

The Ford–Fulkerson algorithm

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This slideshow illustrates the Ford–Fulkerson algorithm for finding a maximal flow through a network.

(0) Assign an initial feasible flow $(x_{ij})_{i,j \in N}$ with value v .

(1) Find an augmenting path $1 = i_1, i_2, \dots, i_{r-1}, i_r = n$ from node 1 to node n .

If no augmenting path exists, then STOP. The flow $(x_{ij})_{i,j \in N}$ is optimal and there exists a cut with capacity v .

(2) If an augmenting path exists, then increase the flow along the path by the amount

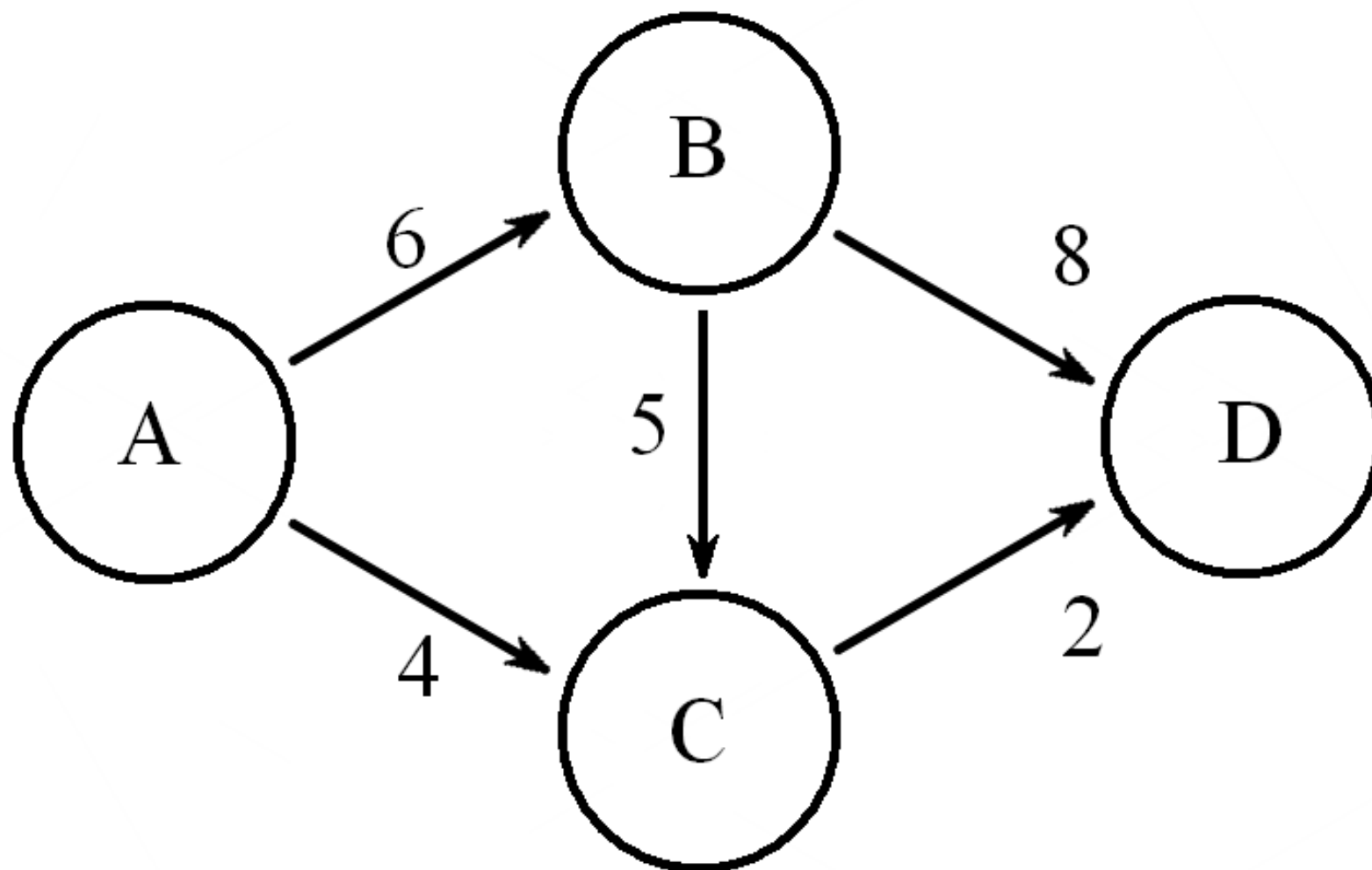
$$\delta = \min_{k=1, \dots, r-1} \max\{c_{i_k i_{k+1}} - x_{i_k i_{k+1}}, x_{i_{k+1} i_k}\}$$

by setting

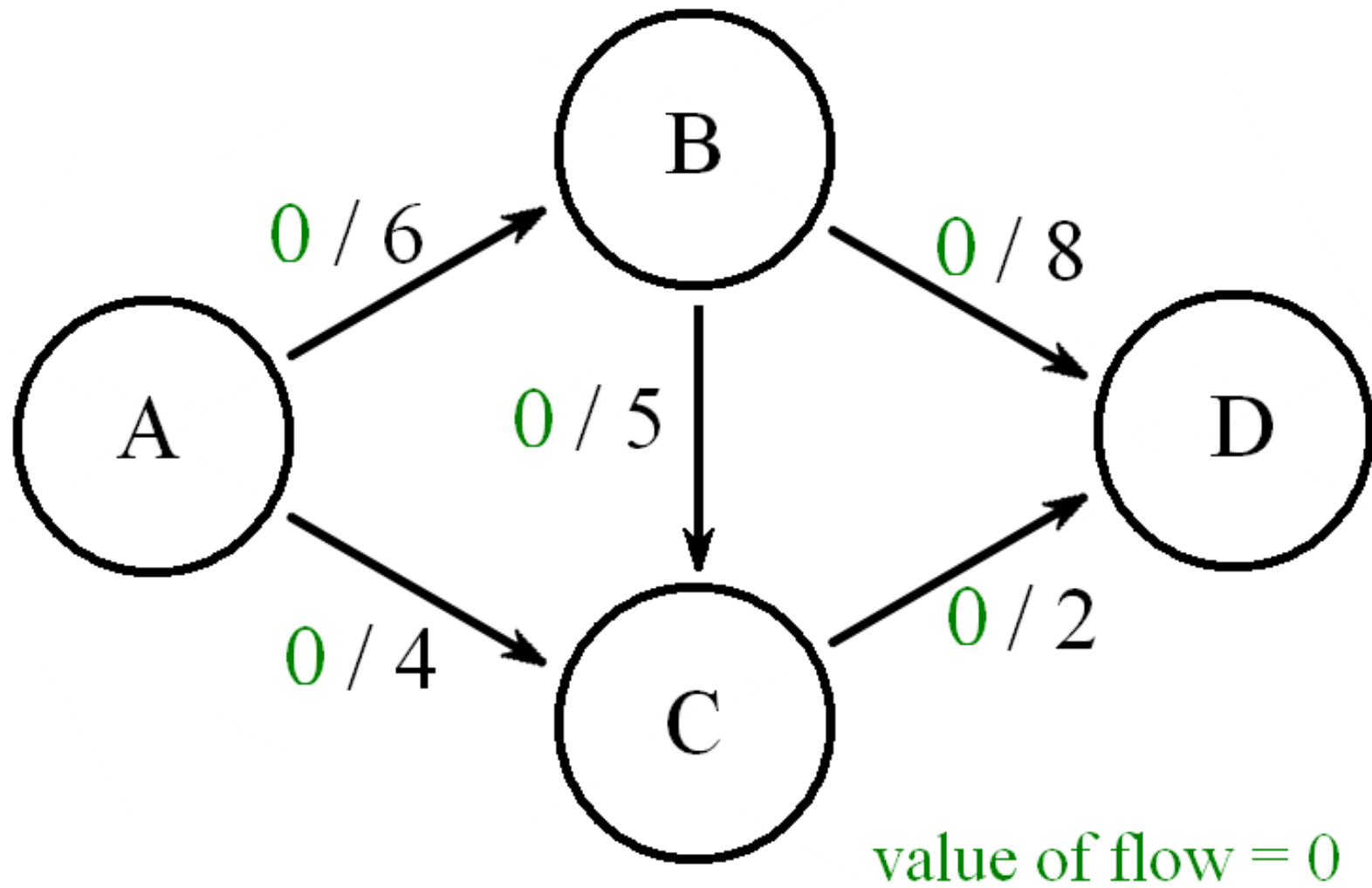
$$x'_{ij} = \begin{cases} x_{ij} + \delta & \text{if } i, j \text{ are consecutive nodes of path} \\ x_{ij} - \delta & \text{if } j, i \text{ are consecutive nodes of path} \\ x_{ij} & \text{otherwise} \end{cases}$$

Then $(x'_{ij})_{i,j \in N}$ is a feasible flow with value $v' = v + \delta$.

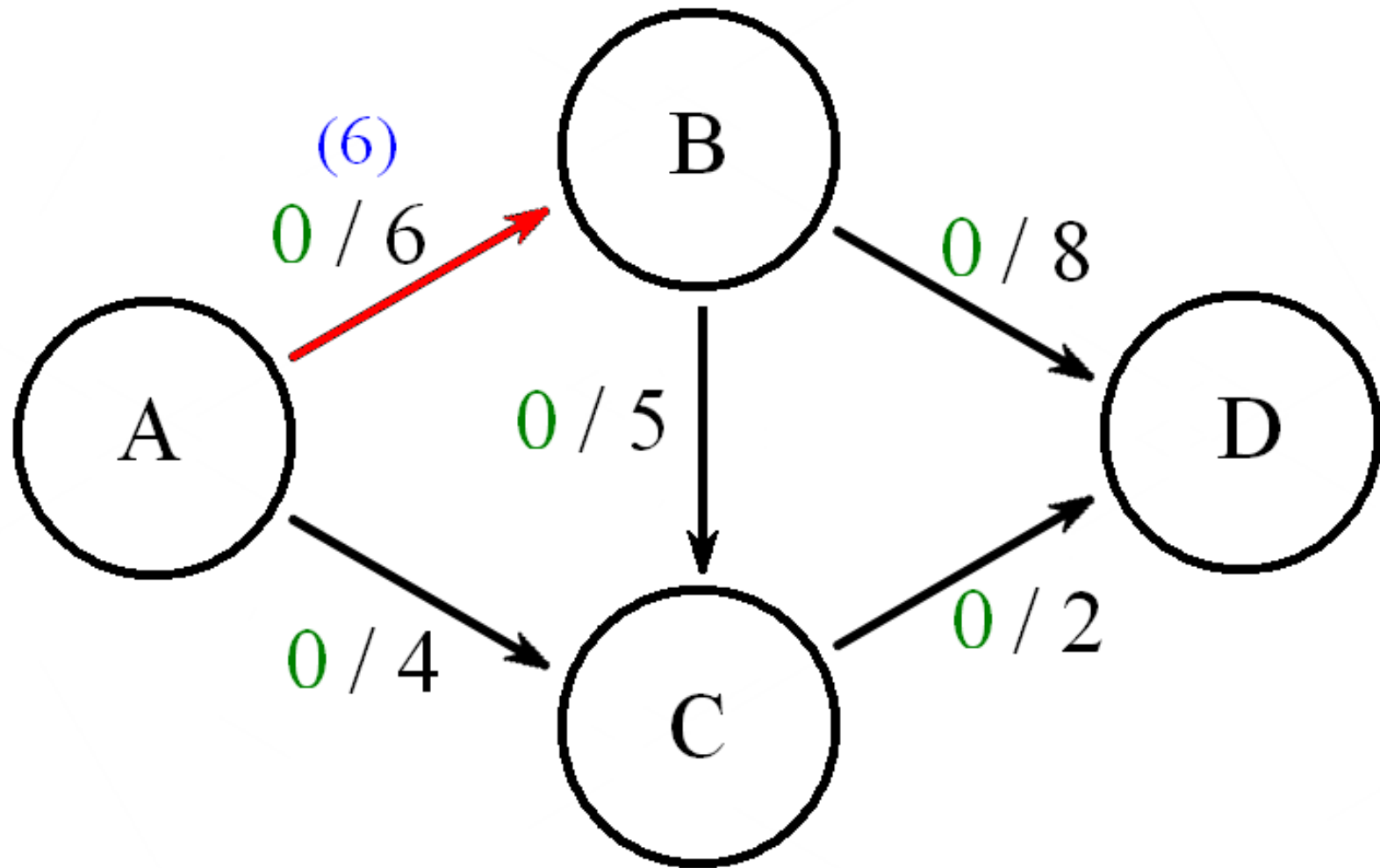
(3) Go to step (1).



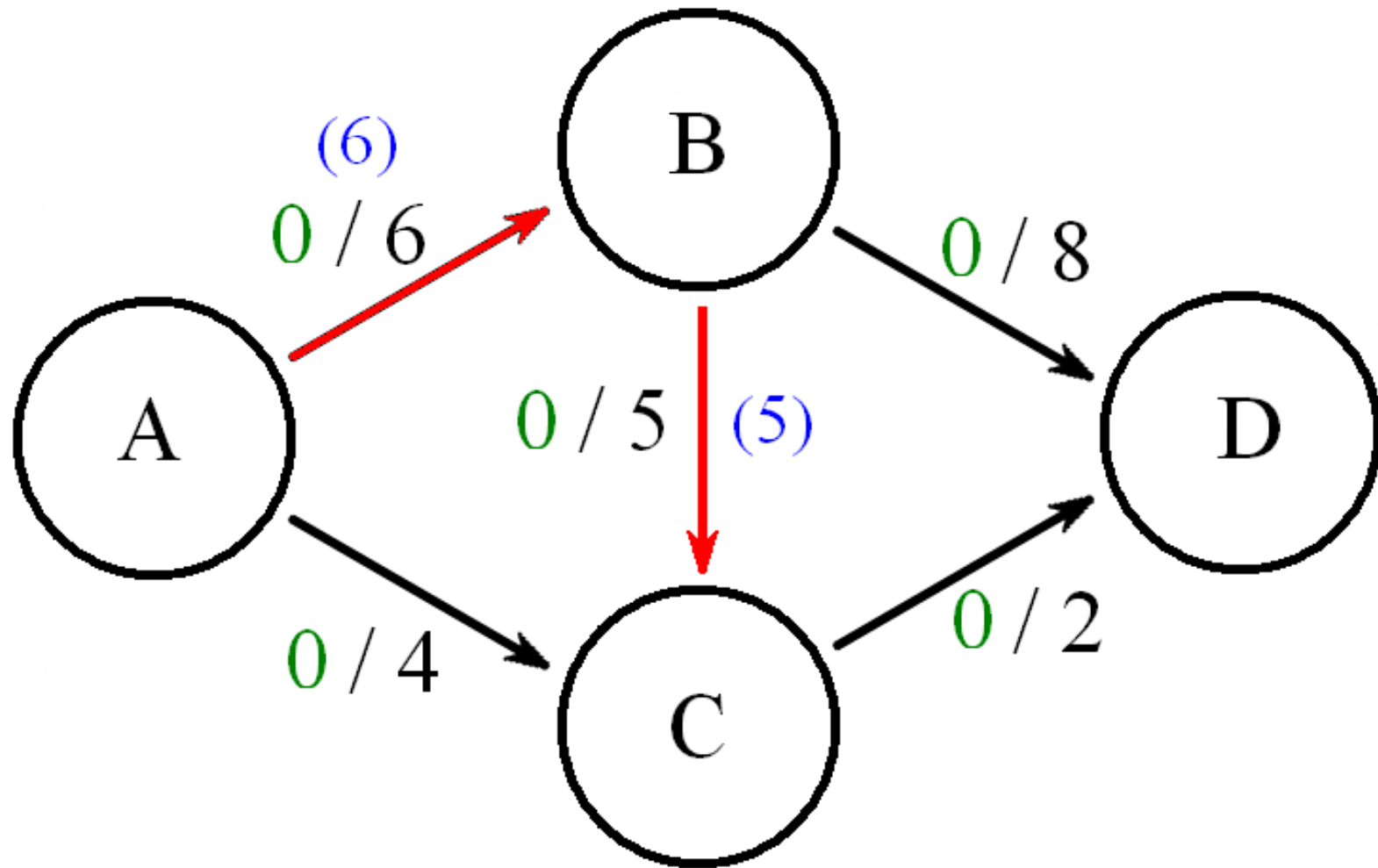
(0) Assign an initial feasible flow



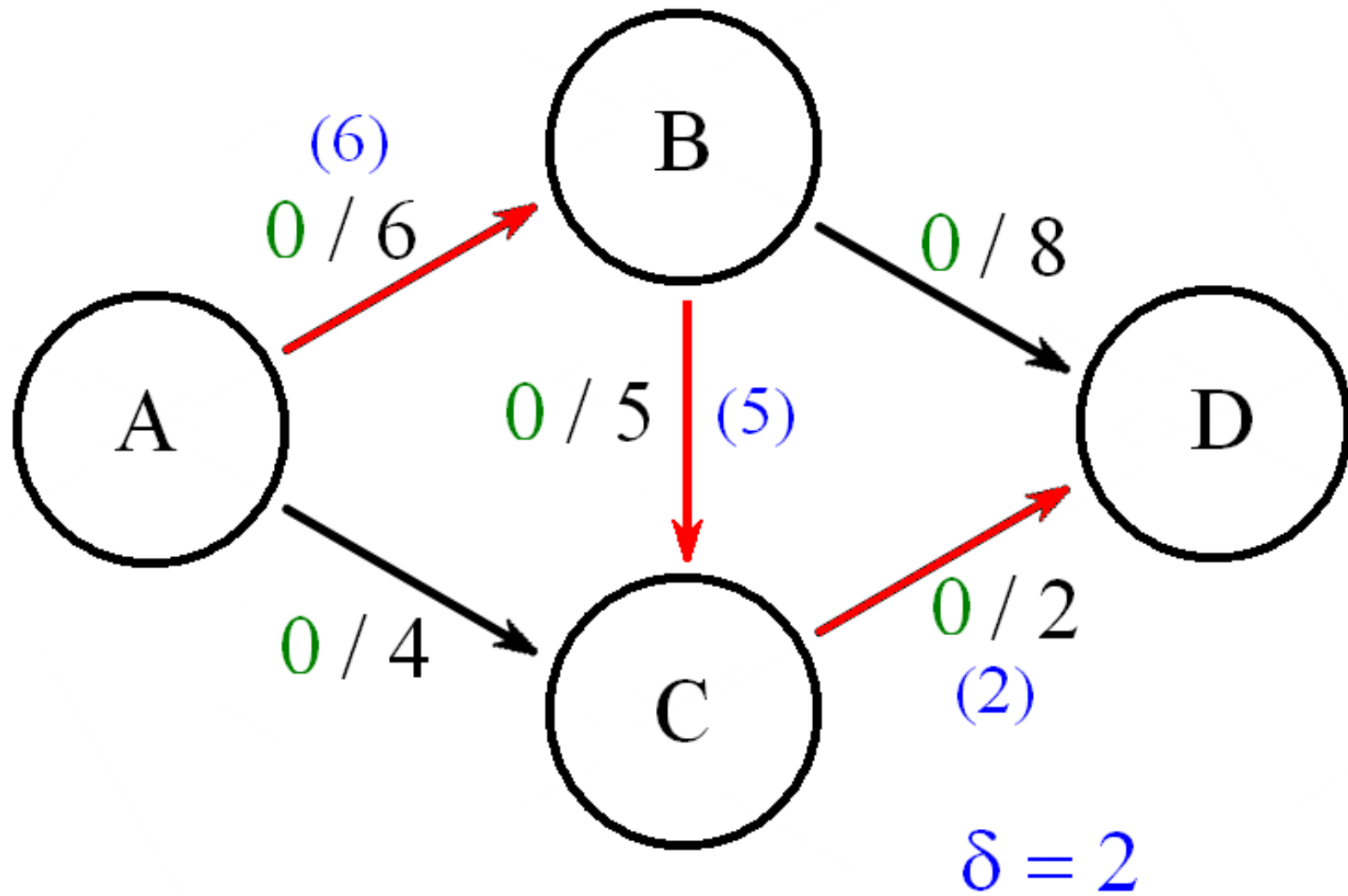
(1) Find an augmenting path



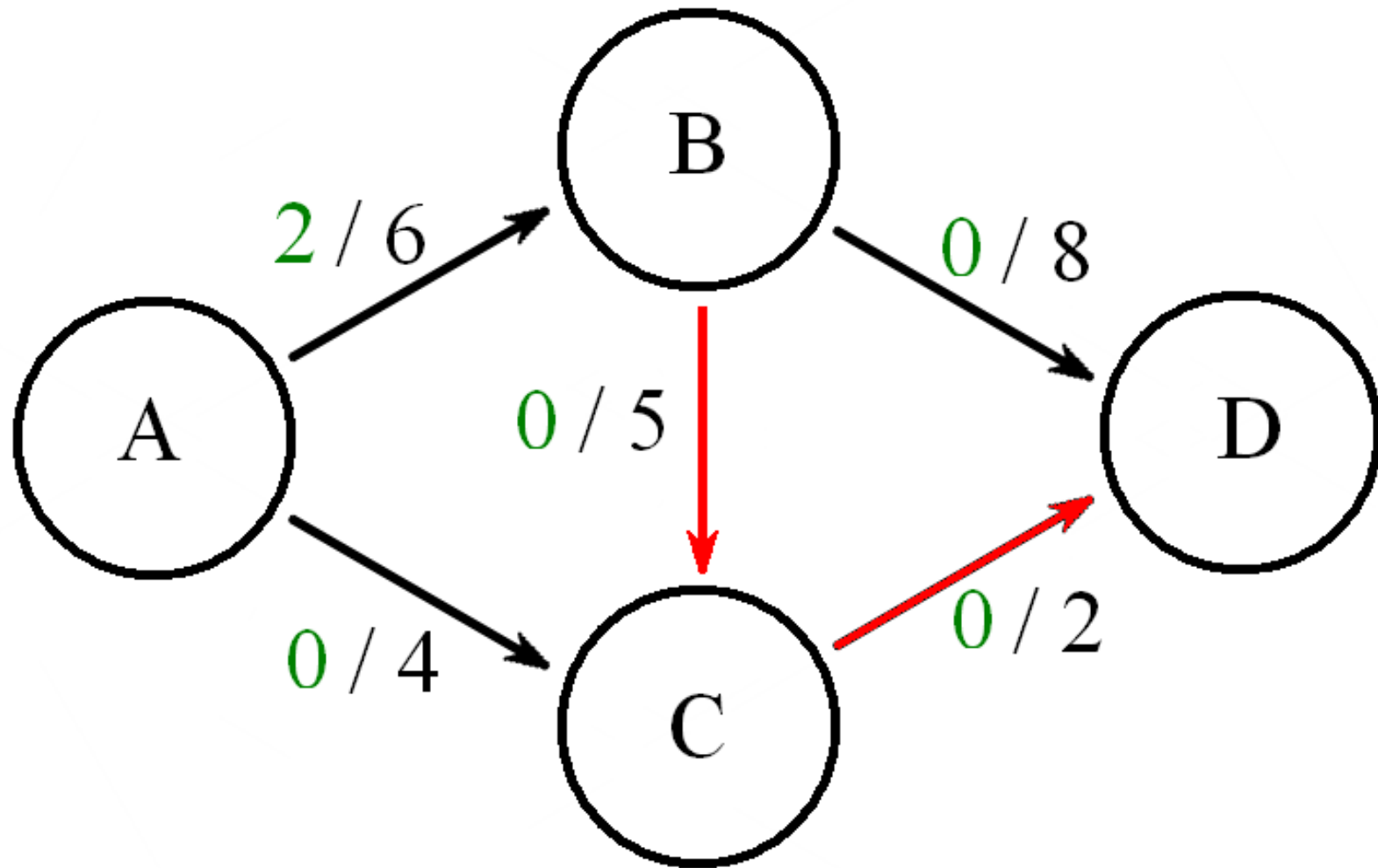
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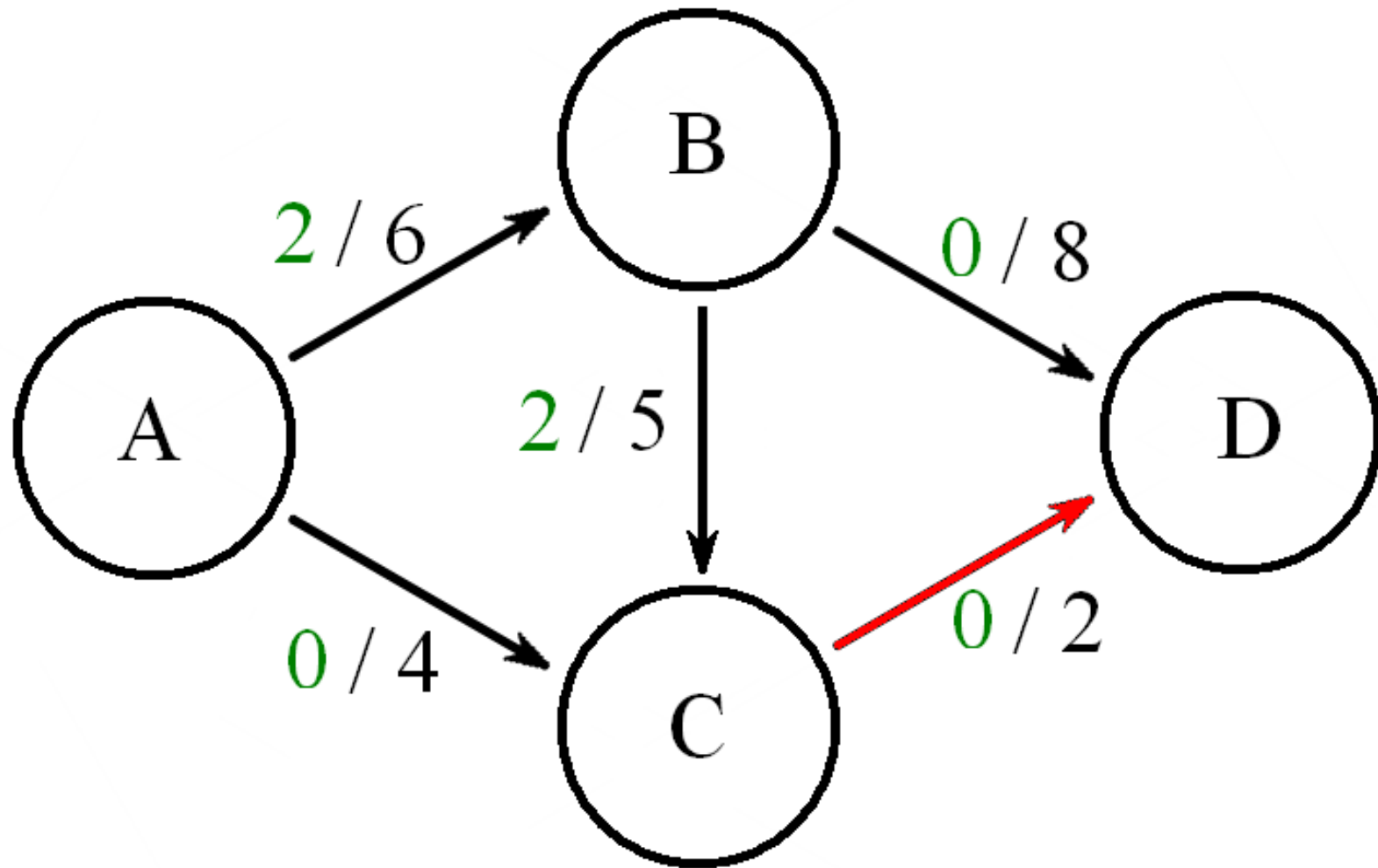
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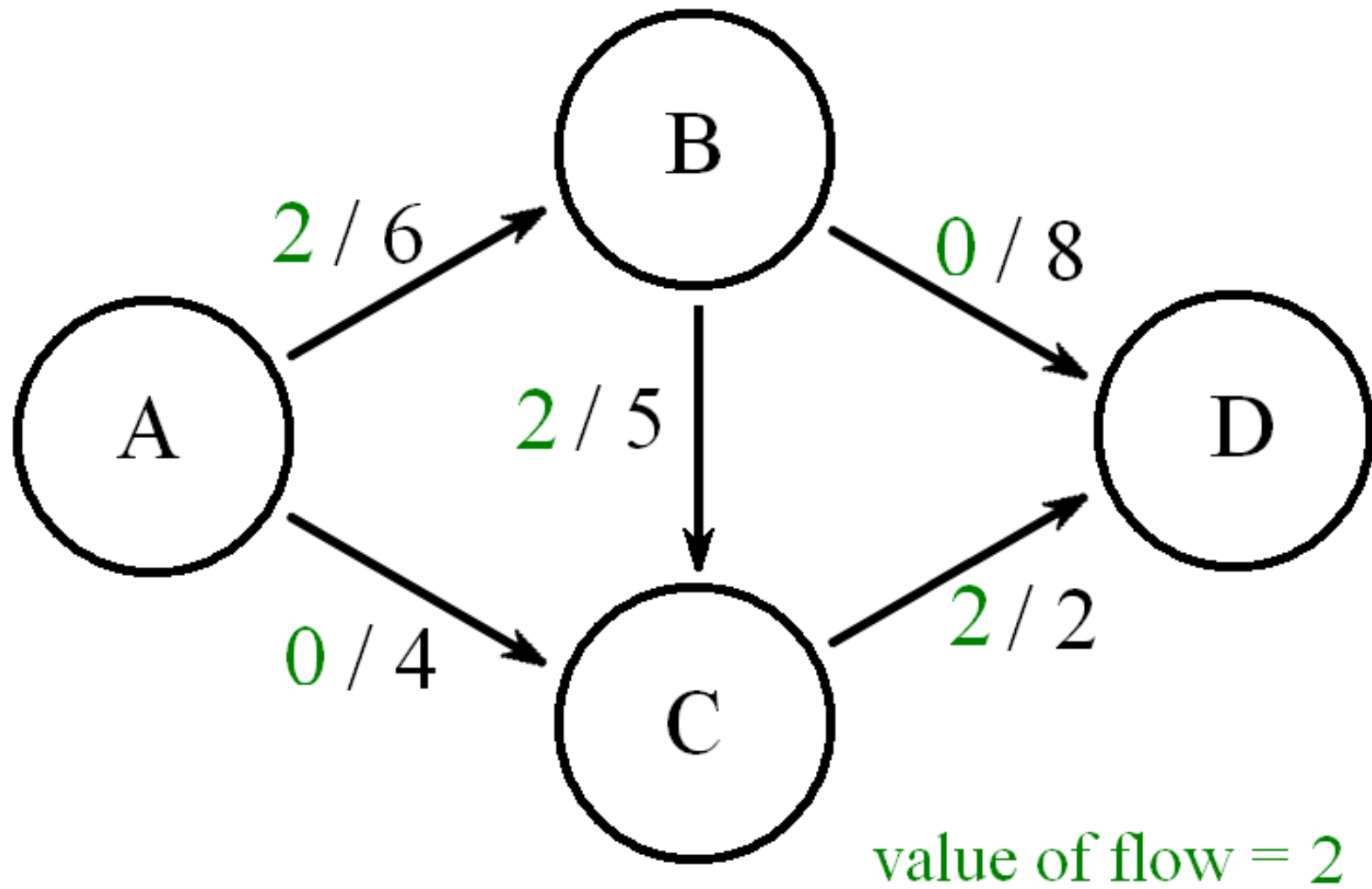
(2) Increase flow along path



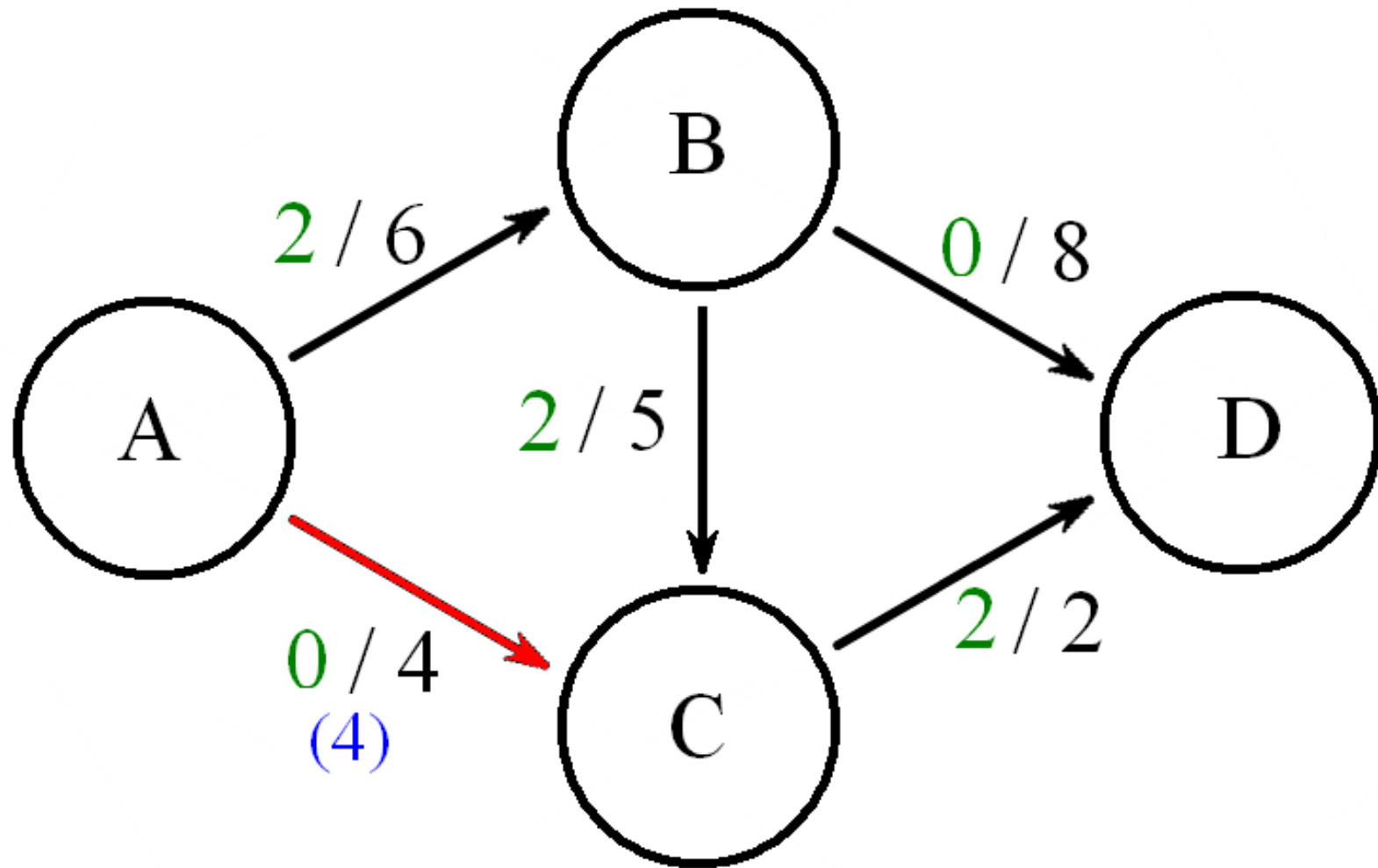
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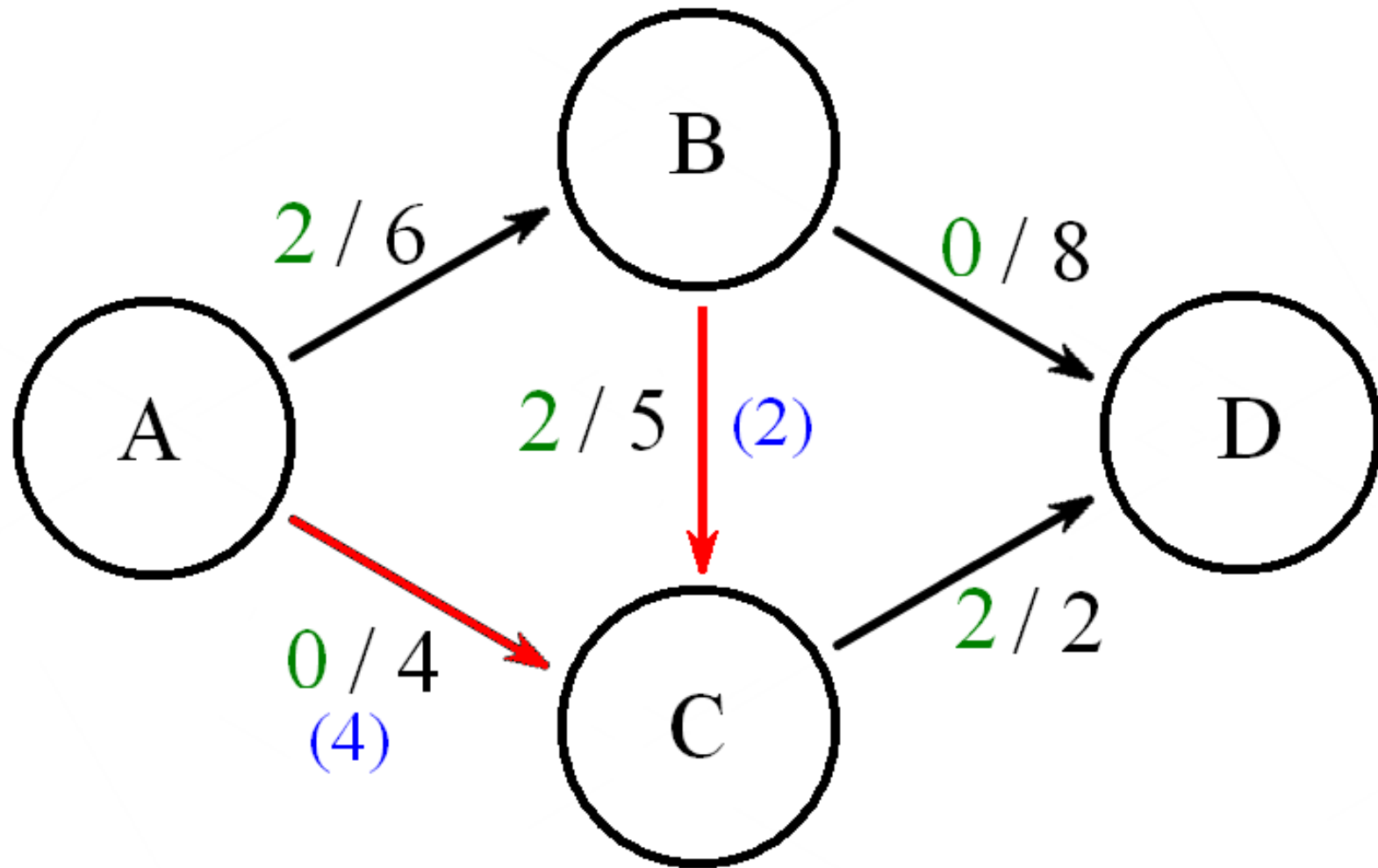
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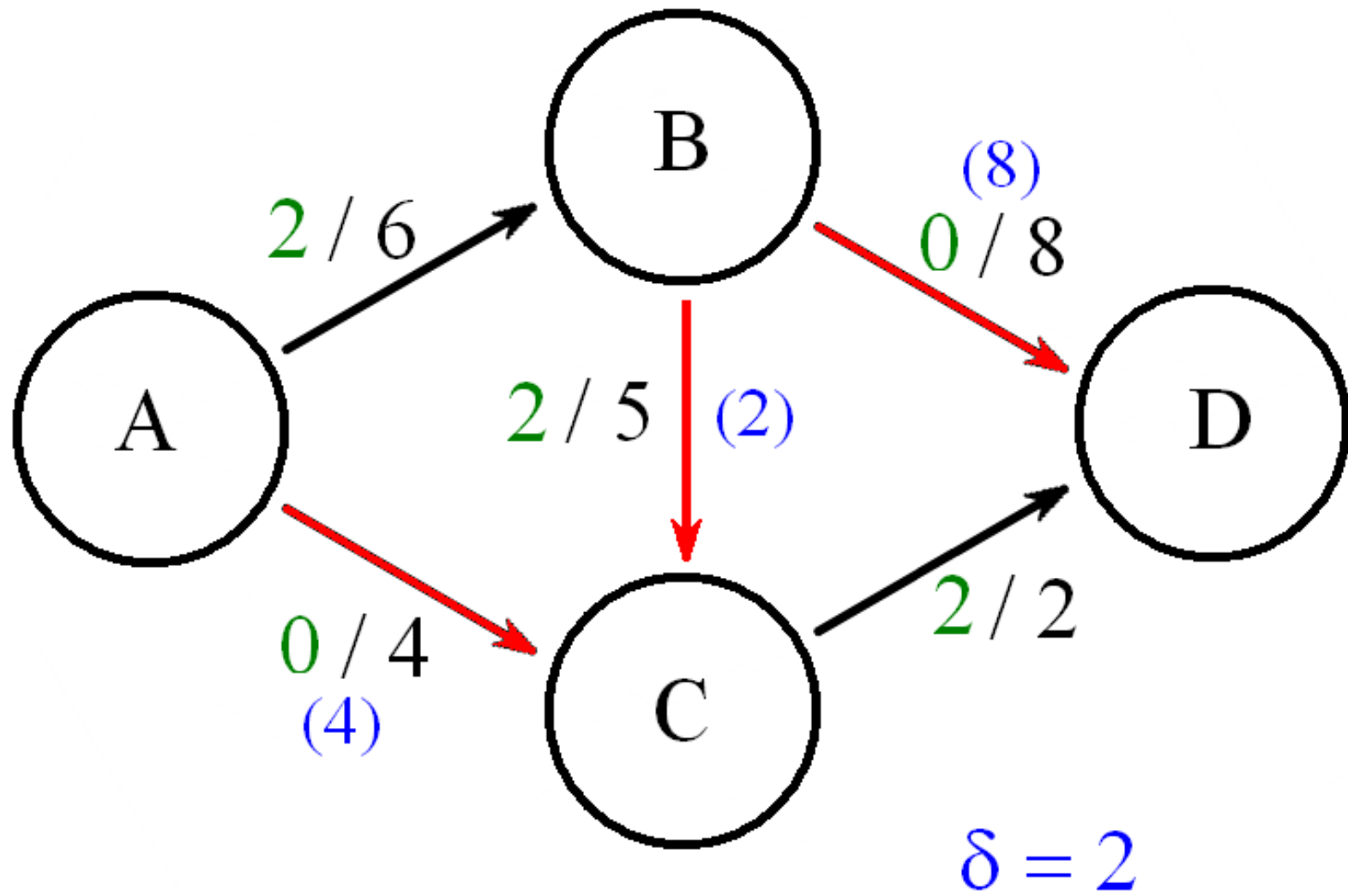
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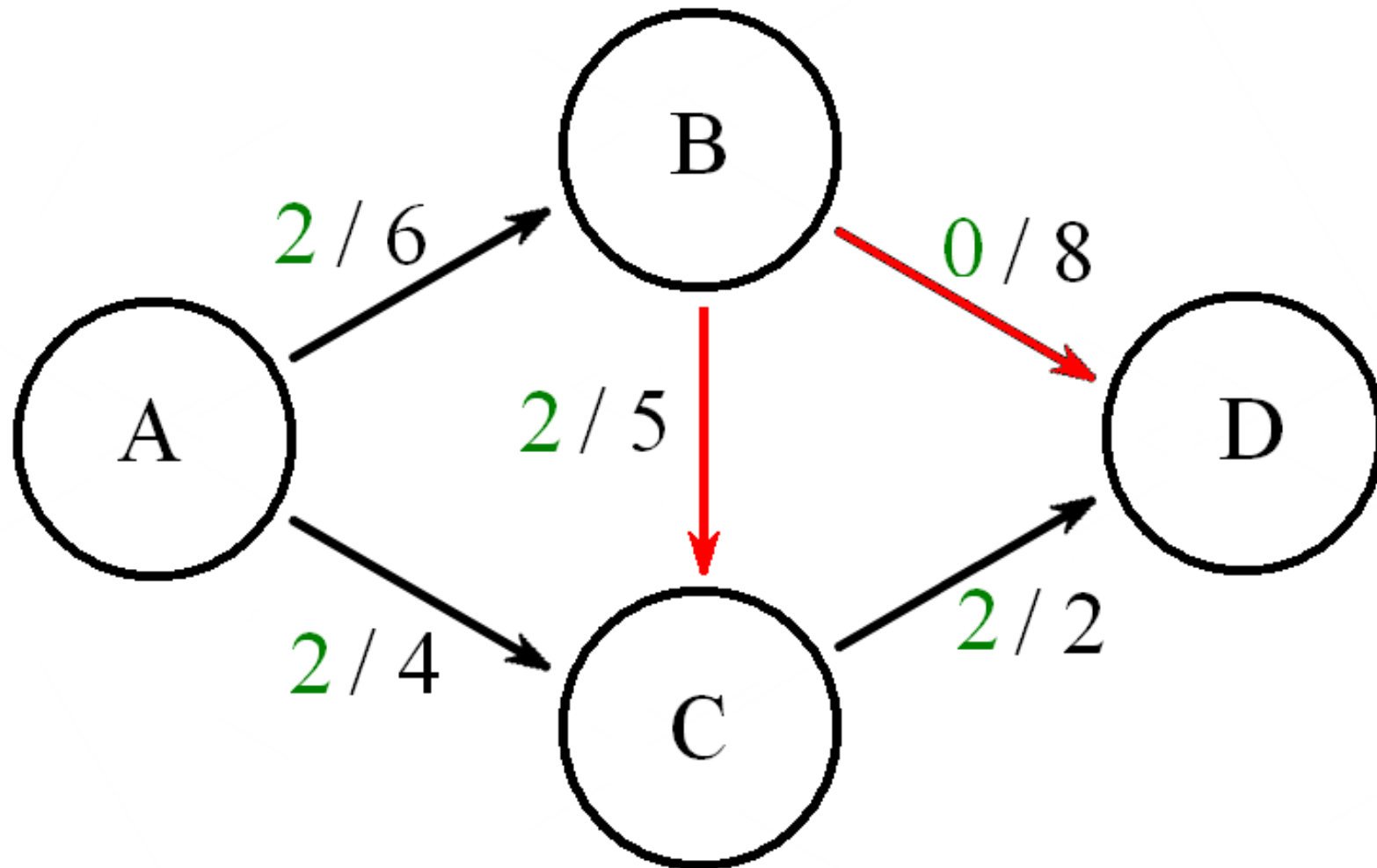
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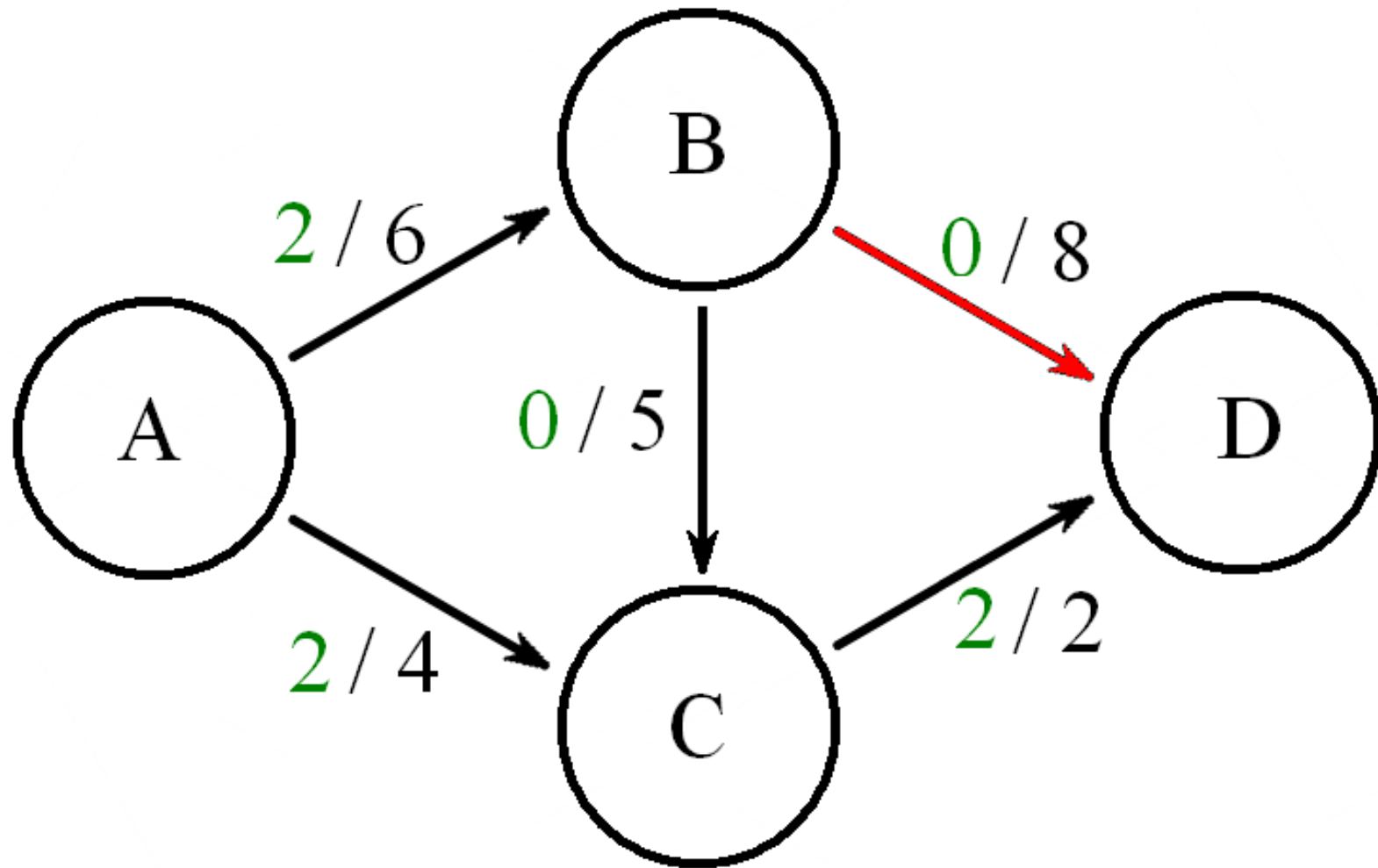
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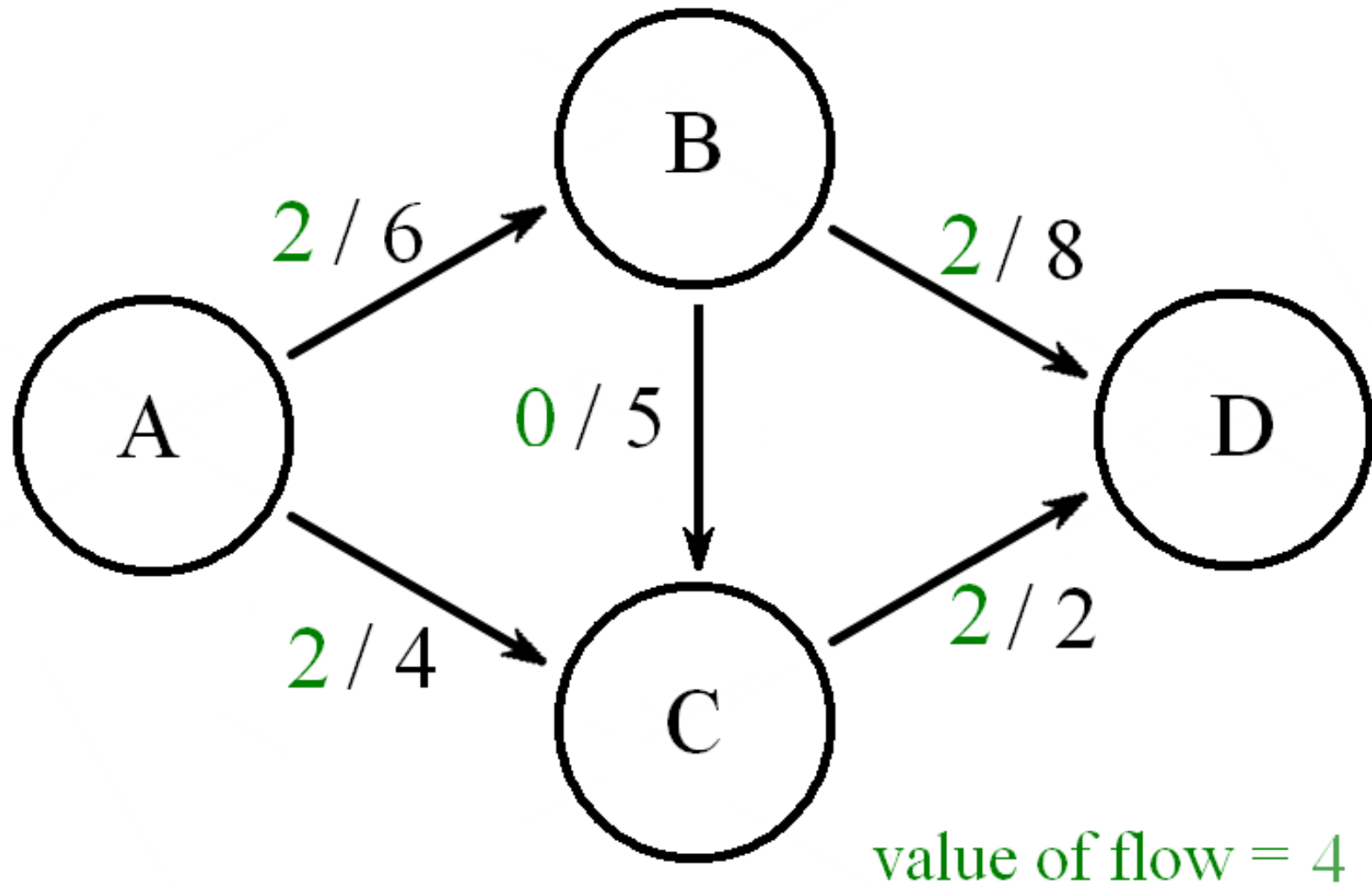
(2) Increase flow along path



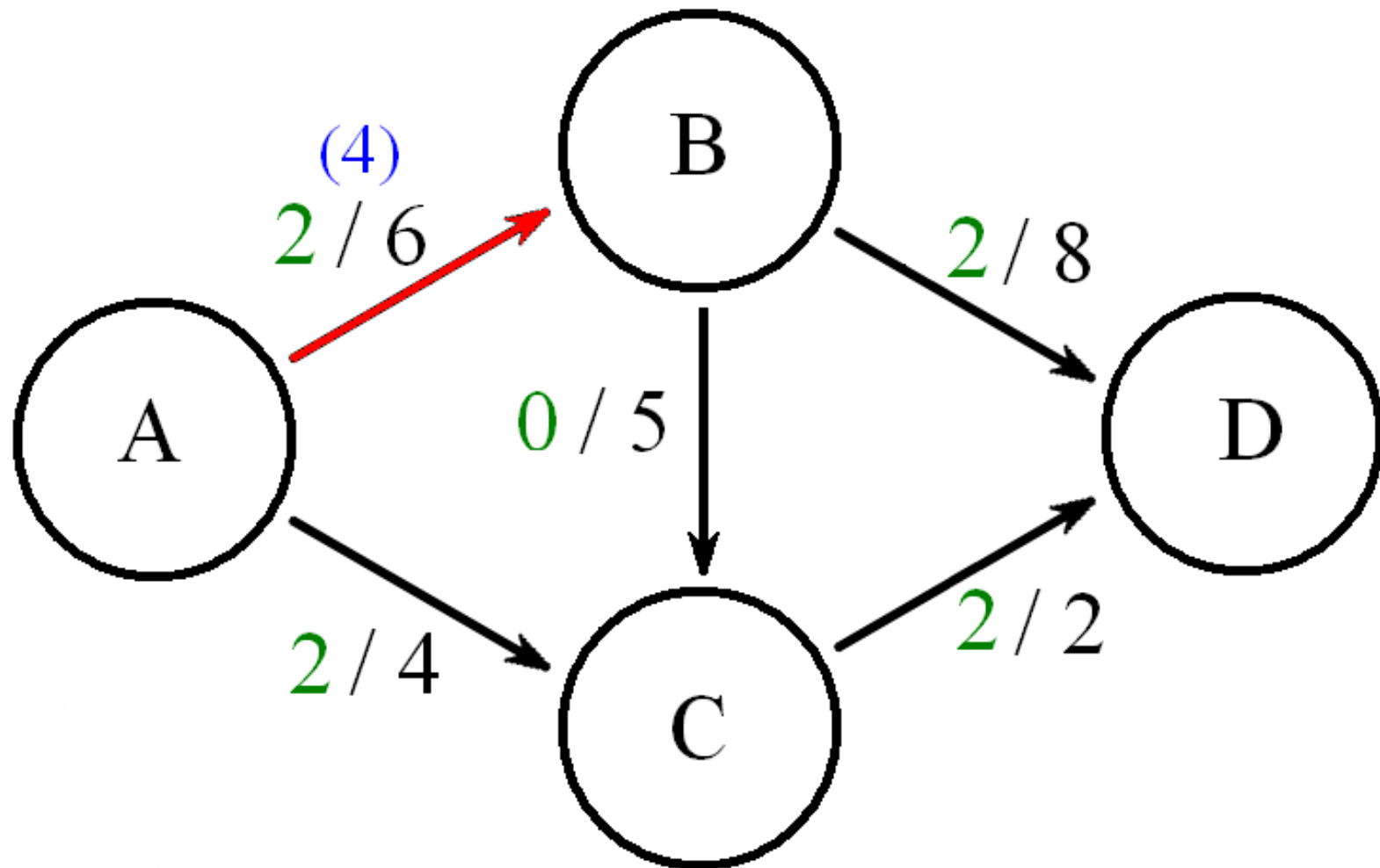
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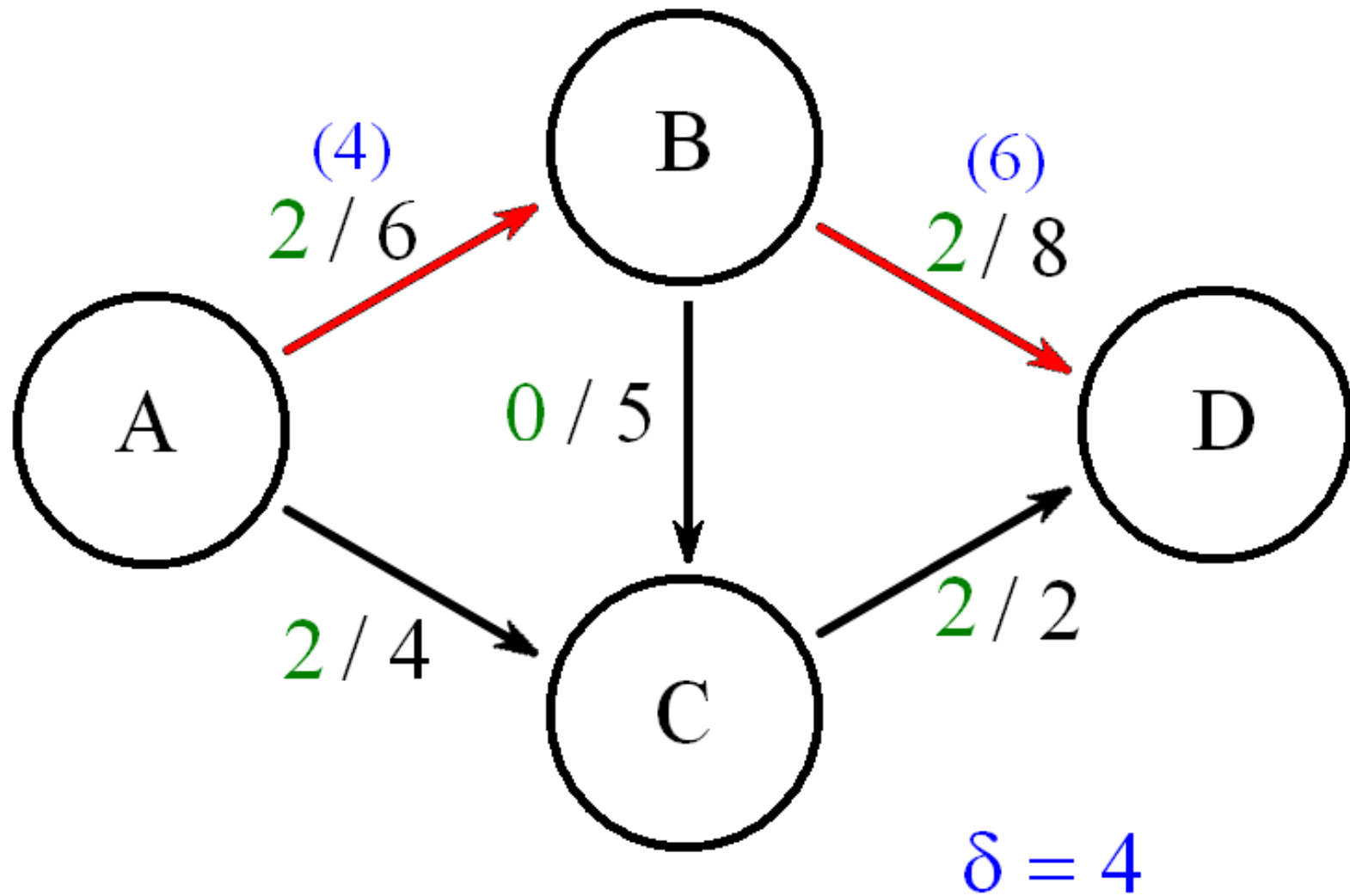
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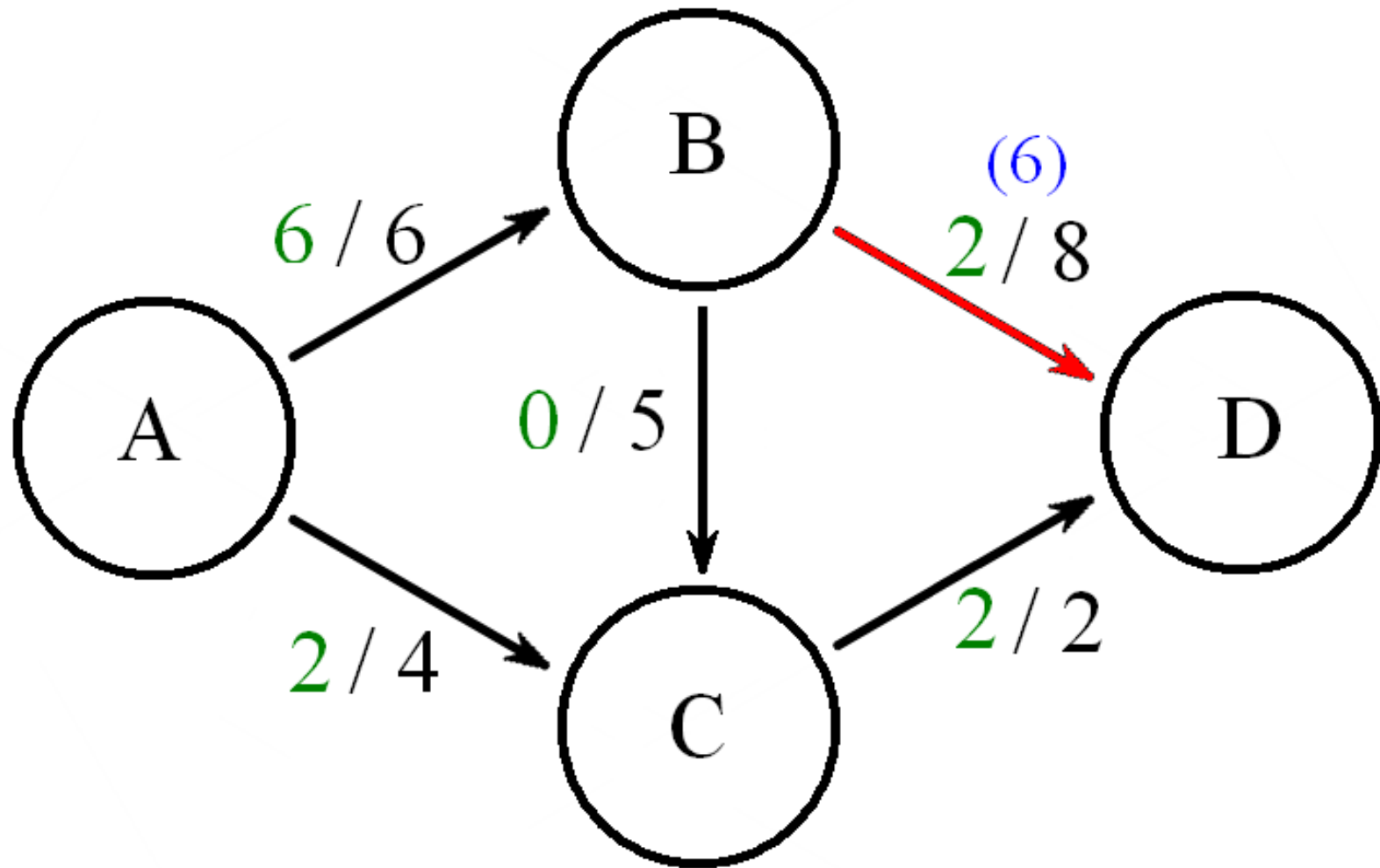
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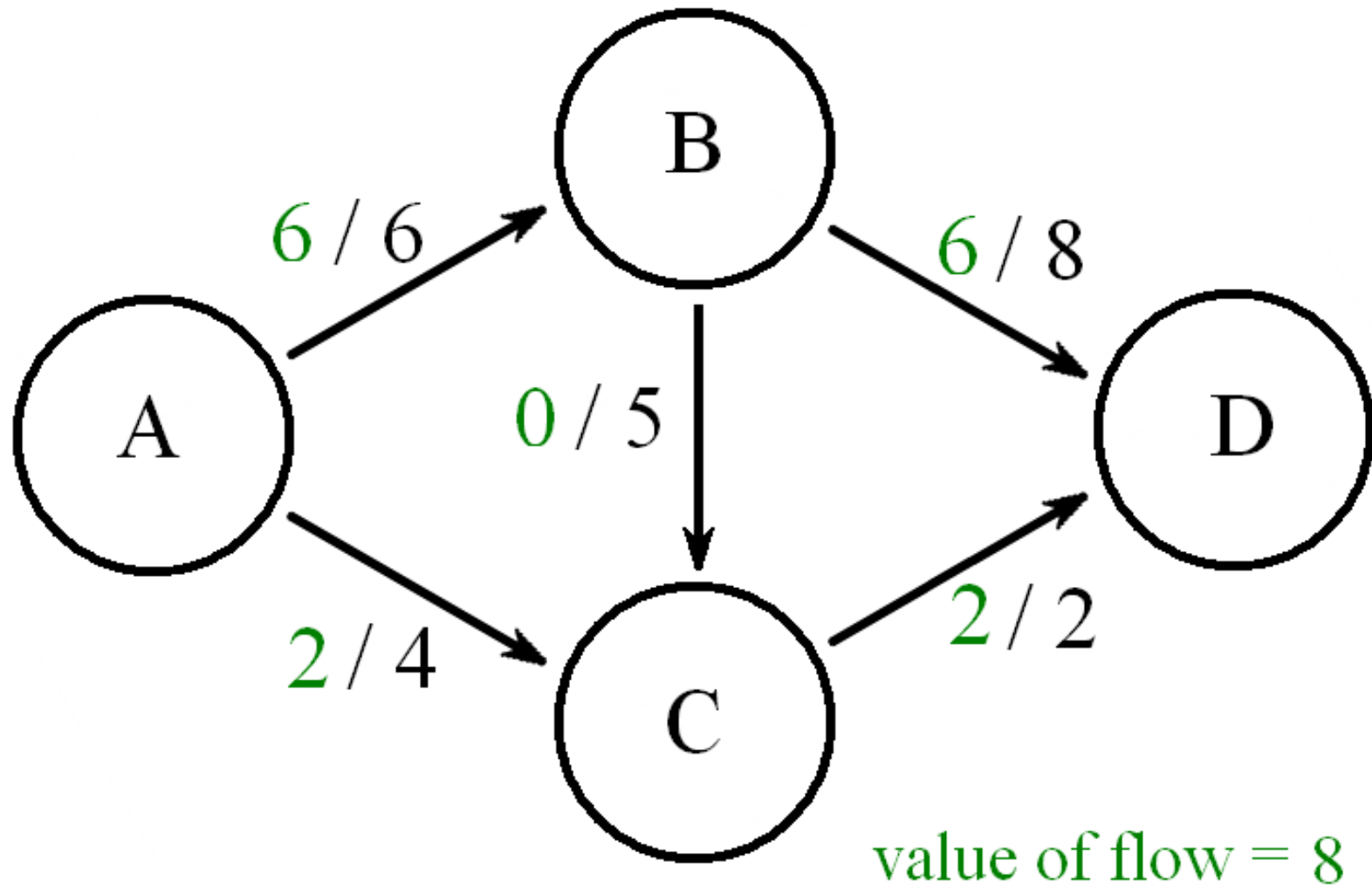
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(2) Increase flow along path



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The minimal cut

