Global Asset Liability Management

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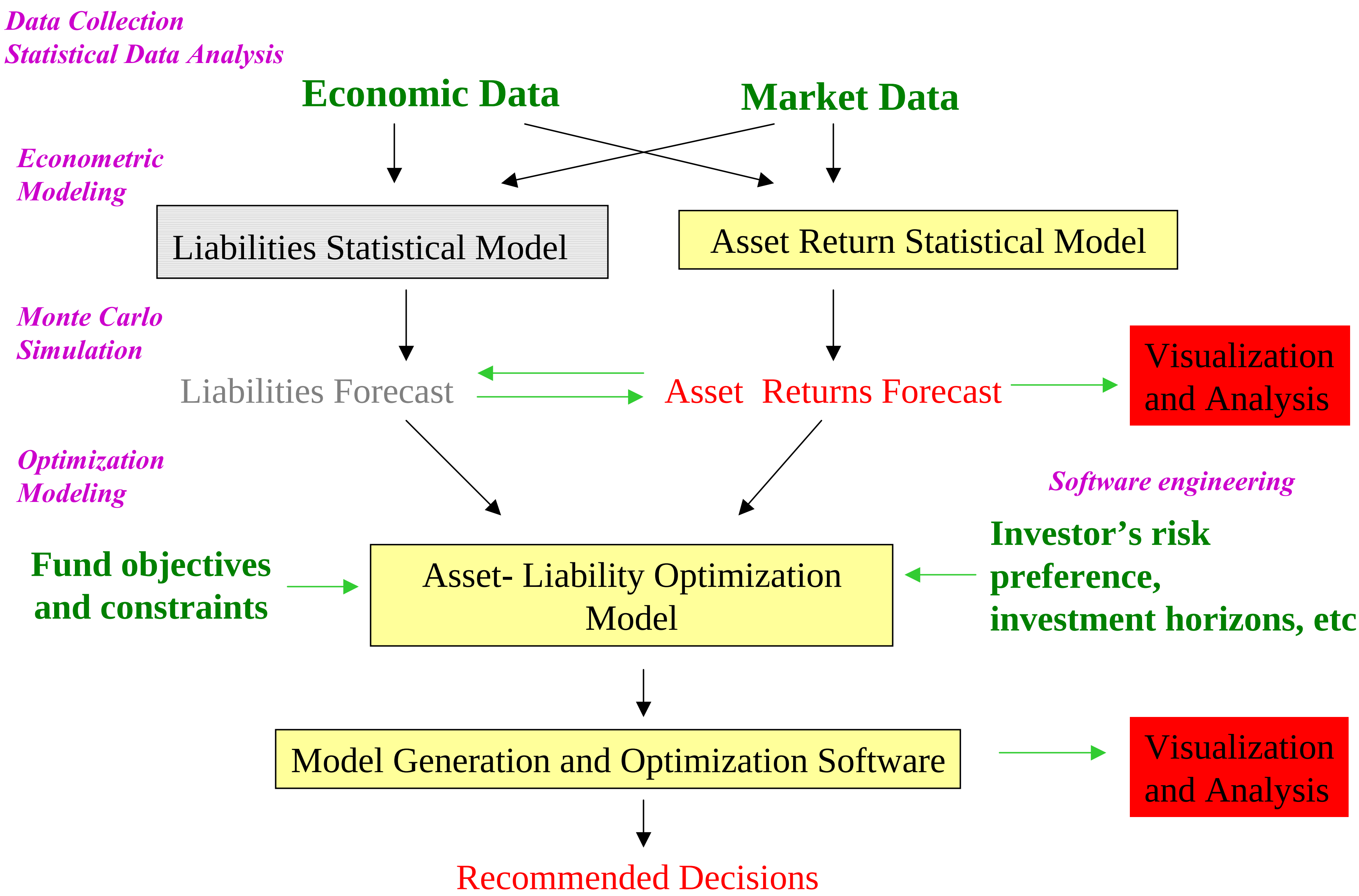
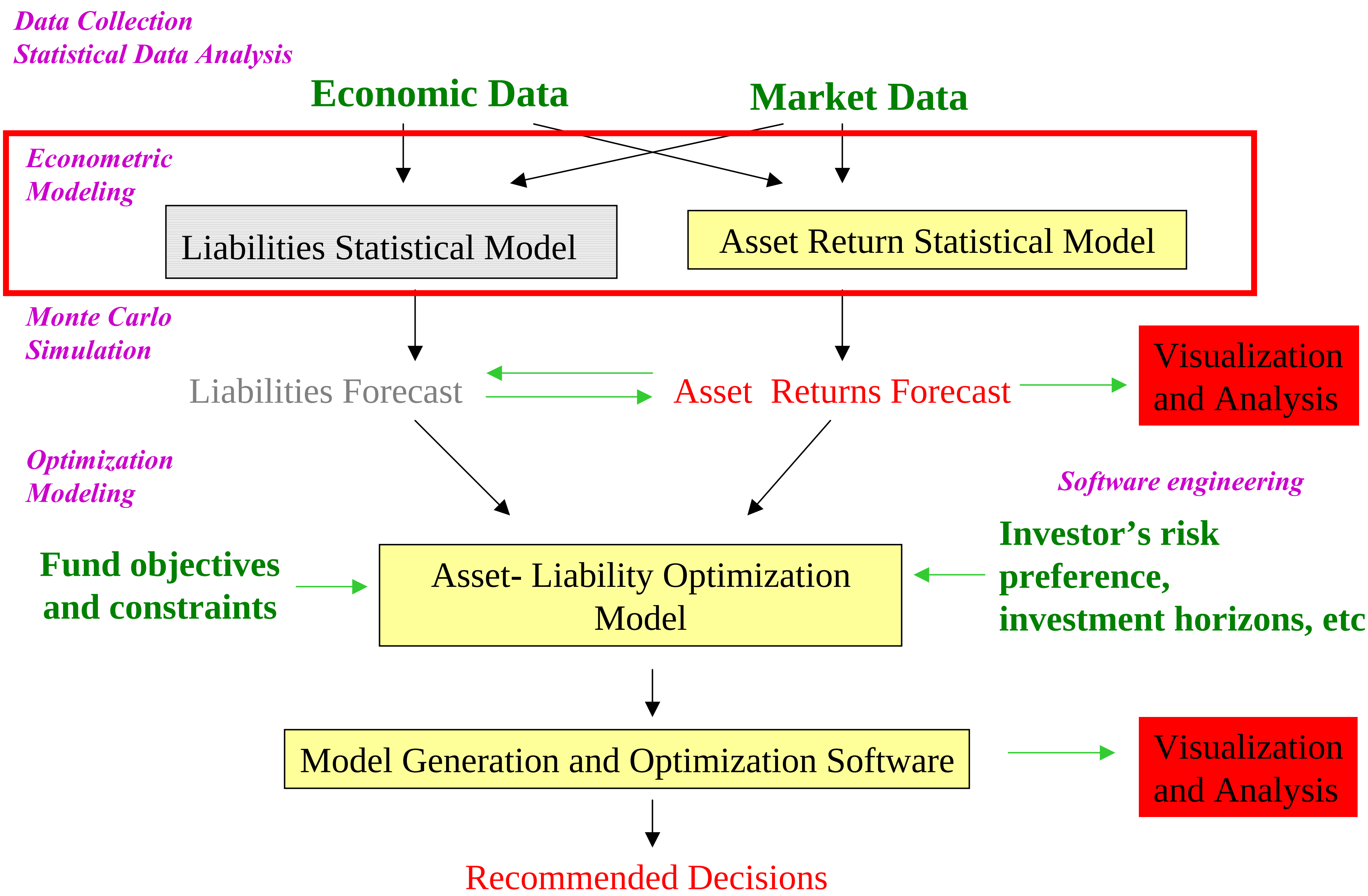
Outline

1. Overview
2. Asset Return Models and Scenario Simulation
3. CALM Model, Problem Generation and Solution Methods
4. Numerical Results
5. Conclusions

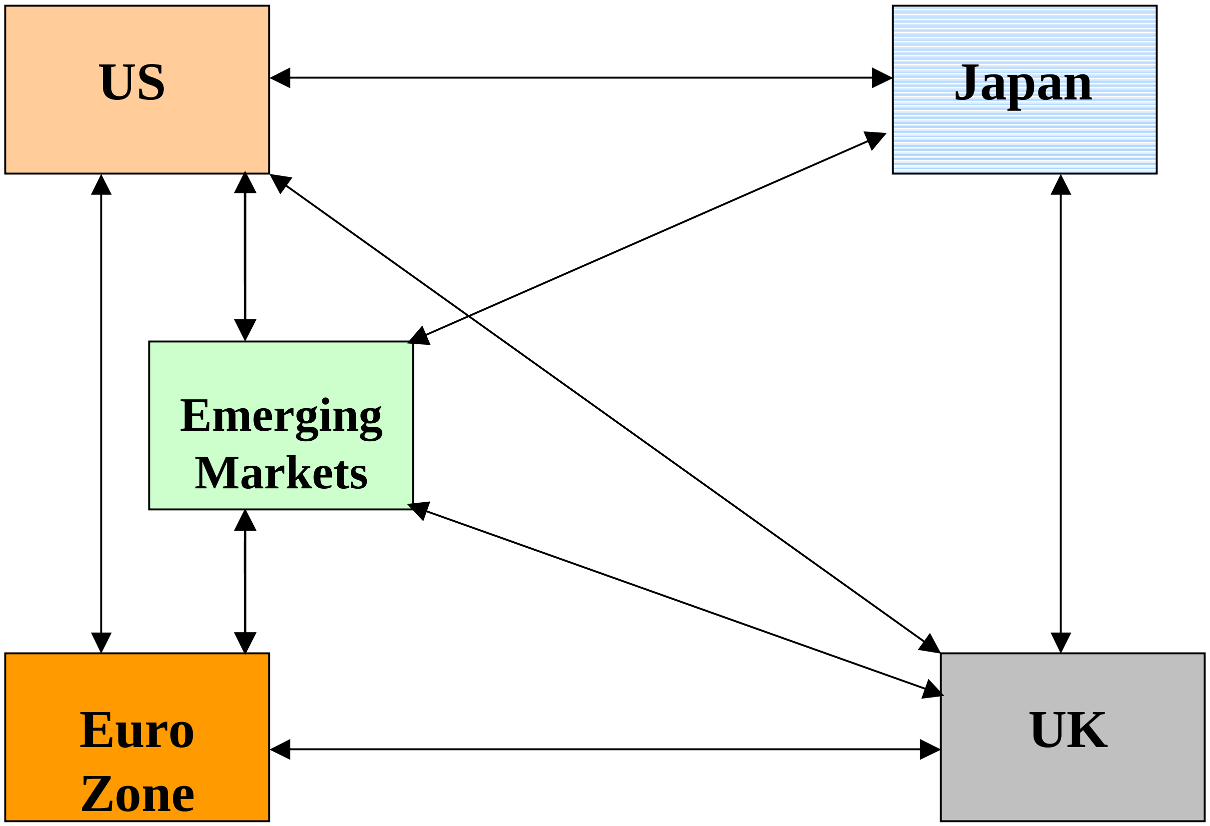
System Overview



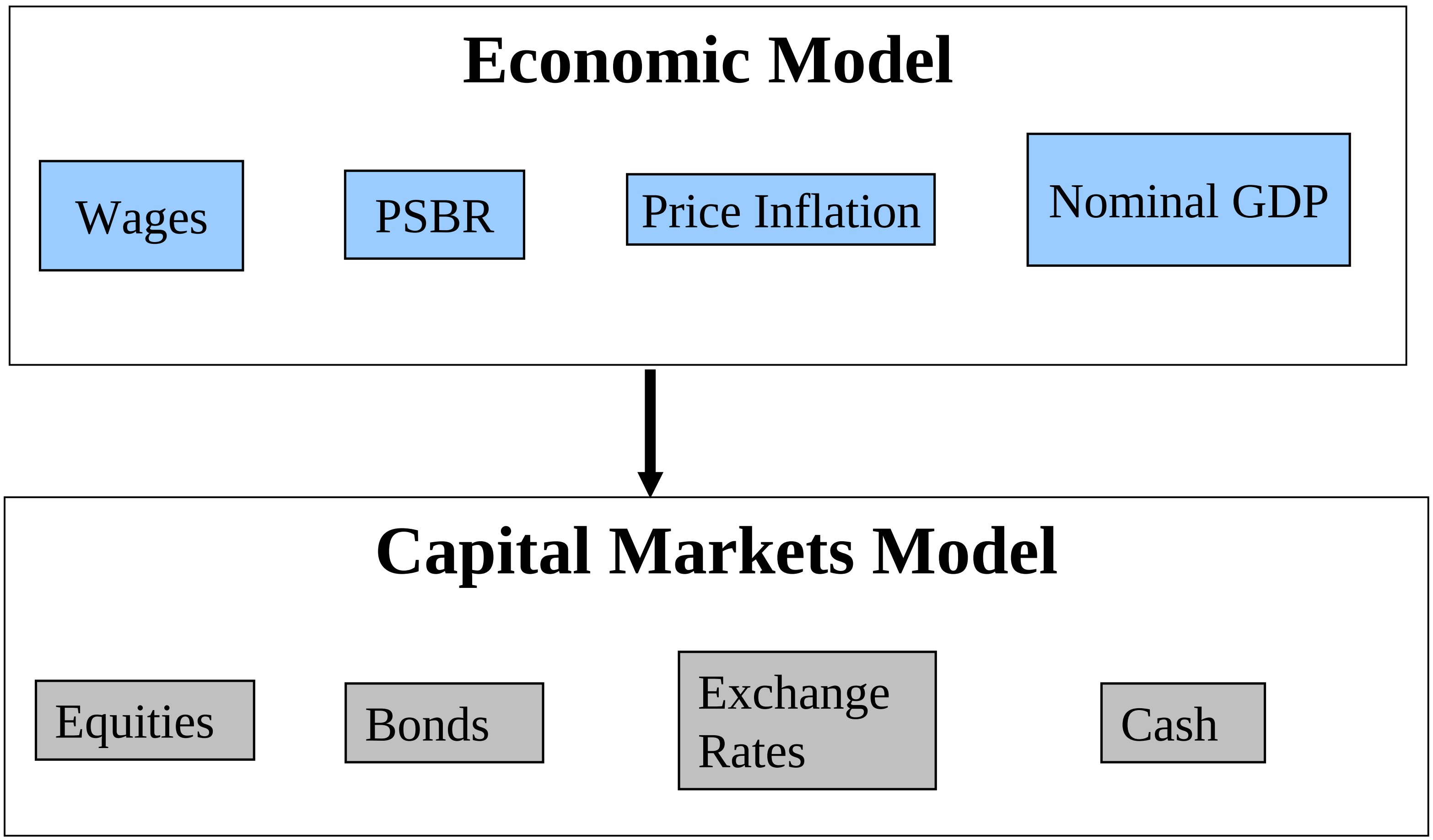
System Overview



Asset Return Statistical Model Global Structure



Asset Return Statistical Model Area Structure (US, UK, EU, Japan)



Asset Return Statistical Model Area Structure (Emerging Markets)

Capital Markets Model

Equities

Bonds

Capital Markets Model (US, UK, EU, Japan)

- Stock (***S***)
- Short term rate (***r***)
- Long term rate (***l***)
- Exchange rate (***x***)

$$\frac{dS_t}{S_t} = \mu_{St}dt + \sigma_S dZ_{St}$$

$$\frac{dr_t}{r_t} = \mu_{rt}dt + \sigma_r dZ_{rt}$$

$$\frac{dl_t}{l_t} = \mu_{lt}dt + \sigma_l dZ_{lt}$$

$$\frac{dx_t}{x_t} = \mu_{xt}dt + \sigma_x dZ_{xt}$$

The *volatility* σ_i parameters are constant. The *drift* μ_{it} parameters can be defined as functions of the state variables

$$\mu_{it}(S_t, S_{t-1}, r_t, r_{t-1}, l_t, l_{t-1}, x_t, x_{t-1})$$

where $d\mathbf{Z}_{it}$ (i = ***S***, ***r***, ***l***, ***x***) are increments of correlated *Wiener processes*

Capital Markets Model (Emerging Markets)

- ARMA model with GARCH error structure

$$\boldsymbol{y}_t = \alpha_0 + \alpha_1 y_{t\text{-}1} - \beta_1 u_{t\text{-}1} + \boldsymbol{u}_t$$

$$H_t = \gamma + \mathrm{p}H_{t\text{-}1} - \mathrm{q}u_{t-1}^2$$

$$\boldsymbol{u_t} := \sqrt{H_t}\,\boldsymbol{\varepsilon}_t$$

- y_t denotes the return on the stock or bond and ε_t are correlated standard normal random variables

Asset Return Statistical Model

- State-dependent systems of equations

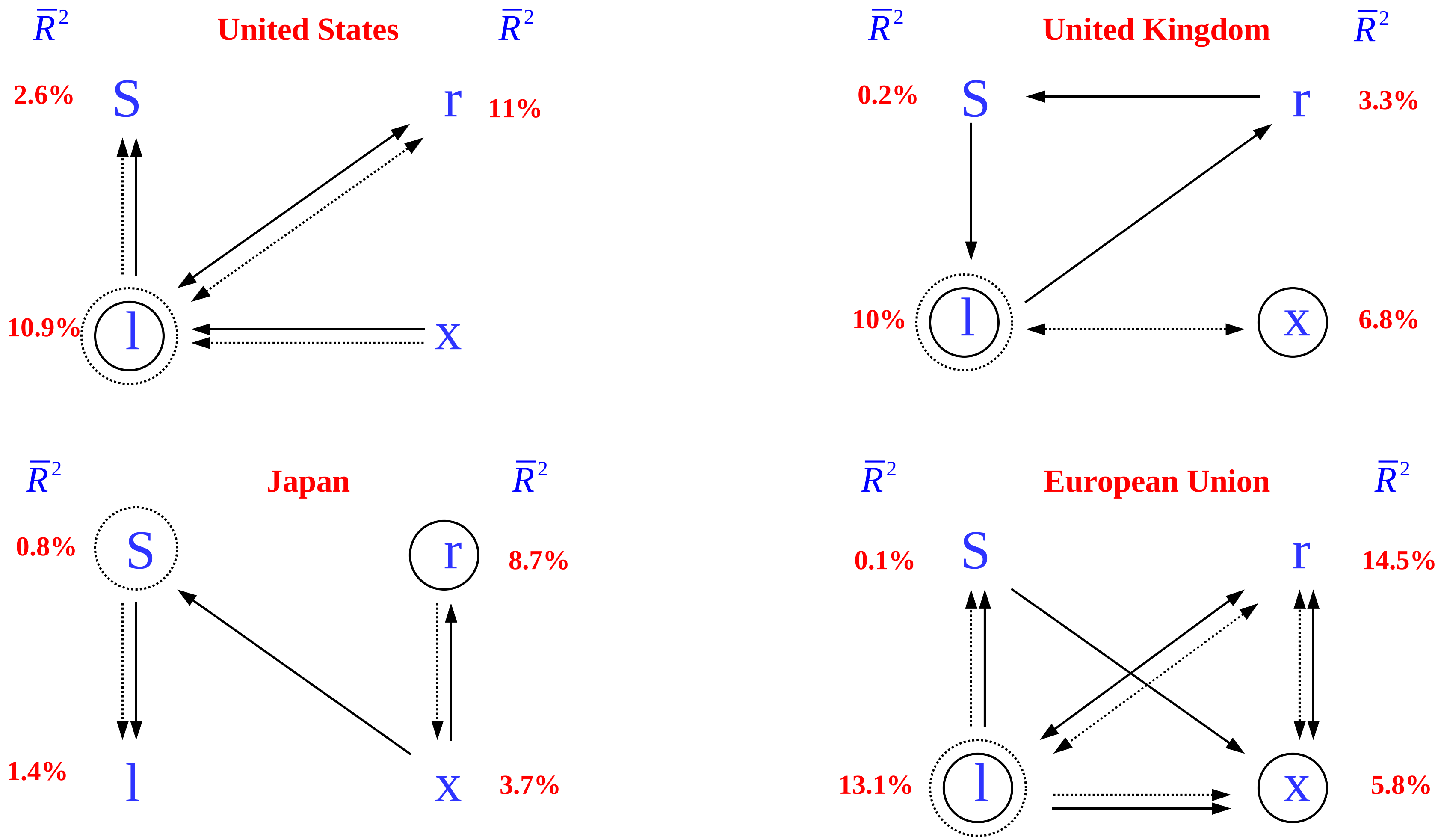
$$d\boldsymbol{x} = diag\left(\boldsymbol{x}\right)[\mu\left(\varphi\left(\boldsymbol{x}\right)\right) + \sqrt{\Sigma}d\boldsymbol{Z}]$$

where φ is a 2nd order non-linear AR filter and $\sqrt{\Sigma}$ is the Cholesky factor containing the correlations of the disturbance terms

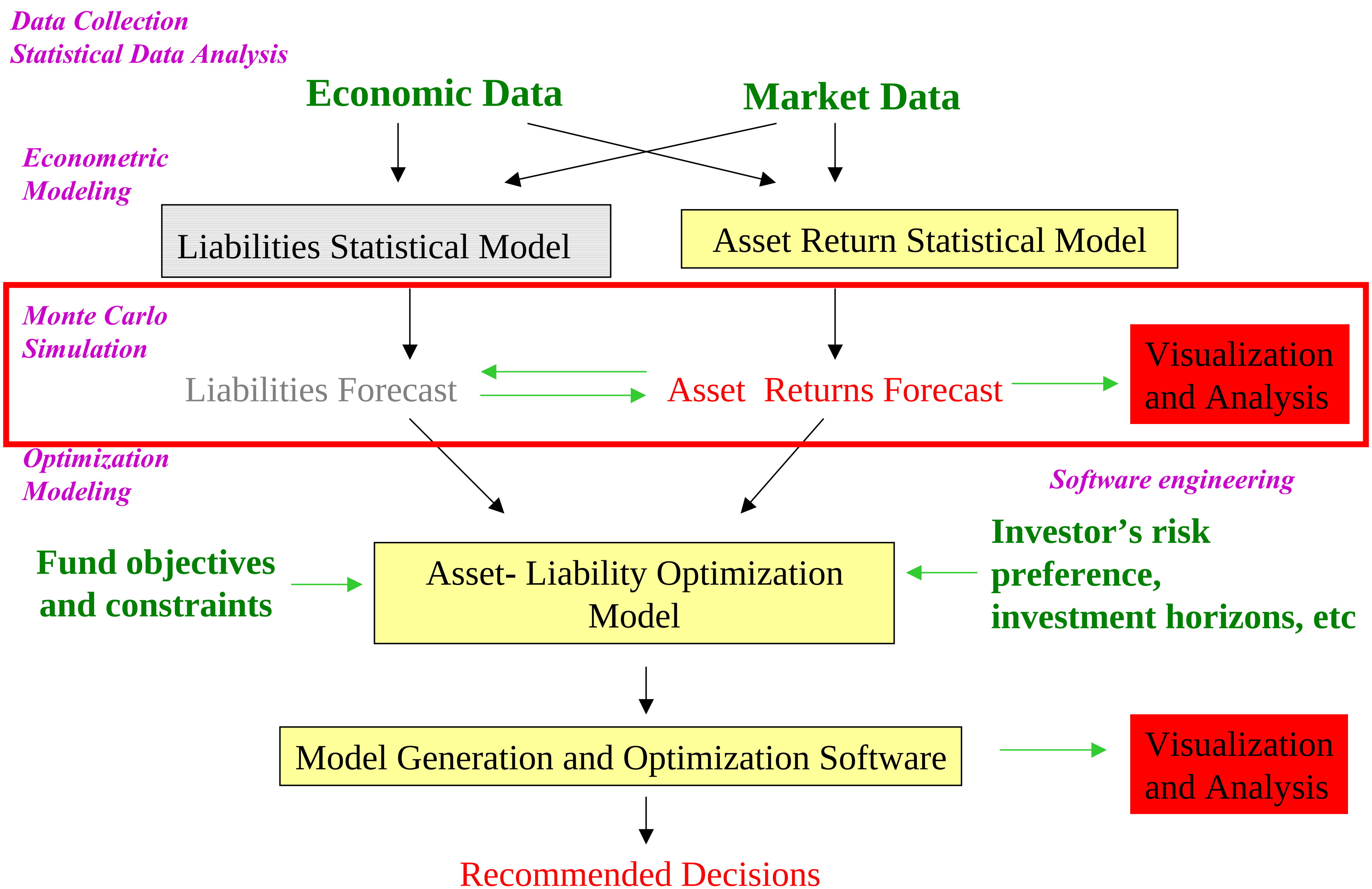
- 21 equations from 4 currency areas and emerging markets

- Full system estimation using Seemingly Unrelated Regression (SUR) and GARCH fitting methods

Influence diagrams: Capital Markets



System Overview



Asset Return Scenario Generation/Simulation

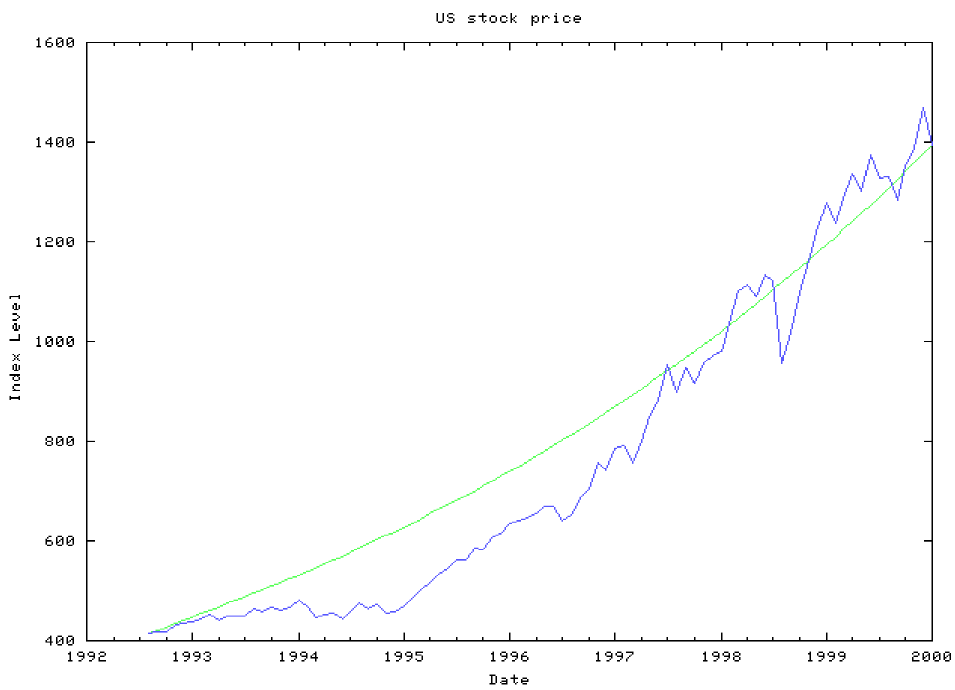
- A C/C++ scenario generator BMSIM has been developed which generates individual paths of the asset return model

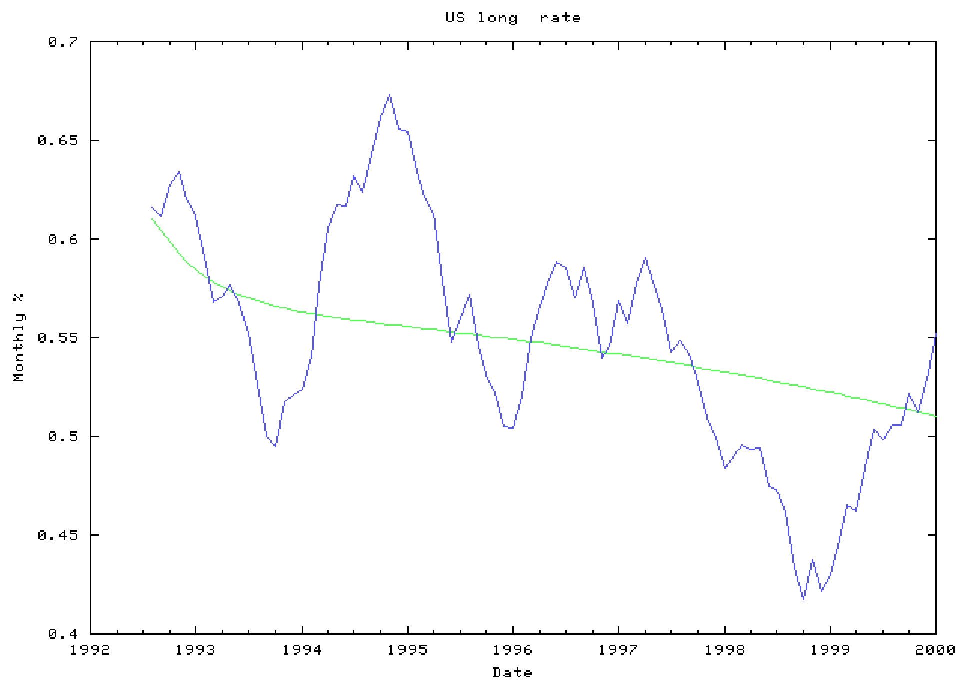
- Parameters input from estimated statistical models

- Paths generated using Monte-Carlo simulation

In-Sample Drift with History

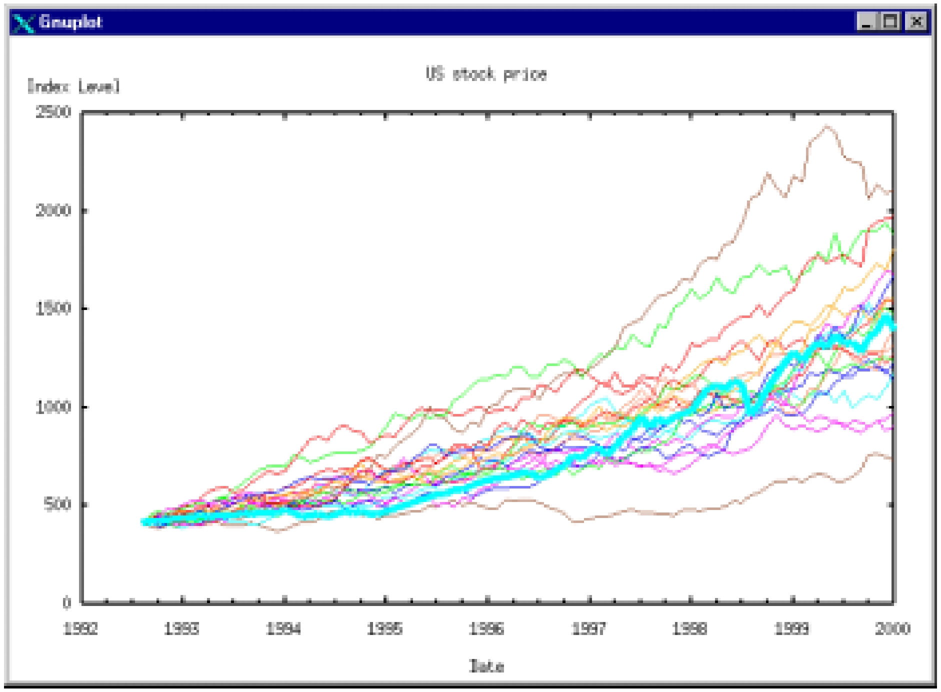
United States: August 1992 - February 2000

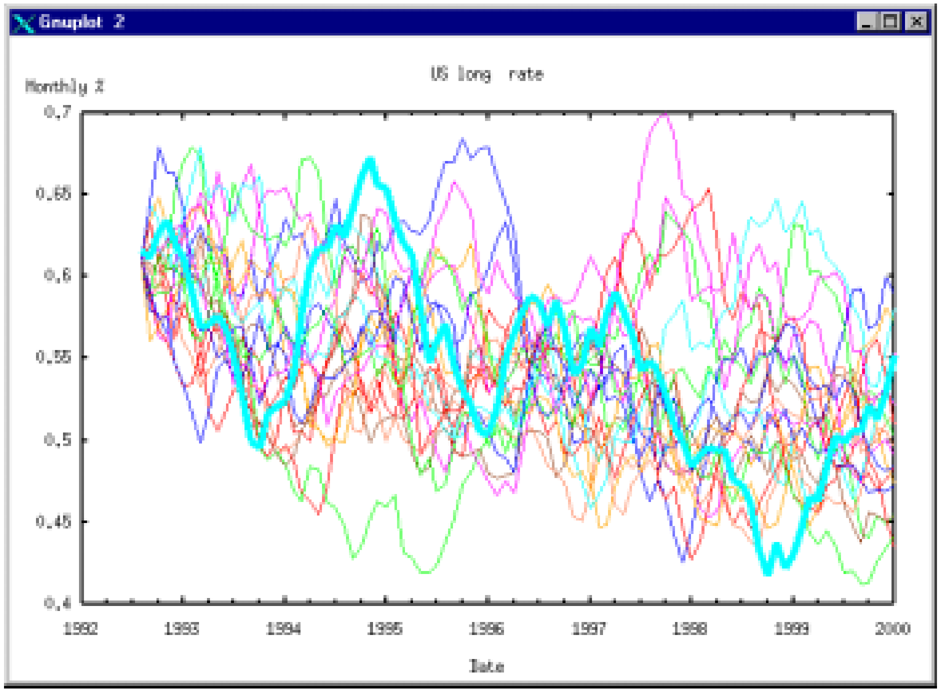




In-Sample Scenarios with History

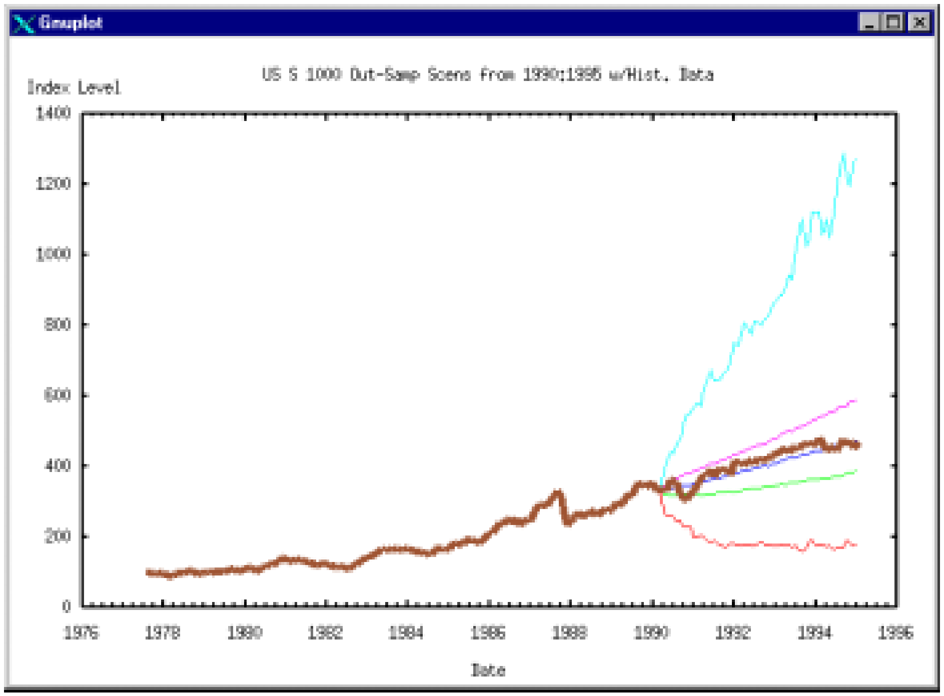
United States: August 1992 - February 2000

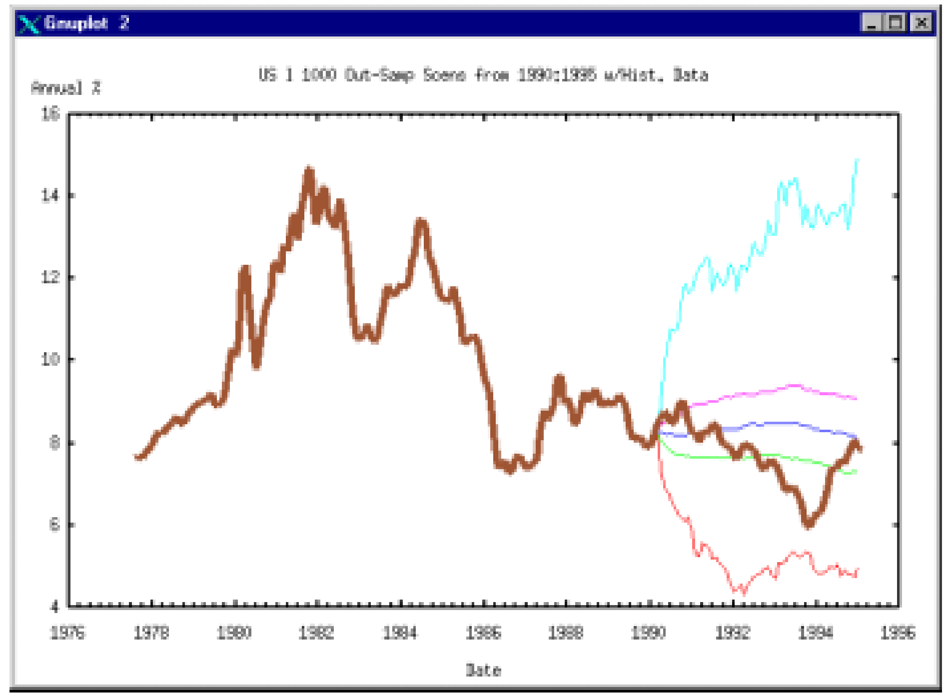




Out-of-Sample Simulation Quantiles with History: US

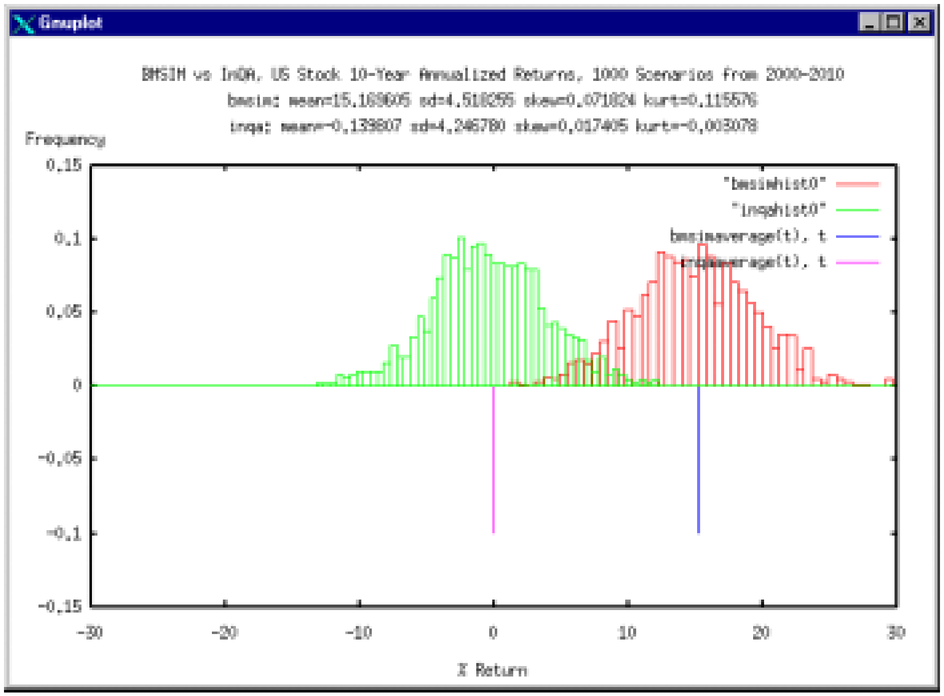
Calibration Period: 77-90 Simulation Period: 90-95

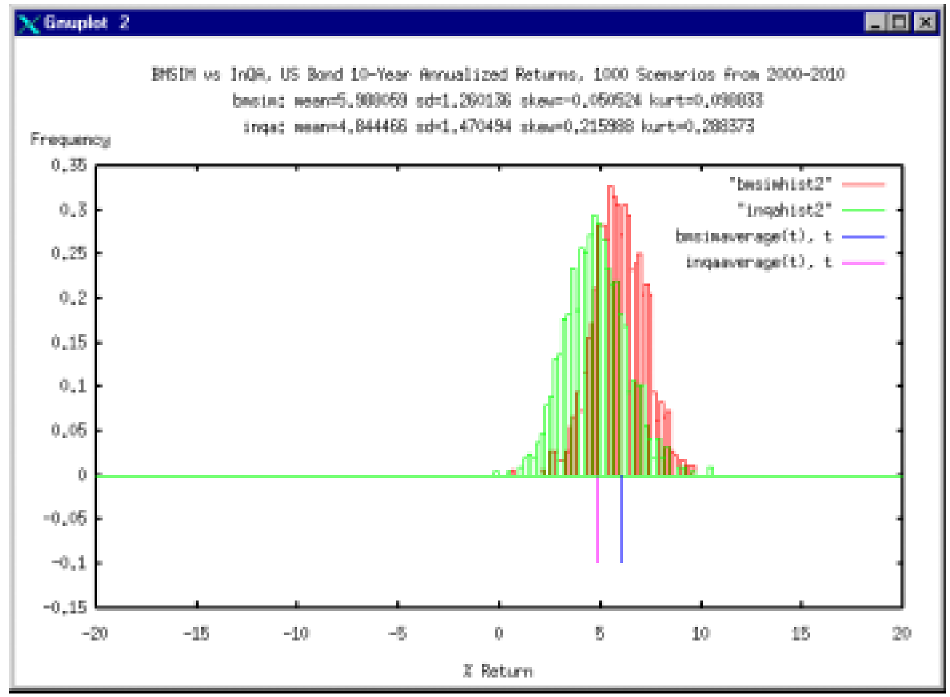




Comparison of 10-Year Annualized Returns with InQA: US

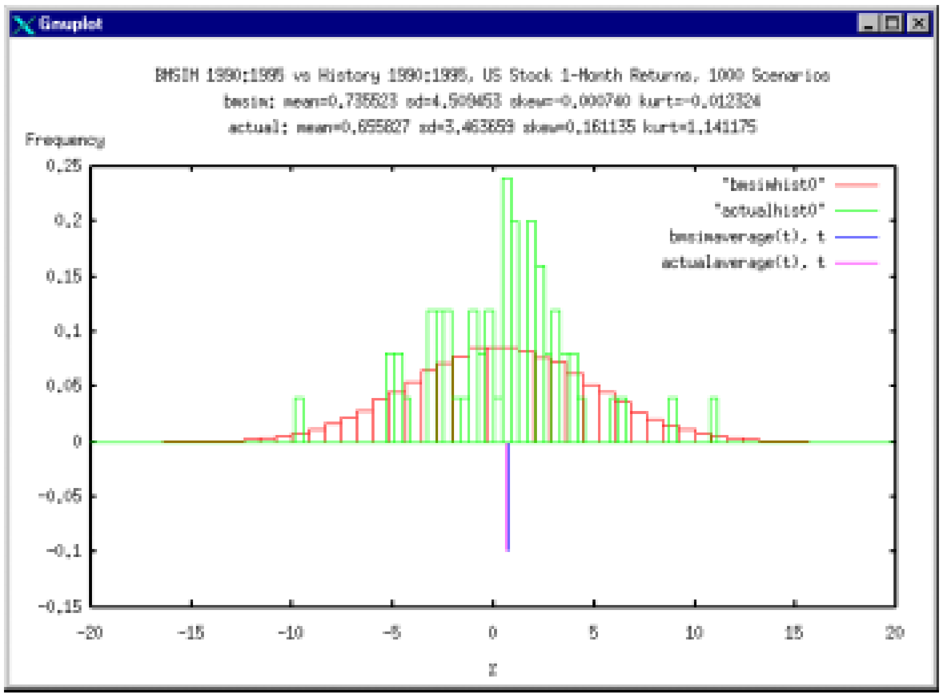
Calibration Period: 92-00 Simulation Period: 00-10

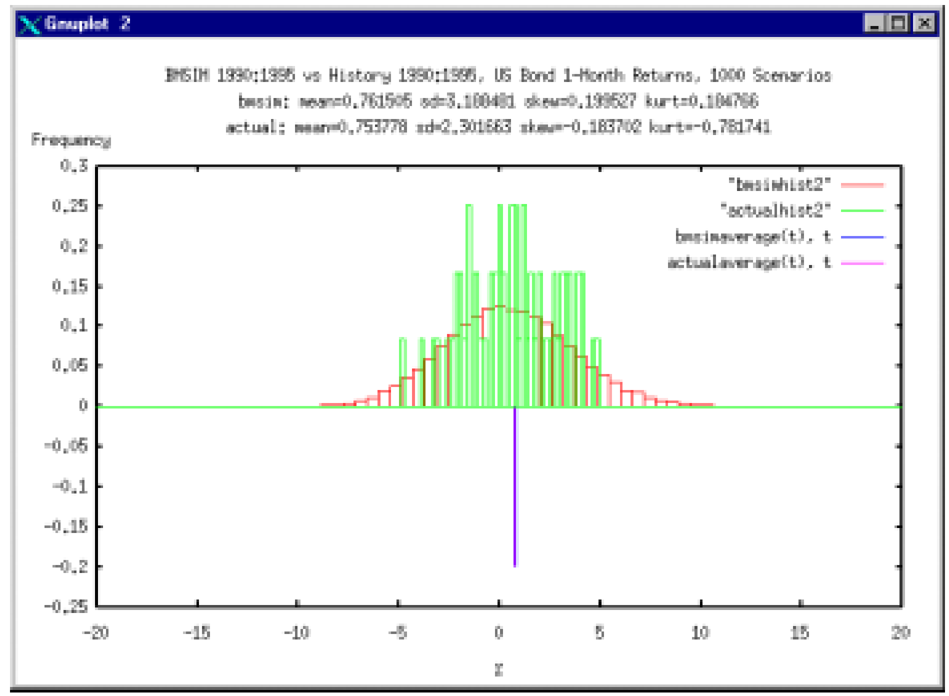




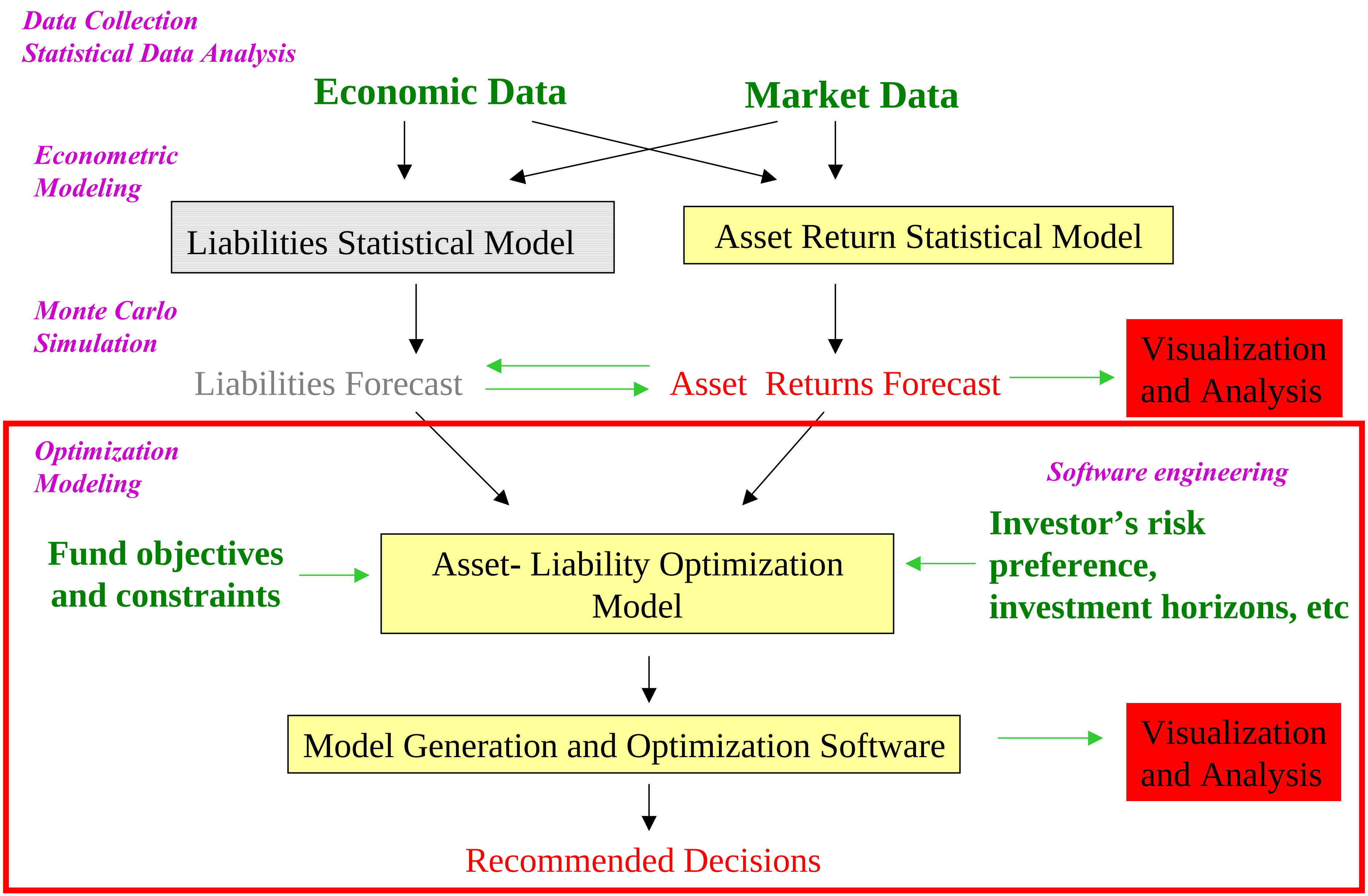
Comparison of 1-Month Returns with History: US

Calibration Period: 77-90 Simulation Period: 90-95





System Overview



Optimal Asset Allocation

Problem formulation

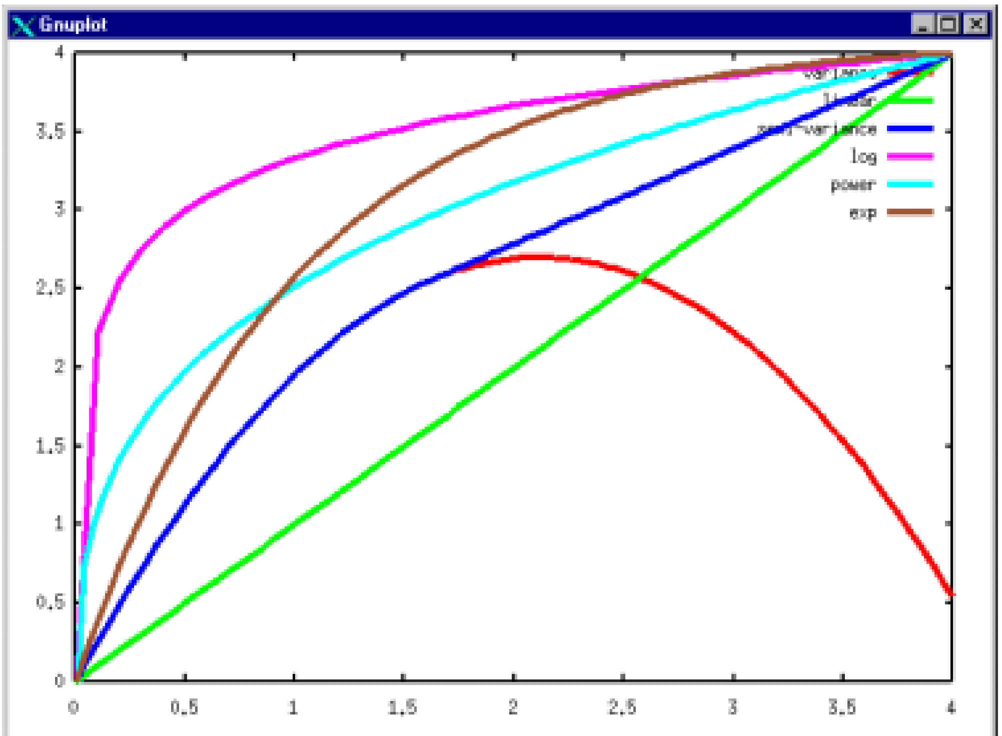
Given a fixed planning horizon and set of portfolio rebalance dates, find the dynamic investment strategy that maximizes the expected utility of terminal fund wealth subject to constraints such as on borrowing and position limits

$$\textit{maximize} \;\; E[u(\mathbf{w}(x))]$$
$$\textit{subject to} \;\; Ax \le b$$

Here u is a utility function and x is a decision process representing the portfolio composition at each rebalance date in each scenario subject to the data (A,b) representing the constraints

Utility Functions

Utility functions are normally increasing to capture the investor's preference for a higher terminal wealth and concave to capture the investor's risk averse attitude -- the greater the curvature the greater the aversion to risk

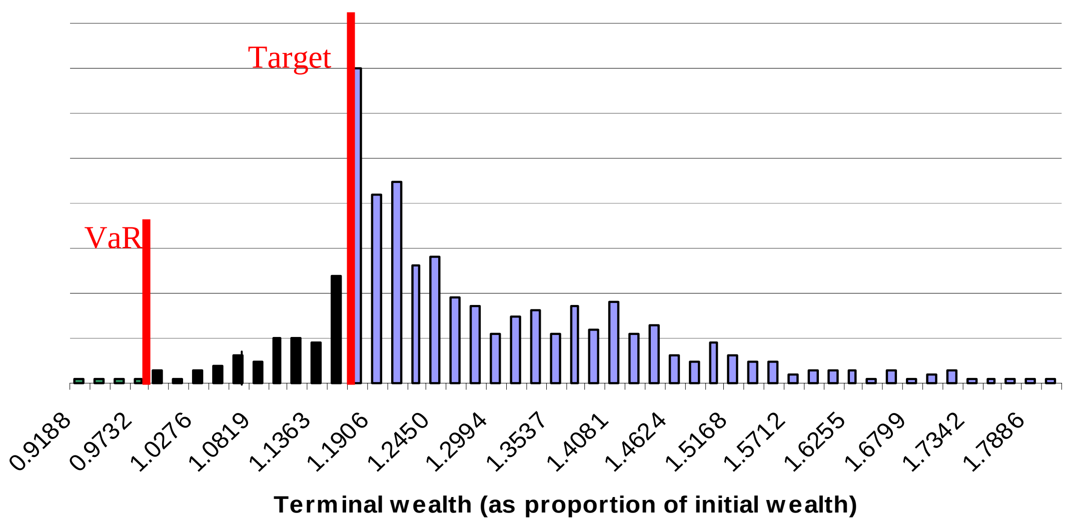


Exponential	$u(w) = -e^{-aw}$
Power	$u(w) = \frac{1}{a} w^{a}$
Log	$u(w) = \log(w)$
Semi-variance	$u(w) = (1-a)w - a(w-tw)_{-}^{2}$
Variance	$u(w) = (1-a)w - a(w-tw)^{2}$
Linear	$u(w) = w$

Semivariance Utility Function

- $(1-a)\mathbf{w} - a(\mathbf{w} - tw)_{-}^{2}$
- Terminal wealth distribution (in-sample)

  Portfolios are penalized for each scenario in which they under-perform relative to the target



Constraints

- **Cash balance constraints**

$$w_1 + \sum_{i=1}^{I} p_{i1}[g_{i1}x_{i1}^- - f_{i1}x_{i1}^+] + \sum_{k=1}^{K} p_{k1}[-z_{k1}^+ + z_{k1}^-] + C_1 - L_1 = 0$$

$$\sum_{i=1}^{I} p_{it}(g_{it}x_{it}^- - f_{it}x_{it}^+) + \sum_{k=1}^{K} p_{kt}[(1+r_{kt}^+)z_{k(t-1)}^+ - (1+r_{kt}^-)z_{k(t-1)}^- - z_{kt}^+ + z_{kt}^-] + C_t - L_t = 0 \quad t = 2,\ldots,T$$

- **Inventory balance constraints**

$$x_{i1} = x_i + x_{i1}^+ - x_{i1}^- \quad \forall i$$

$$x_{it} = x_{i(t-1)}v_{it} + x_{it}^+ - x_{it}^- \quad t = 2,\ldots,T \quad \forall i$$

- **Terminal wealth constraint**

$$\sum_{i=1}^{I} p_{i(T+1)}(1+v_{i(T+1)})x_{iT} + \sum_{k=1}^{K} p_{k(T+1)}[(1+r_{k(T+1)}^+)z_{kT}^+ - (1+r_{k(T+1)}^-)z_{kT}^-] = w$$

Constraints

- **Regulatory constraints**
  - Borrowing and position limits

$$\underline{X}_{it} \le p_{it} x_{it} \le \overline{X}_{it} \qquad \underline{Z}_{kt} \le p_{kt}(z^{+}_{kt} - z^{-}_{kt}) \le \overline{Z}_{kt} \quad t = 1,\ldots,T \quad \forall i$$

- **Performance constraints**
  - Guaranteed return

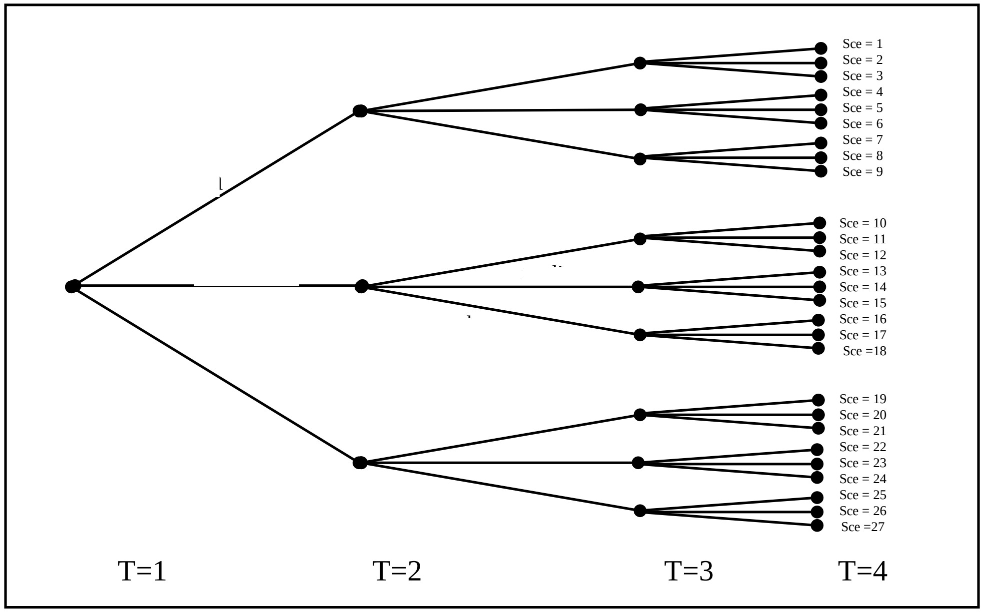
$$\frac{w}{w_1} \ge R$$

- **Fixed mix constraints**
  - The amount invested in each asset is rebalanced to a benchmark proportion of total fund wealth at each trading date

$$p_{it} x_{it} = \lambda_i \left(\sum_{j=1}^{I} p_{jt} x_{jt}\right) \quad t = 1,\ldots,T \quad \forall i$$

Problem Generation

- Scenario tree structure chosen depending on time horizon and rebalance frequency of problem



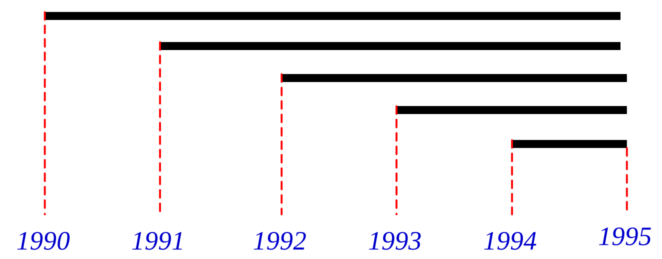
- Scenario tree generated using BMSIM and Stochgen
- SMPS form of the problem constructed by Stochgen from the scenario tree and the deterministic problem formulated in AMPL

Solution Methods

- CPLEX simplex and interior point for convex linear and quadratic problems
- Nested Benders for convex linear and general non-linear problems
- Global optimization methods for non-convex fixed mix and guaranteed return problems

3 Country Backtest: 1990-1995

- Telescoping horizon



- Performance summary

Portfolio Without Position Limits		Portfolio With Position Limits	
Utility Function	**P.A. Return %**	**Utility Function**	**P.A. Return %**
Semivariance (10% target)	10.33	Semivariance (10% target)	9.34
Variance (10% target)	9.21	Variance (10% target)	9.34
Exponential	7.14	Exponential	8.62
Exponential Fixed-Mix	3.89	Exponential Fixed-Mix	--

  – Performance of the S&P 500 from 1990-1995: 7.41% P.A.

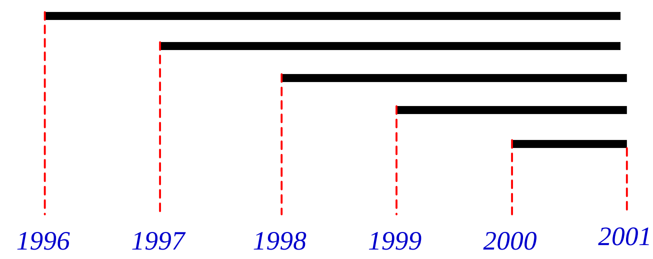
- All optimization problems were generated with 4000 scenarios

Semi-Variance Objective Function (α=.5, Target Return = 10% per annum)

	US Stock	US Cash	US Bonds	UK Stock	UK Cash	UK Bonds	US/UK FX	EU Stock	EU Cash	EU Bonds	US/EU FX
Year 1990											
First Stage Portfolio (% weight)	0.00	5.95	0.00	14.06	20.00	20.00		0.00	20.00	20.00	
Actual % Return (local currency)	4.51	7.65	8.62	-11.22	14.96	11.84		-12.82	8.92	1.26	
Actual % Return ($)	4.51	7.65	8.62	4.07	34.76	31.10	17.22	-0.34	24.52	15.76	14.32
Yearly Return	21.51	Total Return	21.51								
Year 1991											
First Stage Portfolio (% weight)	0.00	0.00	0.00	0.00	20.00	20.00		20.00	20.00	20.00	
Actual % Return (local currency)	18.86	5.33	17.05	18.49	11.07	18.91		20.78	9.67	15.73	
Actual % Return ($)	18.86	5.33	17.05	7.68	0.93	8.06	-9.13	9.40	-0.67	4.83	-9.43
Yearly Return	3.60	Total Return	25.88								
Year 1992											
First Stage Portfolio (% weight)	20.00	0.00	20.00	0.00	20.00	20.00		0.00	20.00	0.00	
Actual % Return (local currency)	7.34	3.45	10.88	11.12	8.83	13.51		1.25	9.86	14.65	
Actual % Return ($)	7.34	3.45	10.88	-7.62	-9.52	-5.63	-16.86	-4.52	3.60	8.12	-5.70
Yearly Return	1.17	Total Return	27.36								
Year 1993											
First Stage Portfolio (% weight)	0.00	0.00	20.00	20.00	20.00	20.00		0.00	20.00	0.00	
Actual % Return (local currency)	9.76	3.06	21.73	27.97	5.44	38.78		51.16	7.30	16.78	
Actual % Return ($)	9.76	3.06	21.73	29.76	6.91	40.72	1.40	35.64	-3.71	4.79	-10.27
Yearly Return	18.94	Total Return	51.48								
Year 1994											
First Stage Portfolio (% weight)	0.00	0.00	0.00	3.83	16.17	20.00		20.00	20.00	20.00	
Actual % Return (local currency)	-2.32	4.60	-11.36	-15.20	5.53	-13.67		-13.81	5.43	-5.84	
Actual % Return ($)	-2.32	4.60	-11.36	-11.11	10.61	-9.51	4.82	-3.43	18.13	5.50	12.04
Yearly Return	3.15	Total Return	56.25								
Total Return (%)	56.25										
Equivalent Annual Return (%)	9.34										

4 Country Backtest: 1996-2001

- Telescoping horizon



- Performance summary

Portfolio Without Position Limits		Portfolio With Position Limits	
Utility Function	**P.A. Return %**	**Utility Function**	**P.A. Return %**
Semivariance (10% target)	13.36	Semivariance (10% target)	7.13
Variance (10% target)	9.44	Variance (10% target)	7.13
Exponential	12.59	Exponential	7.13

  – Performance of the S&P 500 from 1996-2001: 14.28% P.A.

- All optimization problems were generated with 4000 scenarios

Semi-Variance Objective Function (α = .5, Target Return = 10% per annum)

	US S	US R	US L	UK S	UK R	UK L	US/UK X	EU S	EU R	EU L	US/EU X	JP S	JP R	JP L	US/JP X
Year 1996															
First Stage Portfolio (% weight)	0.00	0.00	0.00	0.00	1.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00	
Actual % Return (local currency)	23.61	5.14	-3.67	13.34	5.96	8.77		29.09	3.32	6.86		-14.92	0.59	9.47	
Actual % Return ($)	23.61	5.14	-3.67	19.97	12.17	15.14	5.85	20.63	-3.46	-0.15	-6.56	-23.74	-9.85	-1.88	-10.37
Yearly Return	12.17														
Total Return	12.17														
Year 1997															
First Stage Portfolio (% weight)	1.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00	
Actual % Return (local currency)	24.69	5.19	22.13	21.51	6.97	27.99		43.55	3.41	10.34		-7.65	0.63	7.65	
Actual % Return ($)	24.69	5.19	22.13	23.76	8.95	30.36	1.85	28.61	-7.35	-1.14	-10.41	-15.96	-8.43	-2.04	-9.00
Yearly Return	24.69														
Total Return	38.41														
Year 1998															
First Stage Portfolio (% weight)	1.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00	
Actual % Return (local currency)	30.54	4.86	16.02	6.28	7.19	29.30		15.64	3.56	8.96		-11.22	0.70	-0.15	
Actual % Return ($)	30.54	4.86	16.02	7.12	8.03	30.32	0.79	22.60	9.79	15.52	6.01	1.45	15.08	14.11	14.28
Yearly Return	30.54														
Total Return	78.97														
Year 1999															
First Stage Portfolio (% weight)	1.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00	
Actual % Return (local currency)	8.97	4.85	-13.73	10.38	5.24	-0.91		26.81	3.02	-1.38		51.78	0.20	4.55	
Actual % Return ($)	8.97	4.85	-13.73	8.46	3.41	-2.64	-1.74	8.34	-11.98	-15.74	-14.57	63.16	7.71	12.39	7.49
Yearly Return	8.97														
Total Return	93.10														
Year 2000															
First Stage Portfolio (% weight)	1.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00	
Actual % Return (local currency)	-2.04	5.98	22.89	1.82	6.08	8.94		2.36	4.60	10.63		-23.87	0.28	3.96	
Actual % Return ($)	-2.04	5.98	22.89	-7.73	-3.87	-1.28	-9.38	-1.23	0.93	6.74	-3.51	-31.16	-9.32	-5.99	-9.57
Yearly Return	-2.04														
Total Return (%)	87.20														
Equivalent Annual Return (%)	13.36														

Conclusions & Further Research

- A global asset return simulator (BMSIM) has been developed
- Optimal asset allocations with differing utility functions and constraint structures have been solved
- Asset allocation backtests show positive results
- Future work includes further development of a liability statistical model