Stochastic Calculus and Applications (L24)

M. Tehranchi

This course is an introduction to the theory of continuous-time stochastic processes, with an emphasis on the central role played by Brownian motion. It complements the material in Advanced Probability, Advanced Financial Models, and Schramm–Loewner Evolutions.

- *Review of Brownian motion.* Isonormal process. Wiener's existence theorem. Sample path properties.
- *Continuous stochastic calculus.* Martingales, local martingales and semi-martingales. Quadratic variation and co-variation. Itô's isometry and definition of stochastic integral. Kunita–Watanabe's theorem. Itô's formula.
- *Applications to Brownian motion.* Lévy's characterization of Brownian motion. Dubins–Schwartz theorem. Girsanov's theorem. Transience and recurrence. Martingale representation theorems.
- *Stochastic differential equations.* Strong and weak solutions. Notions of existence and uniqueness. Yamada–Watanabe theorem. Strong Markov property. Kolmogorov, Fokker–Planck and Feynmann–Kac partial differential equations. The one-dimensional case. Stochastic partial differential equations.

Pre-requisites

Knowledge of measure theoretic probability at the level of Part III Advanced Probability will be assumed, especially familiarity with discrete-time martingales and basic properties of Brownian motion.

Literature

1. I. Karatzas and S. Shreve. *Brownian Motion and Stochastic Calculus.* Springer. 1998
2. D. Revuz and M. Yor. *Continuous martingales and Brownian motion.* Springer. 2001
3. L.C. Rogers and D. Williams.*Diffusions, Markov Processes and Martingales. Vol.1 and 2.* Cambridge University Press. 2002

Additional support

Four sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.