t,\omega^t) := \mathbb{E}_{\omega_{t+1}|\omega^t}\left[f_{t+1}(x_{t+1},\omega^{t+1}) + ... + \mathbb{E}_{\mathcal{F}_{T-1}}\left(f_T(x_T,\omega^T)\right)\cdots\right]$.
Michael already in [MAHD(1982)] did fully characterize the information process associated with the non anticipativity constraint $x(t,\omega^t) = \mathbb{E}\{x_t|\mathcal{F}_t\}$ for $t = 1, 2, ..., T$ in explicit form. The Lagrangean of (2) is

$$\mathcal{L}(x_t, y_t', \rho_t') = \mathbb{E}\{\left[f_t(\xi^t, x^{t-1}, \mathbf{x}_t) + v_{t+1}(\xi^t, x^t)\right] + y_t'(\mathbf{B}_t x_{t-1} + \mathbf{A}_t \mathbf{x}_t - \mathbf{b}_t) + \rho_t'(I_t - \Pi_t)\mathbf{x}_t)\} \quad (5)$$

From (5) we see that $\rho' := (\rho_1', \rho_2', ..., \rho_T')$ is the dual process associated with the non-anticipativity constraint. For $1 \leq p \leq \infty$ and $1 \leq t < \infty$ let $\mathcal{N} := \{x \in L_p^n : (I - \Pi_t)x_t = 0, t = 1, 2, ..., T\}$ be a closed subspace, then $\rho' \in L_q^n$, $(1 \geq q = \frac{p}{p-1} \leq \infty)$ is a nonanticipative nonnegative marginal EVPI process [MAHD et al(1983)]:

$$\rho_t' \geq \{\rho_s' | \mathcal{F}_t\} \geq 0' \quad a.s. \quad 1 \leq t \leq s \quad s, t \in \mathcal{T} \quad (6)$$

Both information processes (4) and (6) have relevant computational implications.

Decomposition methods

From (2), thanks to the convexity of the value function we can introduce a decomposition scheme, for $t = 1, .., T - 1$:

$$\max_{x_t \in \mathcal{X}_t} \{ f_t(x_t) + \theta_t : \quad B_t x_{t-1} + A_t x_t = b_t, \ \theta_t \leq v_{t+1}(x^t, \omega^t) \} \tag{7}$$

Relying on (7) and previous work on cutting plane methods and numerical convergence [Barzilai and MAHD(1993), MAHD and Merkovsky(1995)], the development of a decomposition method for large scale problems in which the underlying data tree process is progressively refined according to the described information processes becomes natural.

The introduction of an SMPS standard accepted by the stochastic programming comunity was necessary at this point [MAHD and J. Hutton(1997)].

Sampling methods for dynamic stochastic programming

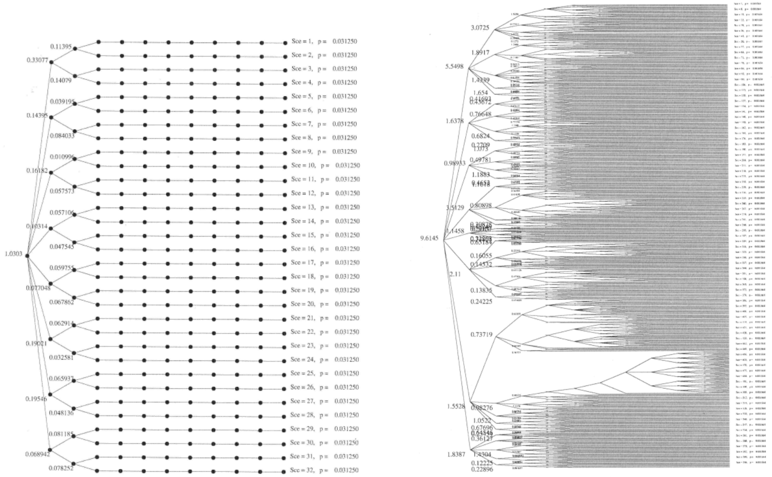


Figure: Tree expansion for FRC 12 stage problem based on marginal EVPI sampling (6 iterations)

Asset-liability management

The *MIDAS* (Manager's Intelligent Debt Advisory System) project [MAHD and Ireland(1988)] addresses an optimal liability management formulated as an MSP problem with recourse. This is the first expert system developed to support a real-world ALM problem.

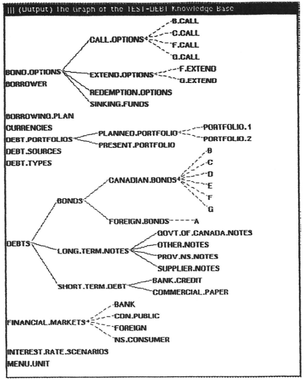


 Figure: Knowledge base for a debt management information system

Institutional ALM

After the MIDAS project, also, partially, thanks to a fruitful cooperation with the UniCredit banking group over the years, a stream of applied research led to a sequence of contributions with relevant financial-economic implications, as discussed below. Particularly relevant from a modeling and methodological perspectives the following works in asset management and pension fund management.

- [Consigli and MAHD(1998)] propose a multistage ALM model, then to become a reference in this domain, with an underlying stochastic model developed by Professor A.D.Wilkie, and a comparison between direct methods and Nested Benders decomposition.
- [MAHD(2003)] provides a comprehensive report of the ALM framework developed for Pioneer Investments: from model specification to estimation and ALM optimization, suitable to different institutional contexts.
- [MAHD et al(2006)] and [MAHD et al(2007b)] develop an extension to model minimum guarantees funds.
- [MAHD et al(2007a)] provides an application to, for the first time to my knowledge, a defined contribution scheme.

Individual ALM

Elena et al, first in [Medova E.A.(2008)] and then jointly with Michael [MAHD and Medova(2011), MAHD(2011)] produced a remarkable sequence of contributions in the specific, highly complex domain of individual or household ALM over a lifetime, resulting into the TradeMark *iALM* decision tool, which roughly 2 decades ago enhanced significantly available operational tools for goal-based individual financial planning. (In this room Woochang Kim is also an expert in this field, which is also of interest to me, and early works can be traced back to the 90's by John M Mulvey in Princeton in whiuch MAHD also collaborated). Specific contributions, primarily of modeling type, but needless to say based on very strong computational power are:

- Individual goal utility function based on reference points,
- Integration of assets, liabilities and events models,
- in [MAHD et al(2016)] a comparison between lifecycle goal achievement and portfolio volatility reduction approaches.

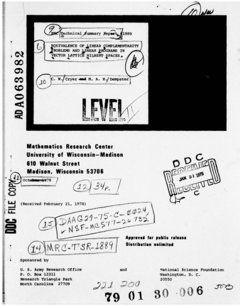
Recent works

To convey the consistency of this research stream over the years, I would like to complete this section by mentioning [MAHD with Medova and Yong(2011), MAHD with Medova and Yong(2017)]. These are two chapters, both to me, of great relevance from a theoretical and a computational perspective, with case studies developed from the Pioneer model.

- The former is titled Comparison of Sampling methods for Dynamic stochastic programming and included in the volume [Bertocchi with Consigli and MAHD(2011)] with an extended and accurate analysis of several scenario reduction schemes developed over the years to support mulitistage stochastic program formulations.
- The latter also appears in a Springer volume I edited in 2017, it is Stabilizing implementable decisions in Dynamic Stochastic programming of primary relevance in operational decision making processes, in which an accurate evaluation of the relationship between tree processes specification and associated MSP optimal ALM policies is conducted with in- and out-of-sample validation.

Research path (2): OCP, VI and derivatives pricing

In 1980 in [MAHD and Cryer(1980)] Michael and Cryer published in the SIAM J.of Control and Opt. an early 1978 research paper, where they established a rigorous set of results on the equivalence between linear order complementarity problems (OCP) and linear programs in vector lattice Hilbert spaces. I show here the back-page with the 1978 stamps of the U.S.Army Research office and the National Science Foundation.


AD A063982

MRC Technical Summary Report #1889

EQUIVALENCE OF LINEAR COMPLEMENTARITY PROBLEMS AND LINEAR PROGRAMS IN VECTOR LATTICE HILBERT SPACES.

C. W. Cryer and M. A. H. Dempster

LEVEL II

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

October 1978

34 p.

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Figure: The 1978 U.S.Army sponsored research on linear complementarity and LP problems

Equivalence between OCP and LP

Michael and Cryer build on early works by R.W. Cottle (1972, 1976) and by O.L.Mangasarian (1976 and 1977) on complementarity problems and by G. Stampacchia's results in the 60's on variational inequalities, to extend the theory to infinite dimensional spaces.

After the work with Cryer, an effective and comprehensive treatment of linear OCP problems is contained in the article published by Michael with Borwein [J.M.Borwein and MAHD(1989)].

The key motivation of this research project in the 70s was related to the possibility to reformulate as linear programs free boundary problems arising in physics, fluid mechanics and other areas, typically formulated as *linear order complementarity problems* (OCP), in which an ordinary or partial differential equation must be solved subject to the solution being non-negative.

Such reformulation brings both numerical advantages as a better understanding of the problem and the interpretability of the solution. This line of research leads in the 90's at the CFR in Cambridge to developing a very efficient LP-based American options valuation scheme [MAHD and Hutton(1997)].

Option pricing by linear programming

Consider the case of a American put option and let $\tau = \inf\{s \in [t, T] : v(S_s, s) = \psi(S_s)\}$ be an optimal stopping time, that is the first time the option value v falls to the payoff value $\psi(S)$, when the conditional expectation of the discounted payoff is maximal, for immediate exercise. Let the log-stock price be ξ, with the put option payoff $\psi(\xi) := (K - e^{\xi})_+$.

Based on the BS PDE operator $\mathcal{L} = \frac{\sigma^2}{2}\frac{\partial^2}{\partial \xi^2} + \frac{1}{2}(r - \frac{\sigma^2}{2})\frac{\partial}{\xi} - r$, the following result is attained.

Theorem (MAHD and James H 1997): *The American put value function is the unique solution of the linear order complementarity problem*

$$
\begin{aligned}
& v(.T) = \psi \\
& v \geq \psi \\
& \mathcal{L}v + \frac{\partial v}{\partial t} \geq 0 \\
& \min\{\mathcal{L}v + \frac{\partial v}{\partial t}, (v - \psi)\} = 0 \quad a.e. \;\; \mathbb{R} \times [0, T].
\end{aligned}
$$

Linear programming

Michael and J. Hutton use another equivalent formulation of the value function problem as a variational inequality (VI) to show the uniqueness of the solution to (OCP) if the differential operator is coercive.

The final stage of the formulation in [MAHD and J. Hutton(1997)] shows that the value function, as the unique solution to (OCP), can be expressed as the unique solution of an abstract linear program:

$$\inf_v < v, c > \quad s.t. \;\; v \in \mathcal{F} \;\; \text{for any} \;\; c > 0 \; a.e. \; on \;\; \mathbb{R} \times [0, T]$$

where $\mathcal{F} := \{v : v(., T) = \psi, v \geq \psi, \mathcal{L}v + \frac{\partial v}{\partial t} \geq 0\}$. The linear operator $\mathcal{L}$ is (type Z) such that $< v, y >= 0 \implies < v, \mathcal{L}y > \leq 0$ for any $v, y \in H$, a Hilbert vector lattice.

A finite LP formulation can then be attained by restricting the domain of the problem and introducing an efficient approximation scheme.

OCP-VI-LP equivalence

The following result in [MAHD and J. Hutton(1999)] establishes Michael's key mathematical and financial contribution.

Let $\mathcal{T}$ be a coercive type Z time homogeneous elliptic differential operator, then there exists a unique solution u to the following equivalent problems:

- (OCP) $v(.T) = \psi,\ v \geq \psi,\ \mathcal{L}v + \frac{\partial v}{\partial t} \geq 0$, and $\min\{\mathcal{L}v + \frac{\partial v}{\partial t}, (v - \psi)\} = 0 \ \ a.e.\ \ \mathbb{R} \times [0, T]$
- (LE) find $v = LE(F)$,
- (LP) $\inf_v < v, c >$ s.t. $v \in F$,

for any $c > 0$ a.e. on $\mathbb{R} \times [0, T]$, where $F := \{v : v(., T) = \psi, v \geq \psi, \mathcal{L}v + \frac{\partial v}{\partial t} \geq 0\}$.

The method is then extended in [Dempster et al(1998)] and [MAHD and D.Richards(2000)] to complex and path dependent options, with the definition of an efficient novel LP solution approach based on a triagonal revised simplex method.

Derivatives pricing by LP

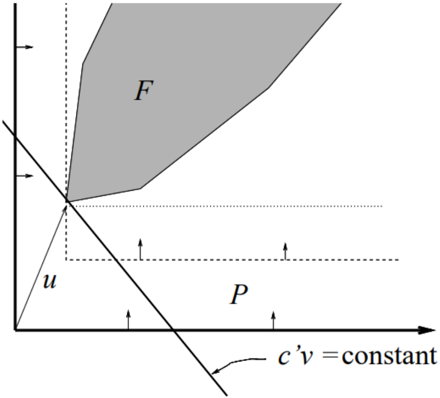

Figure: The least element problem as a linear program

Research path (3): Financial economics

There wouldn't be a research area as applied stochastic optimization to economics and finance without accurate modeling of financial and economic dynamics: here is where Michael's strong statistical and mathematical backgrounds come together with his interest on current financial and economic dynamics as well as emerging decision-making and policy-making problems.

In 1966 he co-authored with Davis and Wildavsky [MAHD with Davis and A.Wildavsky(1966)] the first dynamic stochastic model of the US budgetary process.

The American
Political Science Review

VOL. LX SEPTEMBER, 1966 NO. 3

A THEORY OF THE BUDGETARY PROCESS*

OTTO A. DAVIS, M. A. H. DEMPSTER, AND AARON WILDAVSKY
Carnegie Institute of Technology, Nuffield College, Oxford, and University of California, Berkeley

Figure: The Am.Pol.Sci.Review (3), 1966

Economic and market dynamics

The following concepts (first time ever in this context) are explained in a methodological compendium to [MAHD with Davis and A.Wildavsky(1966)]: linear regression; stochastic difference equation; unstable, evolutionary or explosive process; Markov process; Goodness of Fit; least square estimators and temporally stable process. Follow ups are represented by [MAHD with Davis and Wildavsky(1974), MAHD and Wildavsky(1979), MAHD and Wildavsky(1982)] where in the domain of public economics, I believe Michael in particular (rather than Aaron) clarifies the importance he associates with the treatment of the budgetary process as an evolving stochastic dynamic (indeed *incremental*) process.

Such interest evolves over time but I would overall frame the resulting contributions as being part of Michael's key interest in financial economics. Behind derivatives pricing and ALM applications is the definition of a rich set of underlying stochastic models, whose extent and degree of innovation is difficult to summarize.

We are here at the crossroad between contributions I like to refer to as belonging to the domain of long-term discrete finance and those very much belonging to continuous time finance.

Financial and commodity markets

Without surely pretending to be exhaustive (far from being ...), to convey Michael's originality in this context, I would just focus here next, on few contributions, specifically related to:

- The long standing cooperation with Pioneer Investments the fund manager of the UniCredit banking conglomerate (now, since 2019, part of Amundi) [MAHD(2003)] and [MAHD et al(2006)].
- The yield curve model proposed in [MAHD with E.Medova and M. Villaverde(2010), MAHD et al and Ustinov(2015)]
- The stochastic models and pricing approaches developed for commodity markets [MAHD and K.Tang(2015), MAHD and K.Tang(2022)]

Common to this rather diverse domains is Michael's familiarity with valuation problems specified under a physical or a risk-neutral probability measure and the key role of the market price of risk, whose financial economic rationale is key to the derivation and solution of complex pricing problems.

Pioneer Investments economic model

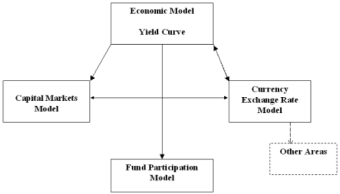

Figure 3.2 Major currency area model structure

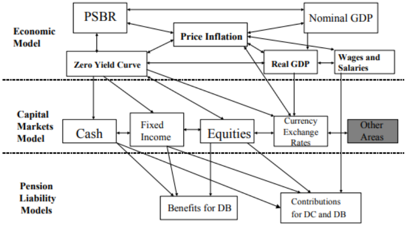

Figure 3.3 Major currency area detailed model structure

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Real world and risk-neutral modeling

Portfolio selection, ALM, fund management all rely on financial and economic scenarios under the empirical measure.
In [MAHD with E.Medova and M. Villaverde(2010)] Michael, Elena Medova and Michael Villaverde propose a 3 factor model of the yield curve: the distinction between the model under the risk-neutral measure for pricing purposes and the equivalent empirical model for forward simulation is clarified upfront. Let the long rate be $\mathbf{X}$, the yield curve slope $\mathbf{Y}$ be the driving processes and the short rate is defined as $\mathbf{R} = \mathbf{X} + \mathbf{Y}$. We have under the two equivalent measures $\mathbb{Q}$ (left) and $\mathbb{P}$ (right):

$$
\begin{aligned}
d\mathbf{X}_t &= (\mu_x - \lambda_x X_t)dt + \sigma_X d\tilde{\mathbf{W}}_t^X & \qquad d\mathbf{X}_t &= (\mu_x - \lambda_x X_t + \gamma_X \sigma_x)dt + \sigma_X d\mathbf{W}_t^X \\
d\mathbf{Y}_t &= (\mu_y - \lambda_y Y_t)dt + \sigma_Y d\tilde{\mathbf{W}}_t^Y & \qquad d\mathbf{Y}_t &= (\mu_y - \lambda_y Y_t + \gamma_Y \sigma_Y)dt + \sigma_Y d\mathbf{W}_t^Y \\
d\mathbf{R}_t &= k(X_t + Y_t - R_t)dt + \sigma_R d\tilde{\mathbf{W}}_t^R & \qquad d\mathbf{R}_t &= [k(X_t + Y_t - R_t) + \gamma_R \sigma_R]\, dt + \sigma_R d\mathbf{W}_t^R
\end{aligned}
$$

It is on these grounds that estimation is possible under $\mathbb{P}$ as simulation and scenario generation for optimization problems, fully consistent with the pricing kernel under the RN measure.

Yield curve modeling

Table 1: Estimated parameters using the Kalman filter

Euro data		*Estimated value*	*SE*
Long-term risk-neutral mean X	μ_X/λ_X	0.199	1.69E-04
Long-term risk-neutral mean Y	μ_Y/λ_Y	−0.134	1.69E-04
Speed of mean reversion X	λ_X	0.161	1.03E-03
Speed of mean reversion Y	λ_Y	1.332	6.87E-03
Speed of mean reversion R	k	0.117	1.64E-03
Volatility X	σ_X	0.030	1.89E-04
Volatility Y	σ_Y	0.186	9.80E-04
Volatility R	σ_R	0.006	2.26E-04
Correlation X and Y	ρ_{XY}	−0.642	6.94E-03
Correlation X and R	ρ_{XR}	0.177	1.82E-02
Correlation Y and R	ρ_{YR}	−0.540	1.81E-02
Market price of risk for X	γ_X	0.556	3.91E-03
Market price of risk for Y	γ_Y	−1.017	5.50E-03
Market price of risk for R	γ_R	0.096	1.65E-02

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Figure: JAM 2010: yield curve modeling

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Commodity markets

A futures market for commodities was established in 1847 in Chicago. I see as very much Michael's interest in this field as resulting from his long term interest in economics as well as political sciences: commodity markets drive International relations and may cause dramatic political events. The commodity futures market is extraordinarily liquid and economically relevant.

Last year the second edition of the CRC FinMath Series volume on *Commodities* [MAHD and K.Tang(2022)] was released. This research project started in the late 90s leading to the following contributions:

- [MAHD et al(2000)] with the proposal of the multistage stochastic programming version of a *Depot and and Refinery Optimization Problem* (DROP): a consortium constrained cost minimization problem resulting into a large scale mixed integer program.
- [MAHD and S.Hong(2000)] with the proposal of a spread options pricing model based on a Fast Fourier Transform (FFT) approach.
- [MAHD with Medova and Tang(2008)], then [MAHD with Medova and Y.Yong(2012)] and [MAHD with Medova and Tang(2018)] focusing on commodity futures.

Pricing commodity futures

In 2012 Michael, Elena and Ke Tang publish a three factor model for commodity futures [MAHD with Medova and Y.Yong(2012)]. Few years after [MAHD with Medova and Tang(2018)] propose an *importance state space form* (ISSF) and develop a new econometric approach to isolate jumps in a latent factor model. The methodology is applied to oil and copper futures prices with data from 2000 to 2012 and MAHD et al isolate jumps generated by long and short term factors **p** and **x**, respectively. Again, the data generating process is specified both in the RN measure for pricing purposes and in the physical measure for estimation.

$$
\begin{aligned}
ln(S_t) &= x_t + p_t & ln(S_t) &= x_t + p_t \\
dx_t &= (k_x x_t + \phi)dt + \sigma_x dW_x + J_x dN_x & dx_t &= -(c + k_x x_t)dt + \sigma_x dW_x + J_x dN_x \\
dp_t &= (\eta - \theta_p)dt + \sigma_p dW_p + J_p dN_p & dp_t &= \eta dt + \sigma_p dW_p + J_p dN_p
\end{aligned} \tag{8}
$$

Futures prices are derived through an affine model and they find that many large long term jumps (permanent jumps) are associated with the long end movements or parallel shifts of the futures term structure, while short term (temporary) jumps are usually accompanied with twists of the futures term structure. Both x and p factor jumps for oil futures are caused by world political and macroeconomic events.

Decision support: *STOCHASTICS*^{TM}

In the area of Operations Research and Management Science decision tools, the development and realization of *STOCHASTICS* [MAHD with Scott and Thompson(2005)], a trademark of Cambridge Systems Associates Ltd (C.S.A.), should in my view also be regarded as a relevant contribution, linking rigorous modeling and mathematics to computationally efficient decision making.

Following the discussion so far, the successful interaction of Michael with the financial industry throughout the years was supported by the modules integrated in the software. Specifically in the domains of

1. long term asset allocation, asset liability management
2. derivative structured product valuation, derivative portfolio pricing and hedging strategies
3. risk management and capital allocation
4. real options.

Given all the above, not surprisingly the macro interfaces of the system are between STOCH-Sim,-Gen,-Opt and STOCH-View, a fair paradigm of optimal financial decision making in finance!

The books

The extent and depth of Michael's research domain and contributions are well summarized by (some of!) his book editorial projects:

- *Mathematics of Financial Derivatives* in cooperation with S.Pliska [MAHD and Pliska(1997)],
- *Risk management: value at risk and beyond* [MAHD(2002)],
- *Quantitative Fund Management* in cooperation with G.Mitra and G.Pflug [MAHD with Mitra and Plug(2008)]
- *Handbook on Stochastic Optimization methods in Finance and Energy* with myself and M.I.Bertocchi [Bertocchi with Consigli and MAHD(2011)],
- *Commodities* in cooperation with Elena and Ke Tang, 2 editions [MAHD and K.Tang(2015), MAHD and K.Tang(2022)].
- *High performance computing in Finance: problems, methods and solutions* in cooperation with Kenniainen, Keane and Vynckier [MAHD et al(2018)].

Left out

I would like to mention few research areas that are relevant but somehow couldn't find space here above. Together they define Michael's originality and ahead-of-time scientific attitude.

- Control problems and the value of information in continuous time models [Davis et al.(1991)Davis, MAHD and Elliott, MAHD(1991), MAHD and Ye(1996)].
- In 2001 Michael published an application of computational learning techniques in the FX market [MAHD et al(2001)]. The year after, he proposed an evolutionary reinforcement learning algorithm [MAHD and Romahi(2002)] in the same context.
- Volatility and its interaction with portfolio management [MAHD with Evstigneev and Schenk-Hoppé(2007)], (2008) and (2011) with the study of fixed-mix as well as Kelly's portfolio strategies in a dynamic context.
- Credit risk and structured products focusing on the adoption of copula functions and a pricing method based on relative entropy [MAHD and Yang(2007)].

An extraordinary forward looking scientist and mentor

In very many contexts, primarily thanks to his solid mathematical backgrounds and persistent interest in current financial and political subjects, Michael has been ahead of the times. I would like to conclude with my taking on of all this:

- Already in the 70s in Oxford he was working on multistage stochastic programs while most of the SP community was focusing on 2-stage problems.
- From which a very early characterization of stochastic dynamic problems in primal and dual form with far reaching methodological implications.
- The characterization of information processes and their integration in computational developments was so far ahead in the area of operations research and management science.
- An open minded and unique ability to understand industry problems and advance the theory while providing solutions, also came very early.
- A unique sequence of results in which stochastic programming, mathematical finance, decision theory and management science were jointly benefited at once!

Conclusion

As supervisor and mentor, Michael's primary quality has been somehow to identify in a very gifted way every collaborator's key qualities as well as weaknesses and build on that by emphasizing the former and helping to reduce the latter!

I hope I conveyed to you all the extent and depth of Michael's contributions over the years and his outstanding achievements as a mathematician and specifically in an applied context as a financial economist.

What to say: **thank you very much indeed Michael for your scientific legacy and (I would add) for being such a kind and generous colleague, supervisor and teacher !**

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