

The transportation algorithm

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This slideshow illustrates the transportation algorithm for finding the minimal cost of transport.

(0) Find an initial feasible assignment.

(1) Assign Lagrange multipliers. Set $\lambda_1 = 0$ and $\lambda_i + \mu_j = d_{ij}$ for basic cells.

(2) Test for optimality. If $\lambda_i + \mu_j \leq d_{ij}$ for each i, j then STOP!

(3) Pivot. Pick one of the cells such that $\lambda_i + \mu_j > d_{ij}$ to add flow.

Put an amount $\varepsilon > 0$ units of flow into the pivot cell. At the same time, add or subtract ε from the basic cells to maintain feasibility.

Now choose the largest ε possible such that the flow is feasible.

(4) Go to (1).

(0) Find an initial feasible assignment.

8	6	7	5	12
4	3	5	4	8
9	8	6	7	11
7	6	10	8	

The North-West method

7							
	8		6		7	5	12
							8
	4		3		5		4
							11
	9		8		6		7
7		6		10		8	

The North-West method

7		5						12
	8		6		7		5	
								8
	4		3		5		4	
								11
	9		8		6		7	
	7		6		10		8	

The North-West method

7	5			12
8	6	7	5	
4	1	7	4	8
9	8	6	7	11
7	6	10	8	

The North-West method

7	5				12
8	6	7	5		
4	1	7	5	4	8
9	8	3	6	7	11
7	6	10	8		

The North-West method

7	5			12
8	6	7	5	
4	1	7		8
9	8	3	8	11
7	6	10	8	

total cost = 198

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8				
0	7	5			
	8	6	7	5	12
	4	1	7	5	4
	9	8	3	8	7
	7	6	10	8	

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	6			
0	7	5			
	8	6	7	5	12
	4	1	7	5	4
	9	8	3	8	7
	7	6	10	8	

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	6			
0	7	5			
	8	6	7	5	12
-3		1	7		
	4	3	5	4	8
			3	8	
	9	8	6	7	11
	7	6	10	8	

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	6	8		
0	7	5			12
	8	6	7	5	
-3		1	7		8
	4	3	5	4	
			3	8	11
	9	8	6	7	
	7	6	10	8	

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	6	8	
0	7	5		12
	8	6	7	5
-3		1	7	8
	4	3	5	4
-2			3	8
	9	8	6	7
	7	6	10	8

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	6	8	9	
0	7	5			12
	8	6	7	5	
-3		1	7		8
	4	3	5	4	
-2			3	8	11
	9	8	6	7	
	7	6	10	8	

(2) Test for optimality.

$\lambda_i \backslash \mu_j$	8	6	8	9	
0	7	5	1	4	12
	8	6	7	5	
-3	1	1	7	2	8
	4	3	5	4	
-2			3	8	11
	9	8	6	7	
	7	6	10	8	

(3) Pivot.

7		5			ε					12
	8		6		7				5	
		1		7						8
	4		3		5				4	
				3				8		11
	9		8		6				7	
	7		6		10				8	

(3) Pivot.

7		5			ε					12
	8		6		7				5	
		1		7						8
	4		3		5				4	
				3				$8-\varepsilon$		11
	9		8		6				7	
	7		6		10				8	

(3) Pivot.

7		5		ε		12		
	8		6		7		5	
		1		$7-\varepsilon$				8
	4		3		5		4	
				$3+\varepsilon$		$8-\varepsilon$		11
	9		8		6		7	
	7		6		10		8	

(3) Pivot.

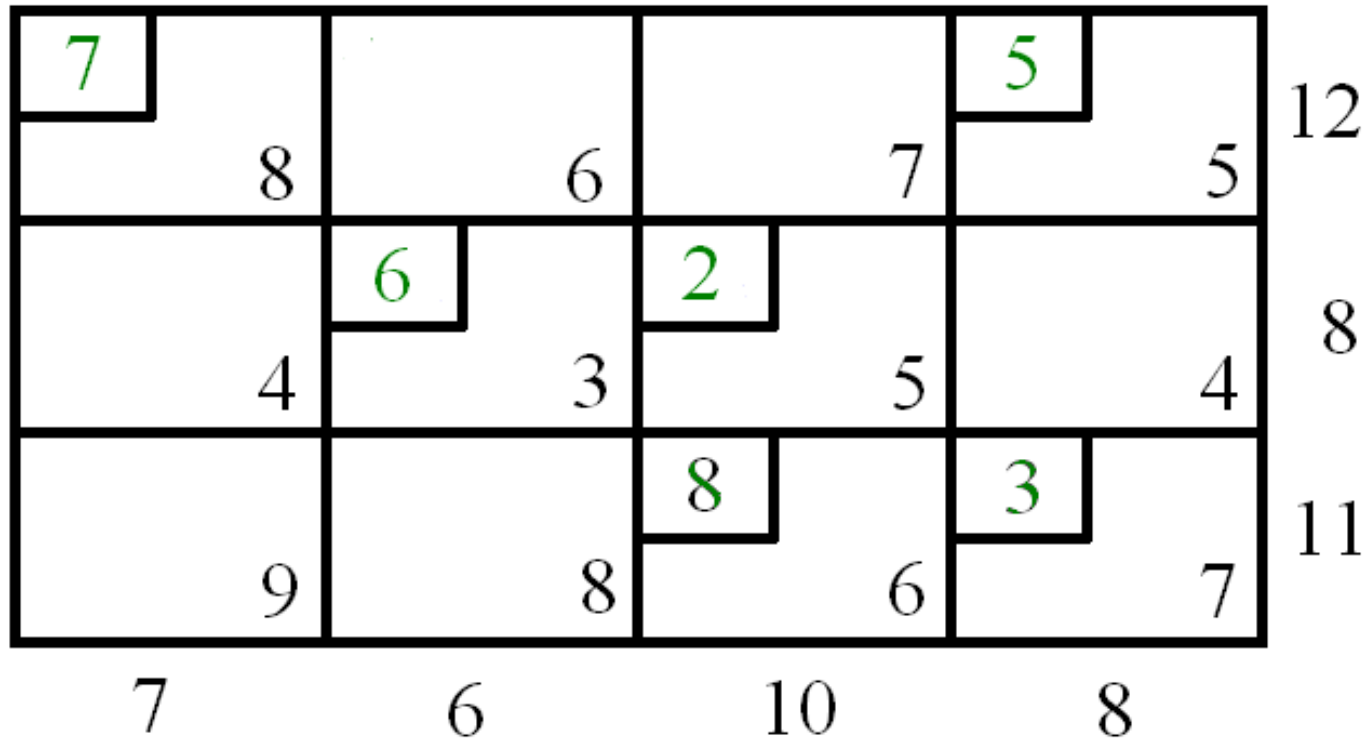
7		5		ε		12		
	8		6		7		5	
		$1+\varepsilon$		$7-\varepsilon$				8
	4		3		5		4	
				$3+\varepsilon$		$8-\varepsilon$		11
	9		8		6		7	
	7		6		10		8	

(3) Pivot.

7		$5-\varepsilon$		ε		12	
	8		6		7	5	
		$1+\varepsilon$		$7-\varepsilon$		8	
	4		3		5	4	
				$3+\varepsilon$		$8-\varepsilon$	11
	9		8		6	7	
7		6		10		8	

(3) Pivot.

$\epsilon=5$



total cost = 178

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	2	4	5	
0	7			5	12
	8	6	7	5	
1		6	2		8
	4	3	5	4	
2			8	3	11
	9	8	6	7	
	7	6	10	8	

(2) Test for optimality.

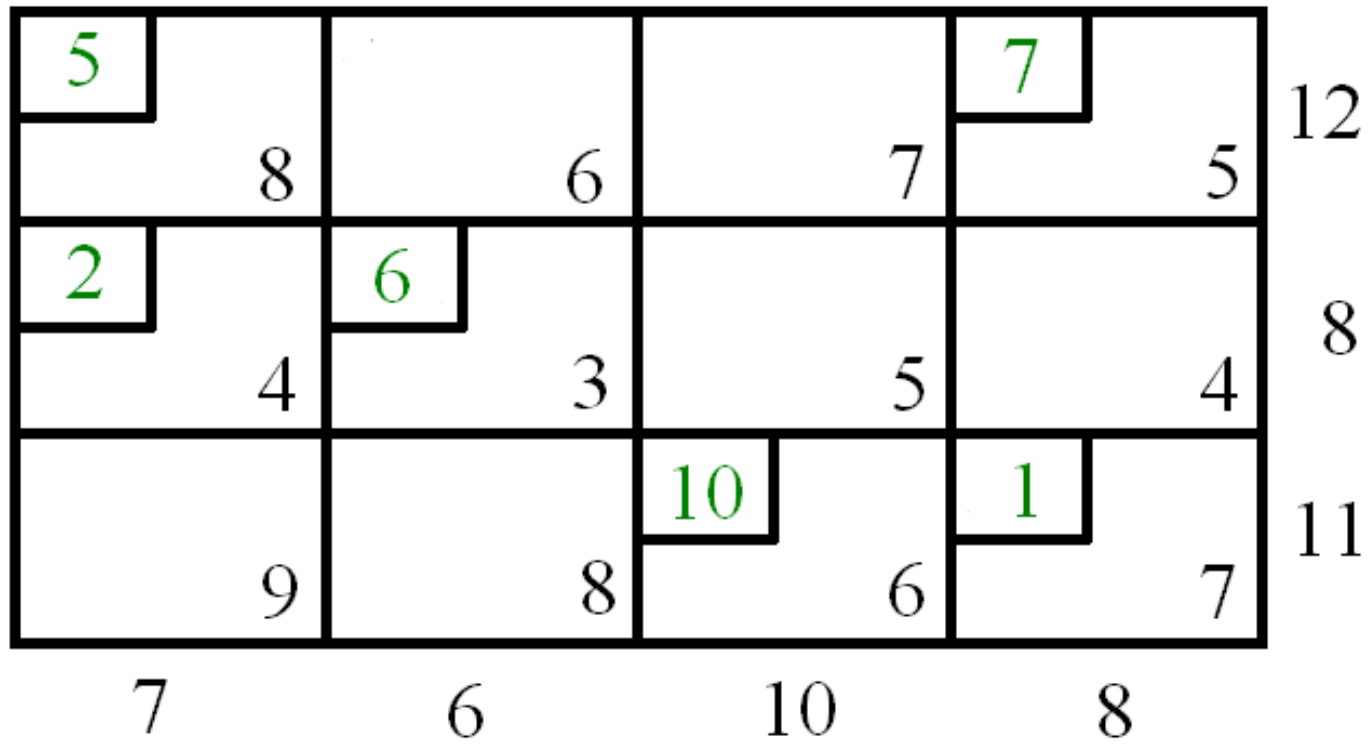
$\lambda_i \backslash \mu_j$	8	2	4	5	
0	7			5	12
	8	6	7	5	
1	5	6	2	2	8
	4	3	5	4	
2	1		8	3	11
	9	8	6	7	
	7	6	10	8	

(3) Pivot.

$7-\varepsilon$			$5+\varepsilon$	12
	8	6	7	5
ε		6	$2-\varepsilon$	8
	4	3	5	4
		$8+\varepsilon$	$3-\varepsilon$	11
	9	8	6	7
7	6	10	8	

(3) Pivot.

$$\varepsilon=2$$



total cost = 168

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	8	7	4	5	
0	5			7	12
	8	6	7	5	
-4	2	6			8
	4	3	5	4	
2			10	1	11
	9	8	6	7	
	7	6	10	8	

(2) Test for optimality.

$\lambda_i \backslash \mu_j$	8	7	4	5	
0	5 8	1 6	7 5	7 5	12
-4	2 4	6 3	5 4	4 4	8
2	1 9	1 8	10 6	1 7	11
	7	6	10	8	

(3) Pivot.

$5-\varepsilon$	ε		7	
8	6	7	5	12
$2+\varepsilon$	$6-\varepsilon$			
4	3	5	4	8
		10	1	
9	8	6	7	11
7	6	10	8	

(1) Assign Lagrange multipliers

$\lambda_i \backslash \mu_j$	7	6	4	5	
0	8	5 6	7	7 5	12
-3	7 4	1 3	5	4	8
2	9	8	10 6	1 7	11
	7	6	10	8	

(2) Test for optimality.

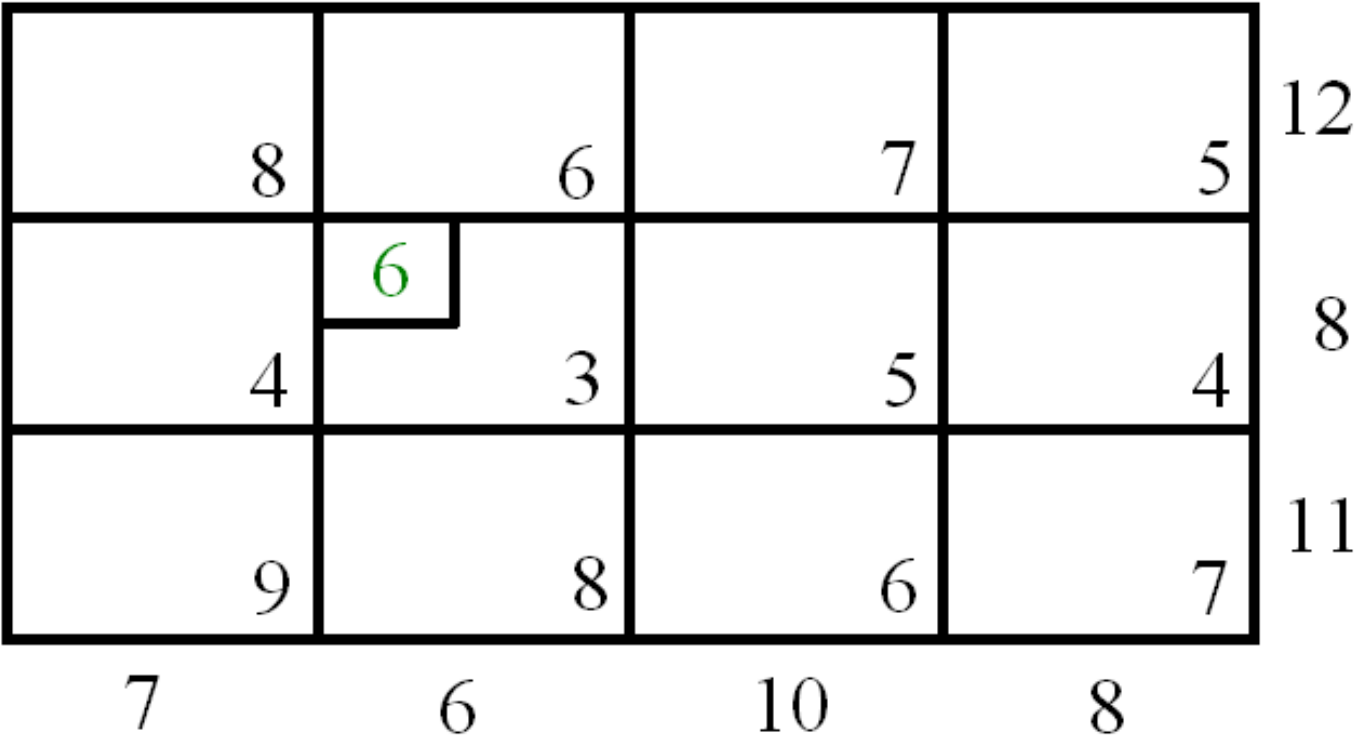
$\lambda_i \backslash \mu_j$	7	6	4	5	
0	8 ✓	5 6	7 ✓	7 5	12
-3	7 4	1 3	5 ✓	4 ✓	8
2	9 ✓	8 ✓	10 6	1 7	11
	7	6	10	8	

Assignment is optimal.

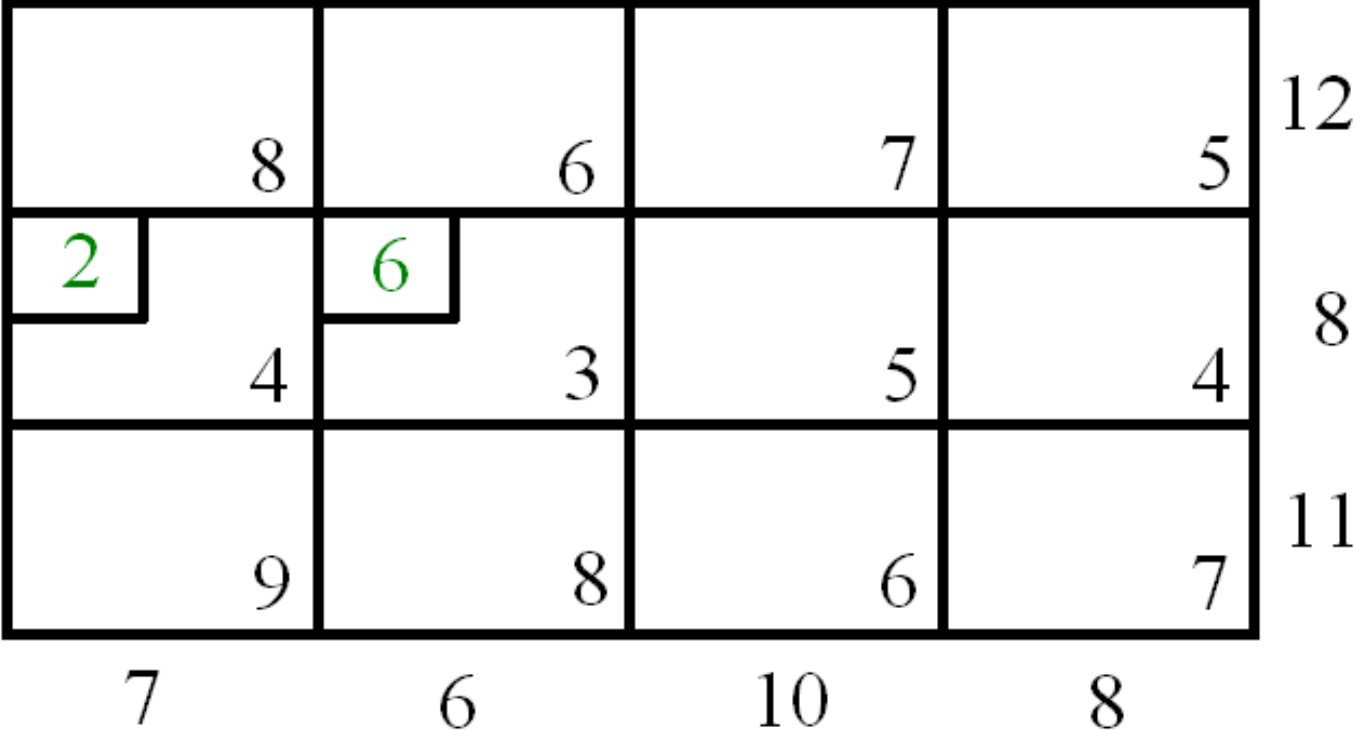
(0) Find an initial feasible assignment.

8	6	7	5	12
4	3	5	4	8
9	8	6	7	11
7	6	10	8	

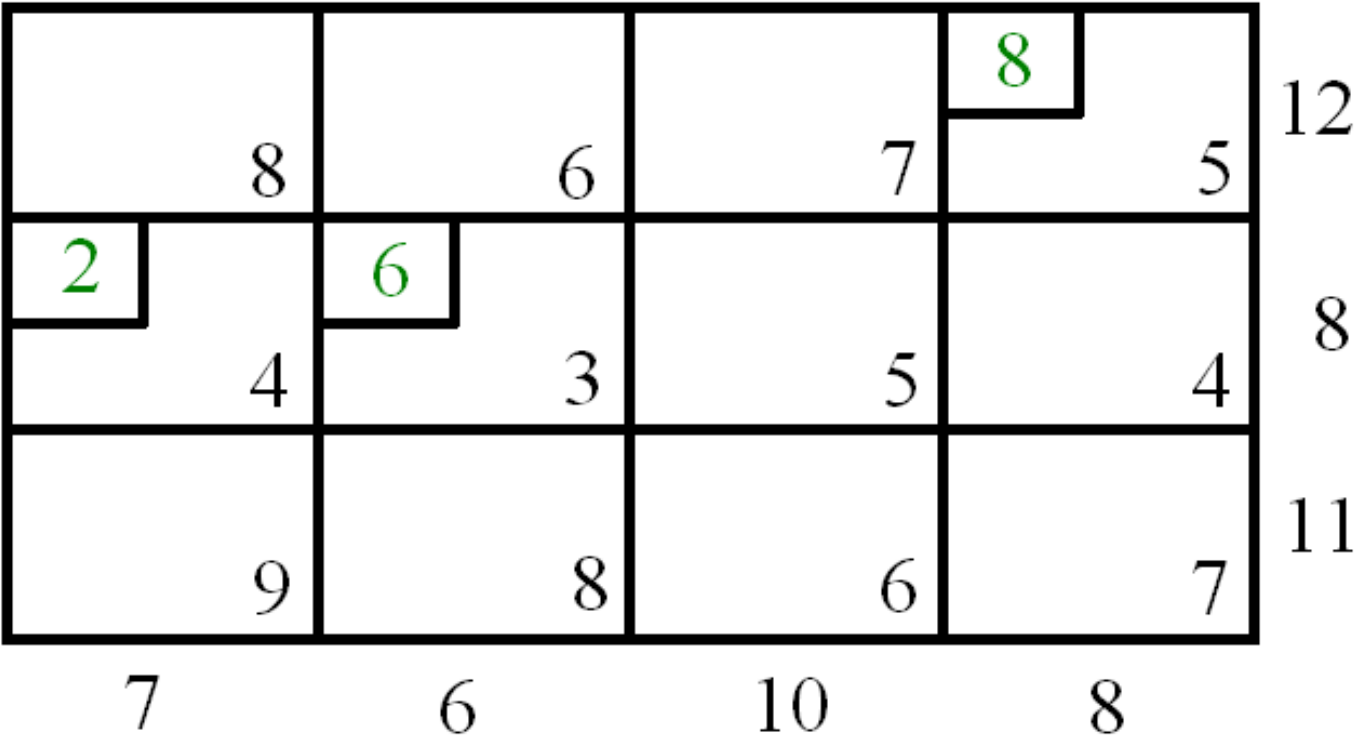
The greedy algorithm



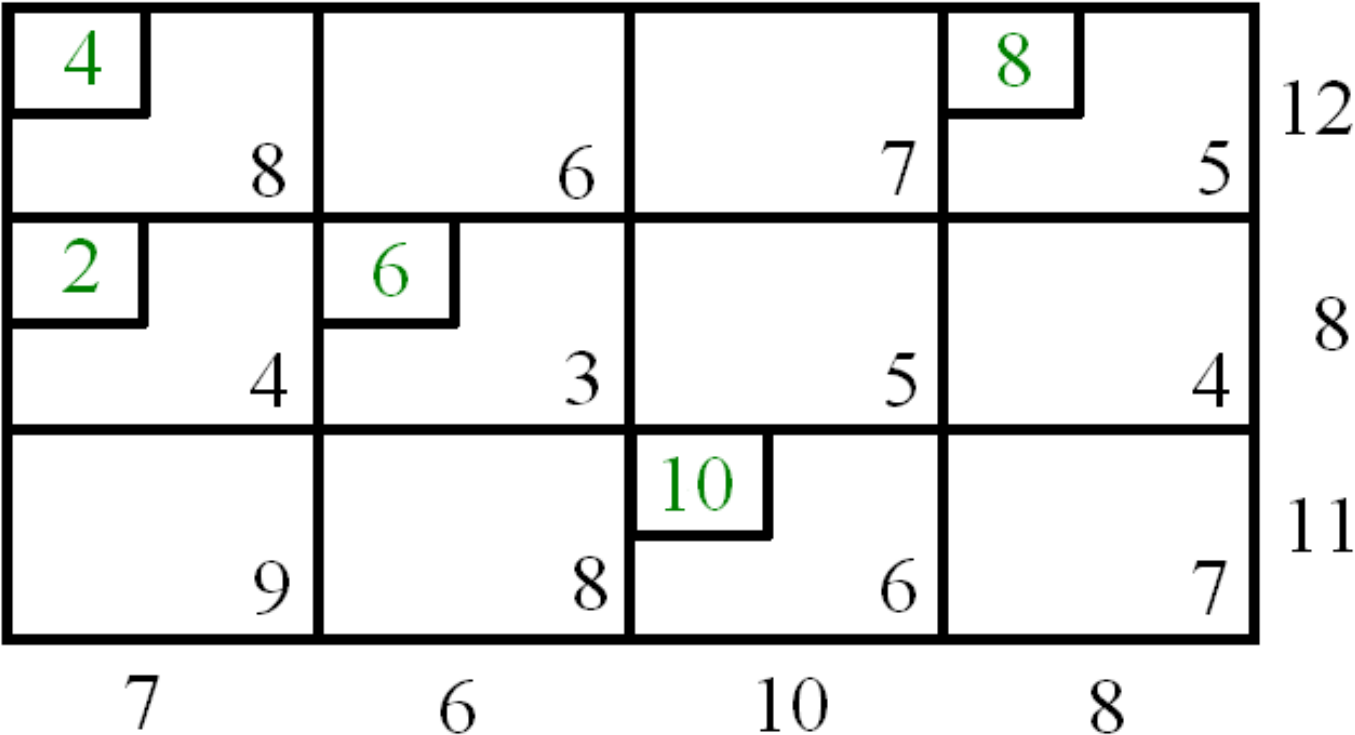
The greedy algorithm



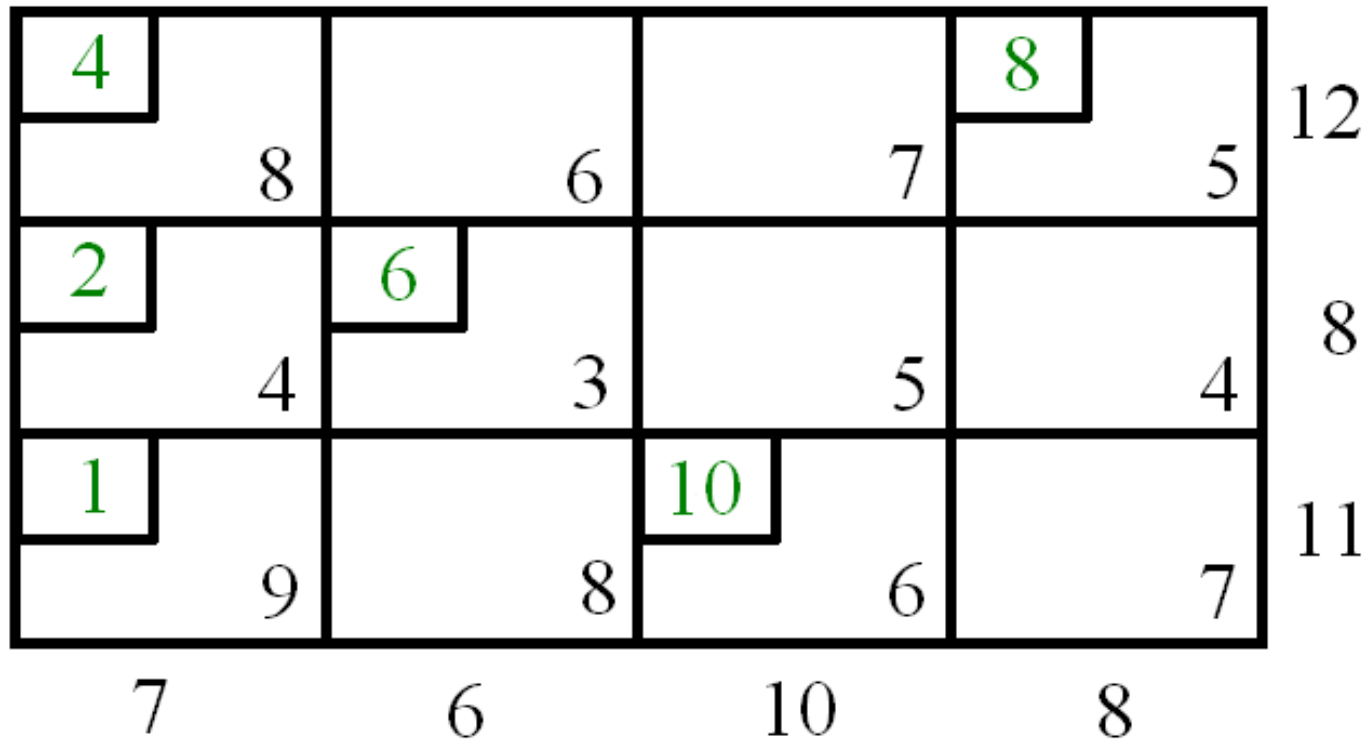
The greedy algorithm



The greedy algorithm



The greedy algorithm

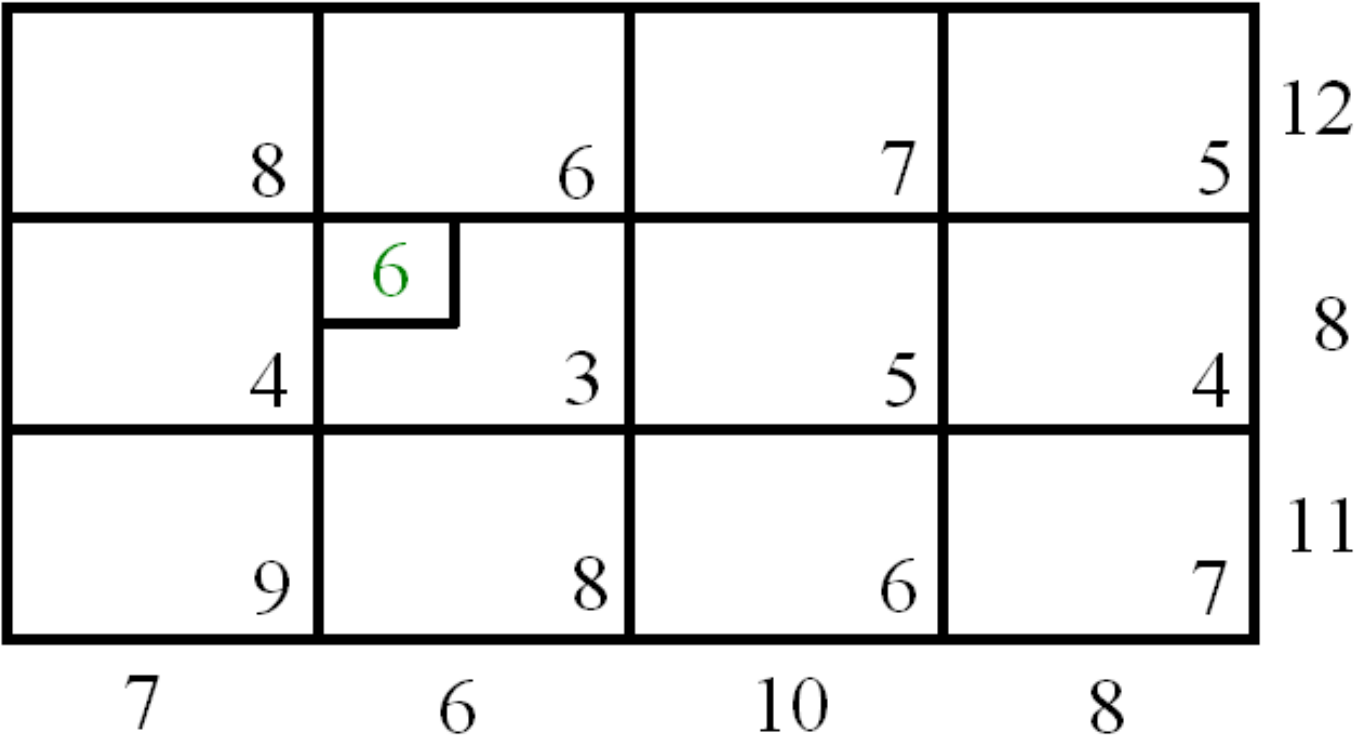


total cost = 167

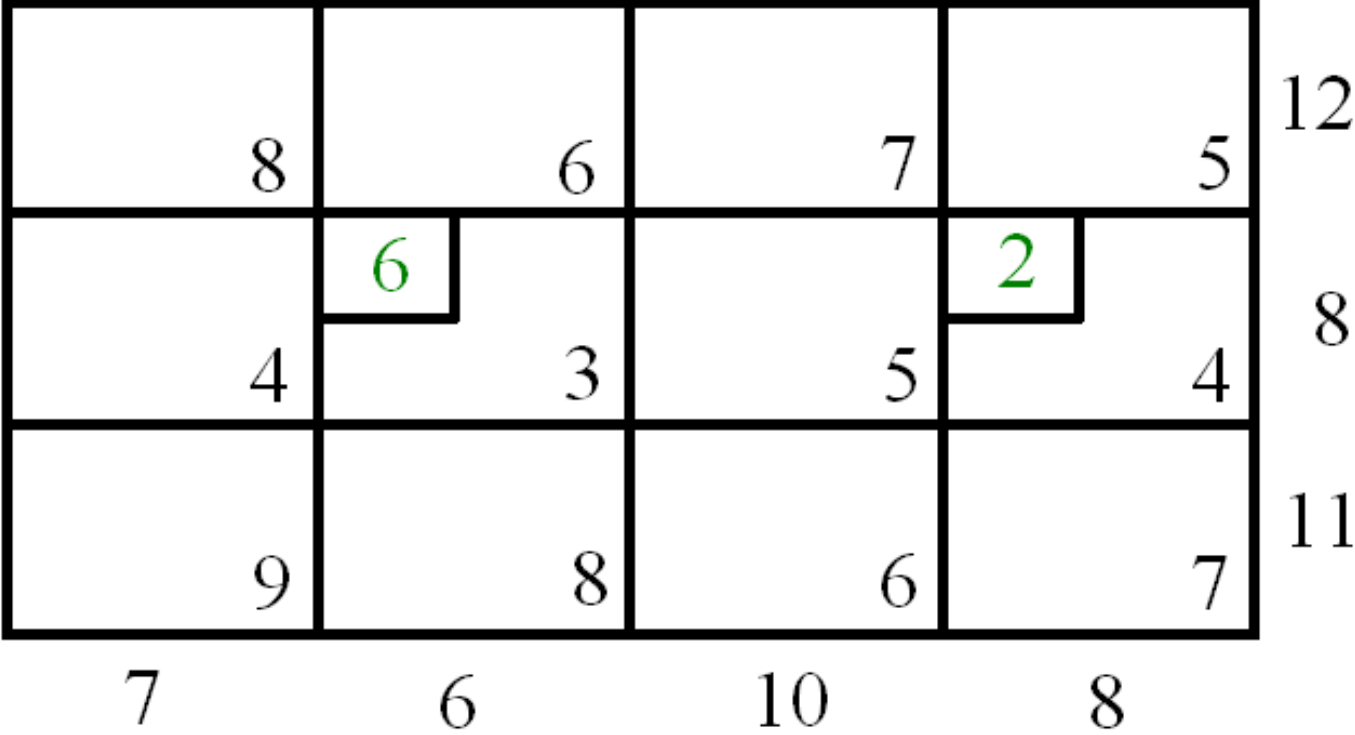
(0) Find an initial feasible assignment.

8	6	7	5	12
4	3	5	4	8
9	8	6	7	11
7	6	10	8	

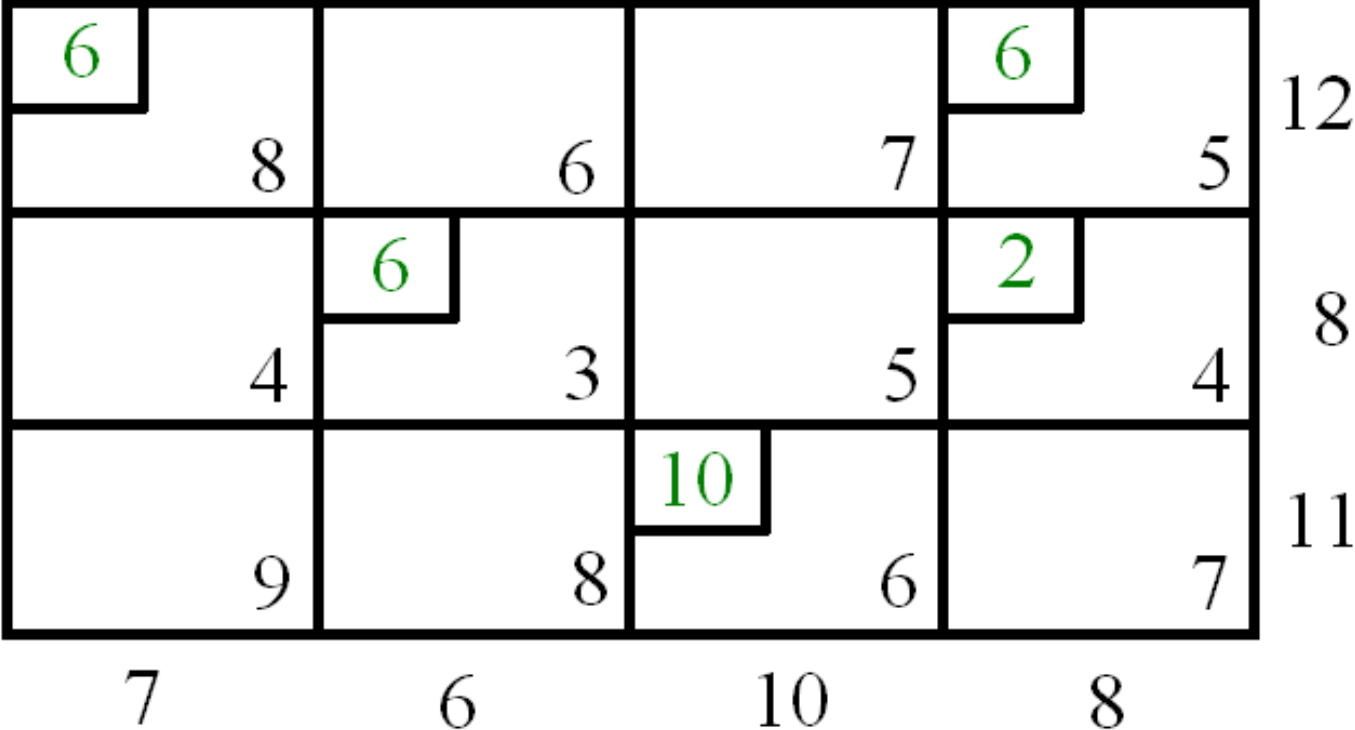
The greedy algorithm



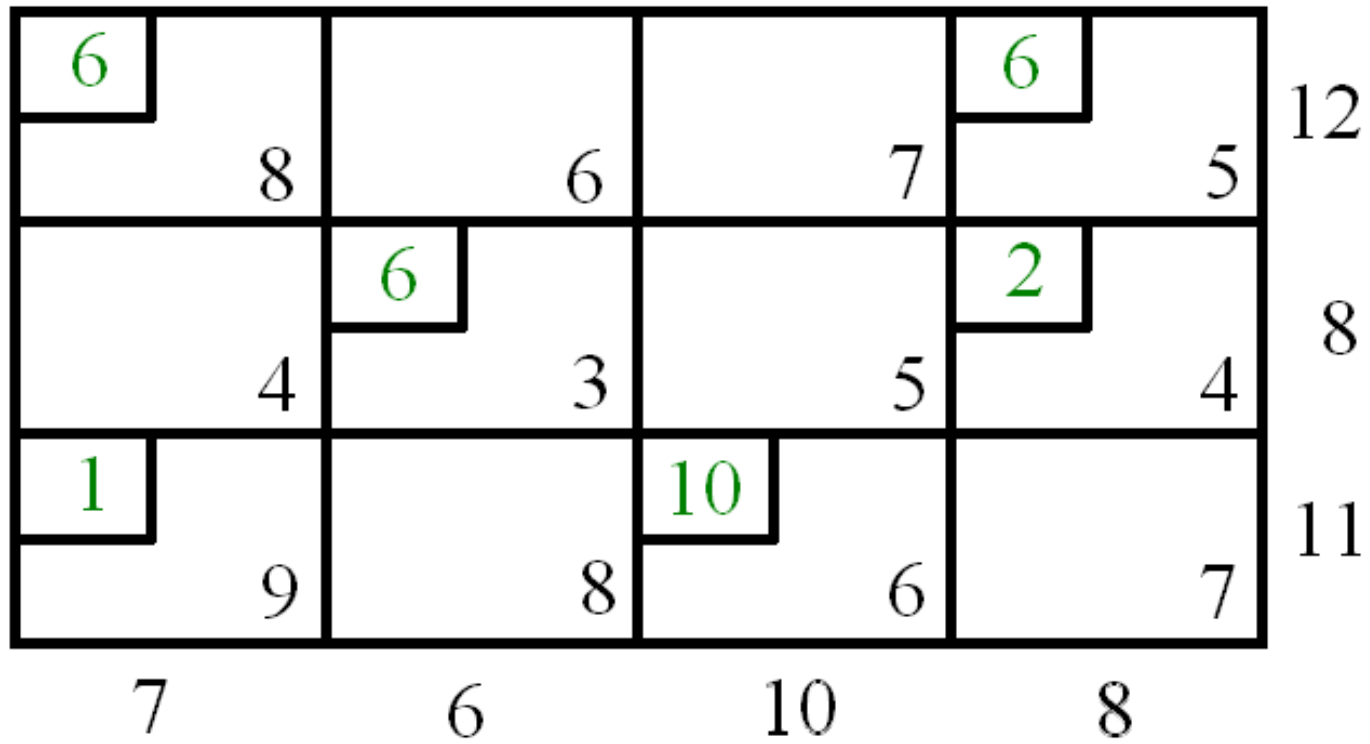
The greedy algorithm



The greedy algorithm



The greedy algorithm



total cost = 173