Correlation scenarios and correlation stress testing

Natalie Packham

joint work with Fabian Woebbeking

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Hochschule für
Wirtschaft und Recht Berlin
Berlin School of Economics and Law

Correlation stress testing

- **What would you consider to be the main challenges in correlation stress testing?**

Correlation stress testing

- **What would you consider to be the main challenges in correlation stress testing?**
  - Obtaining a mathematically valid correlation matrix.
  - Specifying plausible scenarios.
  - Identifying risk factors.
  - Linking risk factors to correlations.
  - More advanced dependence measures, such as copulas, should be used.
  - Other.

Overview

- **Correlation** lies at the heart of many financial applications: portfolio risk-management, diversification, hedging.
- Principal idea: link **economically meaningful scenarios** to **correlation scenarios**
- First paper ("London Whale"):

  Packham, N. and Woebbeking, F.: *A factor-model approach for correlation scenarios and correlation stress-testing*. Journal of Banking and Finance, 101 (2019), 92-103. link

Overview

- Extend the previous setup:
  - **Correlation factor model** for any kind of financial asset portfolio
  - **Bayesian factor selection** to incorporate a priori knowledge
  - **Stress testing**: portfolio effect of adverse correlation scenarios
  - **Reverse stress testing**: identify extreme yet plausible scenarios
- Second paper:

  Packham, N. and Woebbeking, F.: *Correlation scenarios and correlation stress testing* . Journal of Economic Behavior and Organization, 205 (2023), 55-67. link

Regulatory aspects

- EU / Basel-regularion (CRR = Capital Requirments Regulation):
  - CRR Article 386(1)(g):
    "[..]institution shall frequently conduct a rigorous programme of stress testing, including reverse stress tests[..]"
  - CRR Article 375(1):
    "[..]potential for significant basis risks in hedging strategies[..]"
  - CRR Article 376(3)(b):
    "[..] assess [..] internal model, particularly with regard to the treatment of concentrations."
  - CRR Article 377:
    "Requirements for an internal model for correlation trading"

Overview

The “London Whale”

- **“London Whale”**: 2012 Loss at JPMorgan Chase & Co. of approx. 6.2 bn USD on a credit derivatives portfolio
- **Authorised trading position**, hence risk management problem
- **Synthetic credit portfolio (SCP)**: portfolio of credit index derivatives to manage credit risk
- Approx. 120 long and short positions, **CDX** and **iTraxx** index + tranche products, investment grade and high-yield
- Roughly 157 bn USD peak net notional
- JPMorgan is naturally exposed to (long) credit risk, hence SCP as “Tail hedge to protect the firm against adverse credit scenarios”

The "London Whale" strategy

- **"Smart short" strategy**: credit protection on high yield is financed by selling protection on investment grade indices.
- Timeline:
  - End of 2011: decision to reduce SCP's risk-weighted assets (RWA's).
  - Avoid liquidation losses by **increasing** positions with opposite market sensitivity (hedges).
  - 23 March 2012: Senior executives ordered to stop trading on SCP; net notional of 157 bn USD (up 260% from September 2011).
- **Risk management** of SCP focussed on **value-at-risk (VaR)** and **CSW-10** (credit spread widening of 10 basis points).
- Publicly available information: JPMorgan, 2013; United-States-Senate, 2013a,b

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The “London Whale”

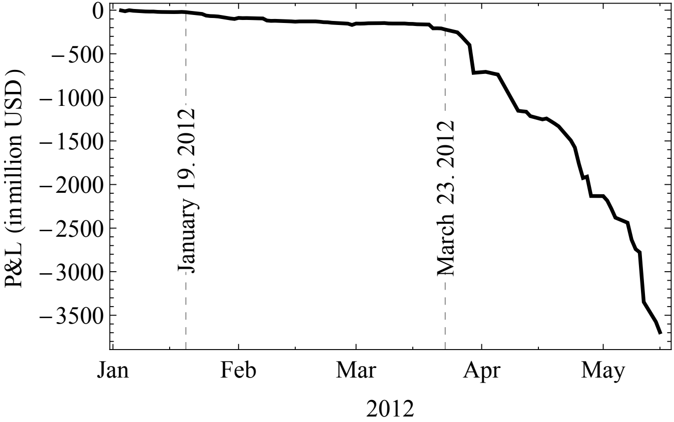


Figure: Cumulative PnL of the SCP in USD (2012). Single day loss of 50mln USD on 19 January 2012, due to Kodak default. Phones down on March 23, 2012. Data source: JPMorgan (2013).

The “London Whale” positions

Table: Top 10 Positions of SCP, 23 March 2012, USD net notional; several positions have a market share close to 50%.

	Index				
Name	Series	Tenor	Tranche (%)	Protection	Net Notional ($)
CDX.IG	9	10yr	Untranched	Seller	72,772,508,000
	9	7yr	Untranched	Seller	32,783,985,000
	9	5yr	Untranched	Buyer	31,675,380,000
iTraxx.EU	9	5yr	Untranched	Seller	23,944,939,583
	9	10yr	22 – 100	Seller	21,083,785,713
	16	5yr	Untranched	Seller	19,220,289,557
CDX.IG	16	5yr	Untranched	Buyer	18,478,750,000
	9	10yr	30 – 100	Seller	18,132,248,430
	15	5yr	Untranched	Buyer	17,520,500,000
iTraxx.EU	9	10yr	Untranched	Seller	17,254,807,398
Net Total					137,517,933,681

Data source: United-States-Senate (2013a, Exhibit 36) and DTCC (2014, Section 1, Table 7).

Overview

Interest-rate modelling: Correlation parameterisation

- Parametric correlation models widespread in

  interest-rate modelling / LIBOR market model,

  e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2000); Packham (2005)

- Simplest case: Correlation c_{ij} between two forward LIBOR's is given by

$$c_{ij} = e^{-\beta|i-j|},$$

  where $\beta > 0$ is a parameter, and i, j represent maturities.

- Captures stylised fact that **correlations decay with increasing maturity difference**

Correlation parameterisation

- Idea: Carry over **"distance" measure** to other **risk factors**, such as geographic regions, industries, investment grade vs. high-yield, ...
- C: $n \times n$-correlation matrix of n financial instruments' returns.
- Factors that determine the correlations: $\mathbf{x} = (x^1, \dots, x^m)'$.
- Correlation of securities i and j modelled as

$$c_{ij} = \exp(-(\beta_1|x_i^1 - x_j^1| + \beta_2|x_i^2 - x_j^2| + \cdots + \beta_m|x_i^m - x_j^m|)), \qquad i, j = 1, \dots, n,$$

  with $\beta_1, \dots, \beta_m$ positive coefficients, determined through calibration.
- Functional form implies that the greater "distance" $|x_i^k - x_j^k|$, the greater de-correlation amongst securities i and j.
- If two instruments are identical in all respects, then correlation is 1.

Correlation parameterisation

- Given historical asset returns, parameters $\beta_1, \ldots, \beta_m$ are determined e.g. by **OLS** on transformed correlations $-\ln(c_{ij})$.
- **Scenario** (e.g. "the correlation between investment grade and high-yield securities decreases") is implemented by **increasing corresponding β-parameter**.
- With parameters calibrated on a regular basis, the **parameter history** can be used to obtain reasonable scenarios.

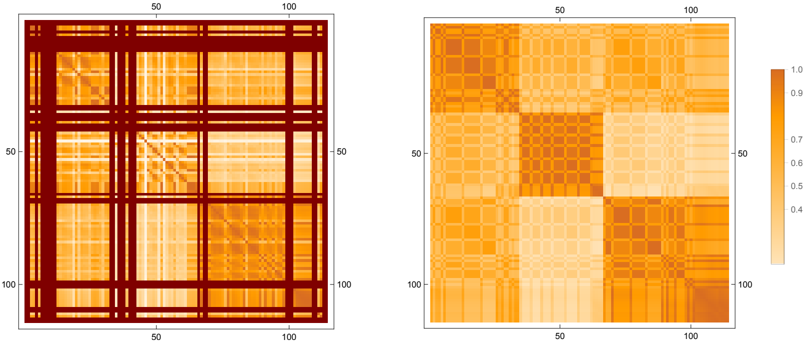
London whale: risk factors and correlation model

- All calculations on SCP portfolio of 23 March 2012 (117 instruments).
- Risk factors:
  - CDX vs. iTraxx
  - investment grade vs. high yield
  - maturity
  - index series
  - index vs. tranche
- Parameterised correlation matrix:

$$c_{ij} = \exp\big(-(\beta_1|\mathsf{isCDX}_i - \mathsf{isCDX}_j| + \beta_2|\mathsf{isIG}_i - \mathsf{isIG}_j| + \beta_3|\mathsf{maturity}_i - \mathsf{maturity}_j| + \beta_4|\mathsf{series}_i - \mathsf{series}_j| + \beta_5|\mathsf{isIndex}_i - \mathsf{isIndex}_j|)\big).$$

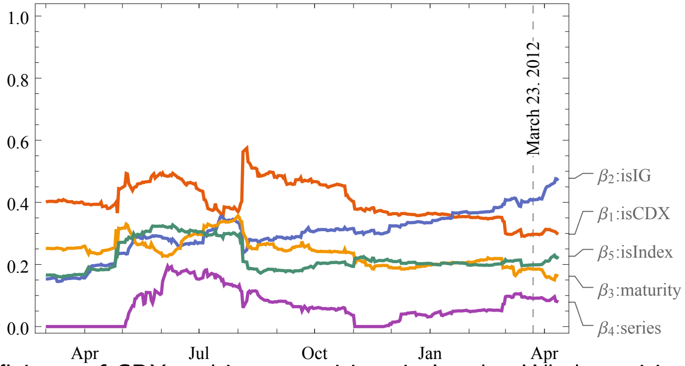
- Daily calibration of $\beta_1, \ldots, \beta_5$ from credit spread returns of 250 days.
- Time period: 1 March 2011 – 12 April 2012. Data source: Markit

London Whale: calibration and results



- Correlation matrices of 23 March 2012.
- Left: Empirical correlation matrix
- Right: parameterised (complete) correlation matrix
- Dark red entries: unavailable correlations
- Blocks of highly correlated data: CDX.IG, CDX.HY and iTraxx

London Whale: calibration and results



- Coefficients of CDX and itraxx positions in London Whale position; 01/03/2011–12/04/2012.
- Distances normalised to $[0, 1]$ to make coefficients comparable.
- (See also (Cont and Wagalath, 2016) who report a correlation break-down after trading was halted.)

Overview

Stress-testing correlations

- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by **value-at-risk (VaR)** in variance-covariance approach:

$$\mathrm{VaR}_\alpha = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \left(\mathbf{w}^\intercal \, \boldsymbol{\Sigma} \, \mathbf{w}\right)^{1/2},$$

  with
  - current position value V_0,
  - $\mathrm{N}_{1-\alpha}$: $(1-\alpha)$-quantile of the standard normal distribution,
  - vector of portfolio weights $\mathbf{w}$ and
  - covariance matrix $\boldsymbol{\Sigma}$.
- For **correlation stress test**, need to consider portfolio variance

$$\mathbf{w}^\intercal \, \boldsymbol{\Sigma} \, \mathbf{w}.$$

Core and peripheral risk factors*

- Following e.g. Kupiec (1998), **stress scenario** comprises
  - **"core"** risk factors (the ones that are stressed)
  - **"peripheral"** risk factors (affected by stress).
- $\boldsymbol{\beta}_s$: $j < m$ core factor parameters that are stressed directly
- $\boldsymbol{\beta}_u$: remaining $m - j$ peripheral risk factor parameters
- In **normal distribution setting**, optimal estimator of $\Delta\boldsymbol{\beta}_u$ conditional on $\Delta\boldsymbol{\beta}_s$:

$$\mathbb{E}(\Delta\boldsymbol{\beta}_u|\Delta\boldsymbol{\beta}_s) = \Sigma_{us}\Sigma_{ss}^{-1}\Delta\boldsymbol{\beta}_s,$$

  where Σ_{us} and Σ_{ss} denote the covariance and variance matrices of $\boldsymbol{\beta}_u$ and $\boldsymbol{\beta}_s$.

Joint stress test of correlation and volatility*

- **Correlation shocks** often coincide with **volatility shocks**, see e.g. (Alexander and Sheedy, 2008; Longin and Solnik, 2001; Loretan and English, 2000).
- Simple model that combines both: **multivariate t-distribution**.
- In this case d-dimensional vector of asset returns $\mathbf{X}$ follows a **normal variance mixture distribution** with decomposition (e.g. Ch. 6.2 of McNeil *et al.* (2015))

$$\mathbf{X} = \sqrt{V} \cdot A \cdot \mathbf{Z},$$

where
  - $\mathbf{Z} \sim \mathrm{N}(0, I_k)$,
  - V is a scalar r.v. independent of $\mathbf{Z}$,
  - $V \sim \mathsf{Ig}(1/2\,\nu, 1/2\,\nu)$, i.e., V follows an inverse gamma distribution,
  - A is a $d \times k$ matrix such that $\tilde{\mathbf{\Sigma}} = AA^T$.

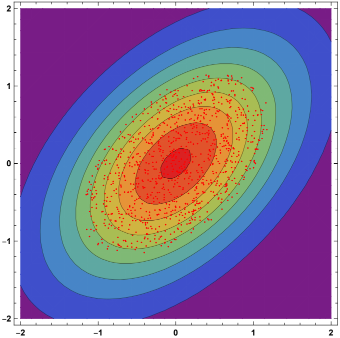
Overview

Reverse stress testing

- **Scenario selection**: What is the worst scenario amongst all scenarios that occur within some pre-given probability?
- Let $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_m)^\intercal$ be a random vector with $\mathbb{E}(\boldsymbol{\beta}) = \overline{\boldsymbol{\beta}}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$.
- **Mahalabonis distance**:

$$D(\boldsymbol{\beta}) = \left((\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^\intercal \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} (\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})\right)^{1/2}.$$

- Maha associated with ellipsoids in normal (or elliptical) distributions.
- Find worst-case scenario within given ellipsoid.



Risk implications from correlation stress-testing

	correlation stress			plus vol stress	
Maha level	$\text{VaR}_{0.99}$	t-$\text{VaR}_{0.99}$	Change(%)	t-$\text{VaR}_{0.99}$	Change(%)
base case	339.32	354.98		354.98	
0.9	372.89	390.10	9.89	464.40	30.83
0.99	381.08	398.67	12.31	617.38	73.92
0.999	386.88	404.74	14.02	780.37	119.84
unconstrained*	620.96	649.62	83.00	1252.53	252.85

*Unconstrained w.r.t. correlation changes; vol stress level at 0.999.

- SCP portfolio's 1-day 99% value-at-risk for different Mahalanobis quantile constraints.
- Percentage changes denote relative distance to base VaR. For joint stress, percentage changes refer to base t-VaR scenario.
- t-distribution parameter ν calibrated to 13.5.
- Vol stress level for joint stress test is set to quantile in column one.

Risk-driver identification (reverse stress test)

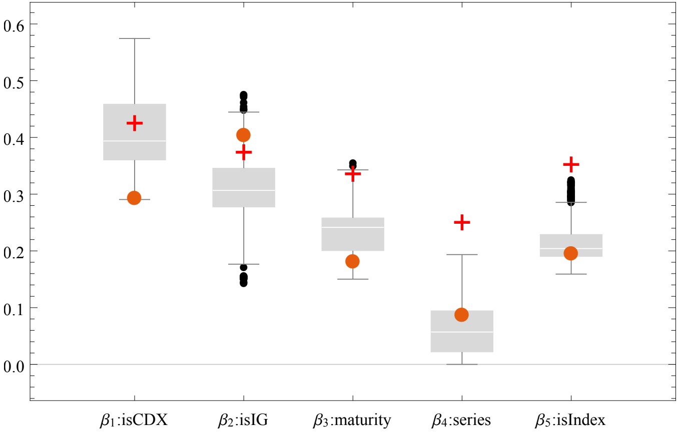


Figure: Box-plots of correlation parameters.

Dots: observed parameters as of 23.03.2012.

Crosses: worst-case scenario under a 99%-quantile Mahalanobis distance.

Overview

Link correlations to risk factors

- Idea: Carry over **"distance" measure** to other **risk factors**, such as geographic regions, industries, investment grade vs. high-yield, ...
- Association of asset $i \in \{1, \ldots, p\}$ with factor $k \in \{1, \ldots, d\}$:

$$\mathbf{1}_{\{k,i\}}$$

[Assume this as given for the time being.]

- Correlation parameterisation:

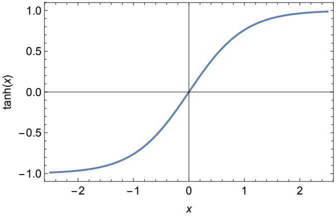
$$c_{ij} = \tanh\Big(\eta + \underbrace{\sum_{k=1}^{d} \lambda_k |\mathbf{1}_{\{k,i\}} - \mathbf{1}_{\{k,j\}}|}_{\text{"inter"-correlations}} + \underbrace{\sum_{k=1}^{d} \nu_k \mathbf{1}_{\{k,i\}} \mathbf{1}_{\{k,j\}}}_{\text{"intra"-correlations}}\Big),$$

with coefficients $\eta, \lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d \in \mathbb{R}$.

Link correlations to risk factors

- tanh : $\mathbb{R} \to [-1, 1]$ allows for negative correlations.
- tanh used in inferential statistics on sample correlation coefficients ($\rightsquigarrow$ Fisher transformation).
- The following summation formula is helpful for a rough interpretation of the coefficients:

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$



Correlation parameterisation

- Given a sample correlation matrix at one time point, the coefficients $\eta, \lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d$ can be determined e.g. by **ordinary least squares** on $\text{arctanh}(c_{ij})$, the inverse of $\tanh$.
- Simple correlation **scenarios** such as "the correlation between assets exposed to factor k and assets not exposed to factor k increases" is then implemented by increasing λ_k (e.g. Europe vs US).
- Likewise, a scenario such as "the correlation of firms exposed to factor k increases" is implemented by increasing ν_k (e.g. within Europe).
- With parameters calibrated on a regular basis, the parameter history can be used to **obtain realistic scenarios** (reverse stress testing).

Overview

Principal ideas

- Risk factors in "London Whale" were tailored to specific portfolio.
- In practice, factor models use **industries** and **countries** as factors to model asset correlations.
- Problem: **How to assign factors to assets?** link
- Number of factors should be **small**, but include all **important** factors.
- **Prior information**: country of firm's headquarter, primary industry
- $\rightsquigarrow$ **Bayesian variable selection** to determine small number of factors driving asset return

Bayesian variable selection

- Different methods, e.g.
  - **Bayesian model selection** compares **posterior probabilities** of different models.
  - **Spike and slab priors** include an indicator variable for each coefficient and determines the indicator variable's **posterior probability** of taking value one.
- In our setting, **Bayesian model selection** worked best.

Bayesian model selection

- Denote candidate models by M_i, $i = 1, \ldots, m$.
- In a linear regression setting, each model M_i includes a specific subset of independent variables ($=$ potential risk factors) and excludes the other variables.
- **Posterior model probability**:

$$p(M_i|\boldsymbol{y}) \propto p(\boldsymbol{y}|M_i)p(M_i),$$

where

- $\boldsymbol{y}$ is the time series of a **firm's asset returns**,
- $p(M_i)$ is the **prior model probability**,
- $p(\boldsymbol{y}|M_i)$ is called the **marginal likelihood**.

(see e.g. Appendix B.5.4 of (Fahrmeir *et al.*, 2013))

Bayesian model comparison

- **Posterior inclusion probabilities (PIP)**:

$$\mathbf{P}(\mathbf{1}_{\{\beta_k \neq 0\}} = 1|\boldsymbol{y}) = \sum_{\beta_k \in M_i} \mathbf{P}(M_i|\boldsymbol{y}).$$

- If number of parameters p is large, then full calculation of 2^p posterior model probabilities is infeasible.
- $\Rightarrow$ Use **Markov Chain Monte Carlo (MCMC)** simulation.
- Factors with PIP greater 0.5 are selected

Example: VW

- Daily returns (2002-2018):
  - VW stock returns
  - MSCI indices; 11 industries and 24 countries as factors
- Factors with PIP greater 0.5 are selected:

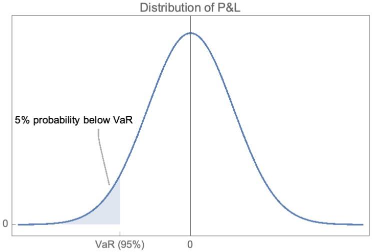
```
>>> print(res[res['PIP']>0.5].round(4))
              coef     PIP     BVS  pvalue
4   MXWO0CD Index  1.0000  1.0000  0.0000
9   MXWO0TC Index  0.9848  0.9900  0.0017
10  MXWO0UT Index  0.9996  1.0000  0.0000
18         MSDUSZ  0.6788  0.4940  0.0105
19         MSDUAT  0.7998  0.7613  0.0000
34         MSDUGR  1.0000  1.0000  0.0000
```

- CD (Consumer Discretionary) and GR (Germany) have prior inclusion probability of 1.
- Other prior inclusion probabilities such that eight factors on average.

Overview

Stress-testing correlations

- As before, use Value-at-risk. link
- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by **value-at-risk (VaR)** sensitive to portfolio variance, which depends on **correlations**.
- **VaR** at level α (e.g. 95%) is the maximum amount that can be lost with a probability of α. Only with a probability of $1-\alpha$, losses exceed VaR.



Reverse stress testing

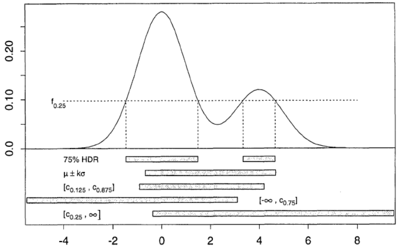
- What is the **worst scenario** amongst all scenarios that occur within some **pre-given range**?
- Restrict **risk-factor distribution** $(\eta, \lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)$
- Univariate setting: quantile
- Multivariate setting:
  - Mahalanobis distance (Mahalanobis, 1936),
  - highest density regions (HDR) (Hyndman, 1996a),
  - concepts based on norms, e.g.(Serfling, 2002).
- Maha is closely tied to the normal or to elliptical distributions.
- HDR allows for more flexibility (e.g. skewness and tail heaviness).

Highest density region (HDR)

- Let $f(x)$ be the density function of a random vector X
- The $100(1-q)\%$ HDR is the subset of $R(f_q)$ of the sample space of X such that

$$R(f_q) = \{x : f(x) \geq f_q\}$$

where f_q is the largest constant such that $\mathbf{P}(X \in R(f_q)) \geq 1 - q$.



(Hyndman, 1996b)

- **Worst-case** scenario within given HDR:

$$\boldsymbol{\beta}^* = \underset{\{\boldsymbol{\beta} \in R(f_q)\}}{\operatorname{argmax}} \ \mathsf{VaR}_\alpha(\boldsymbol{\beta}).$$

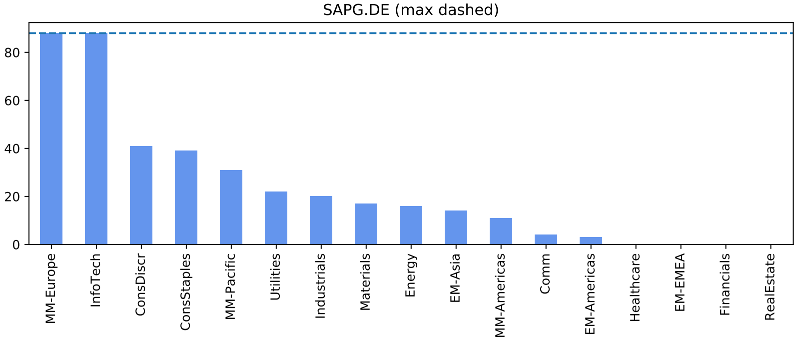
Overview

Factor selection

- Factors: MSCI stock indices representing 11 industries and 6 regional indices (3 mature, 3 EM markets)
- Individual stocks: 505 S&P constituents, 30 DAX constituents
- Daily data from 1999-2018 (Source: Bloomberg, MSCI, Refinitiv Eikon)
- Factor assignment re-calibrated every quarter, based on 3-years of daily data (88 quarters)
- Prior: hard-code primary country and industry; include 6 factors on expectation
- All other calculations are conducted daily on a rolling time window of 250 trading days

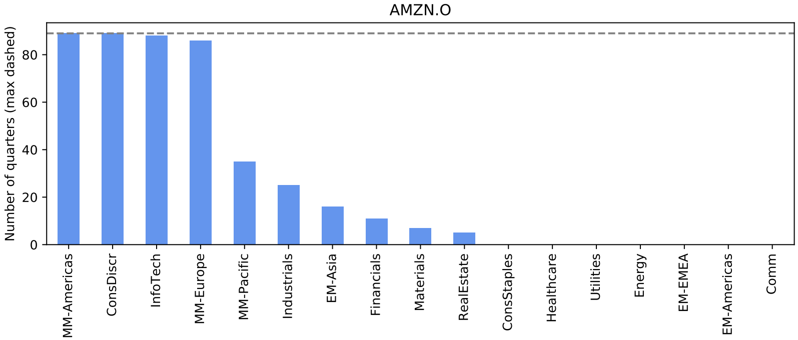
Factor selection

- Number of quarters that each factor is included for SAP
- German IT company

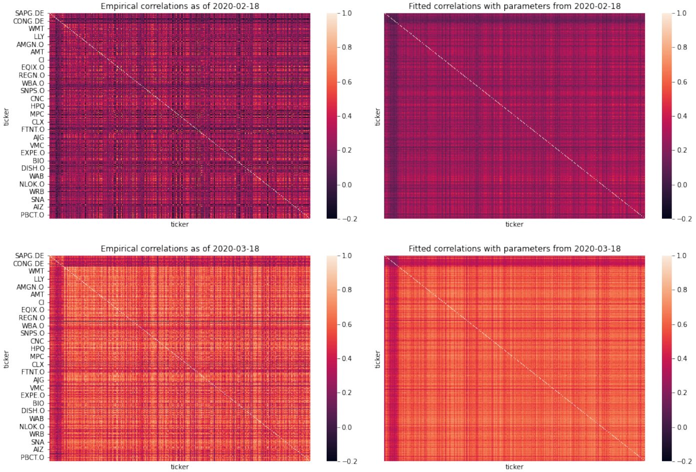


Factor selection

- Number of quarters that each factor is included for Amazon:
- US based online retailer with strong presence in Europe
- World's largest provider of computing services (AWS)

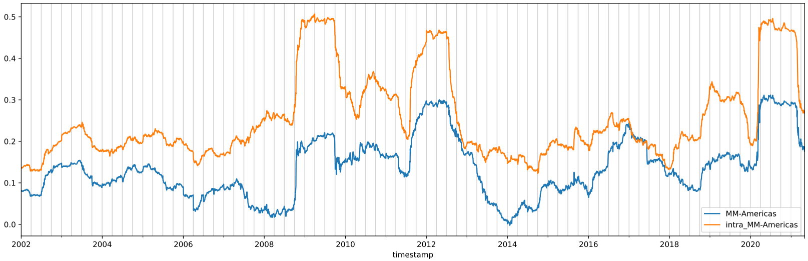


Correlations at beginning of Covid-19 pandemic



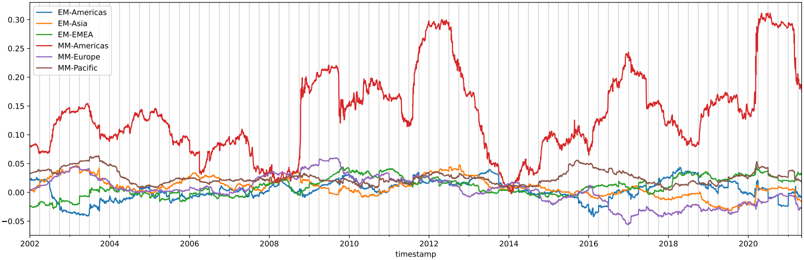
- Empirical & fitted correlations; top: 18 Feb, bottom: 18 Mar 2020.

Factor coefficients



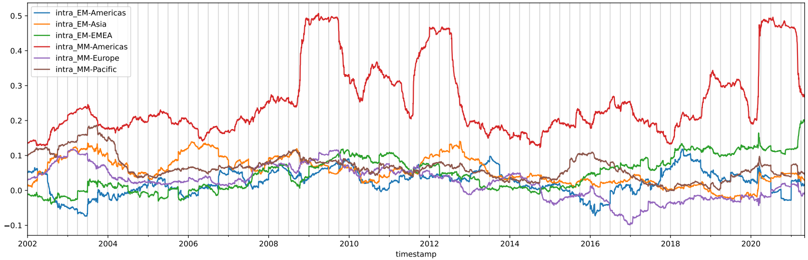
- Fitted parameters for risk factors with high loads.

Factor coefficients



- Fitted "inter" parameters for selected risk factors ("λ_k"'s)

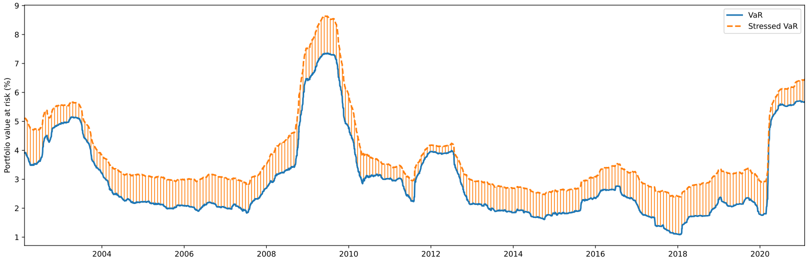
Factor coefficients



- Fitted "intra" parameters for selected risk factors ("ν_k"'s)

Overview

Value-at-risk impact

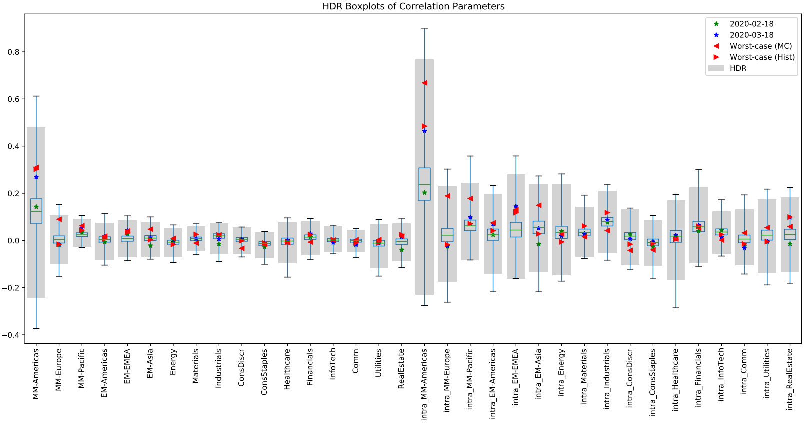


- Blue: $\text{VaR}_{99\%,1\text{ day}}$ on equally-weighted portfolio of DAX and S&P 500
- Orange: Stressed $\text{VaR}_{99\%,1\text{ day}}$ on reverse stress scenario of 5 April 2021.

Risk-factor distribution

- Fit time series of risk factor parameters $(\eta, \lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)$ to **Normal-Inverse Gaussian (NIG)** distribution
- NIG: generalisation of normal dist. that allows for skewness and higher variation in tails
- Calibration via using expectation-maximization (EM) algorithm, (McNeil *et al.*, 2005, Chapter 3) and Dempster *et al.* (1977)

Reverse stress testing (Covid-19 pandemic)



- Worst-case scenario within 95% HDR (18 Feb 2020)
- Triangles: worst-case scenarios (MC sim., Hist. sim.)
- Stars: Scenarios on 18 Feb (green) and 18 March (blue)

Conclusion

- We develop a correlation stress testing framework, linking risk factors with correlations.
- Risk factors (e.g. industries, countries) are linked firms via Bayesian variable selection methods.
- Reverse stress tests are conducted by assigning the factor loadings a distribution and determining the worst-case scenario within a HDR.

Outlook

- Current research focusses on extending universe of stress scenarios (not limited to correlation) by using latent factors.
- For example: Use PCA to build global risk factor.
- More generally: assign economic interpretation to latent factors from dimension reduction methods commonly used in Machine Learning.

References I

Alexander, C. and E. Sheedy. Developing a stress testing framework based on market risk models. *Journal of Banking & Finance*, 32(10):2220–2236, 2008.

Brigo, D. A note on correlation and rank reduction. Working Paper, May 2002.

Cont, R. and L. Wagalath. Risk management for whales. *Risk*, (June):79–82, 2016.

Dempster, A. P., N. M. Laird, and D. B. Rubin. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society: Series B (Methodological)*, 39(1):1–22, 1977.

DTCC. Depository Trust & Clearing Corporation – Trade Information Warehouse, 2014.

Fahrmeir, L., T. Kneib, S. Lang, and B. Marx. *Regression*. Springer, 2013.

Hyndman, R. J. Computing and graphing highest density regions. *The American Statistician*, 50(2):120–126, 1996.

Hyndman, R. J. Computing and graphing highest density regions. *The American Statistician*, 50(2):120–126, 1996.

JPMorgan. Report of JPMorgan Chase & Co. Management Task Force Regarding 2012 CIO Losses, 2013.

References II

Kupiec, P. Stress testing in a Value at Risk framework. *Journal of Derivatives*, 6:7–24, 1998.

Longin, F. and B. Solnik. Extreme correlation of international equity markets. *The Journal of Finance*, 56(2):649–676, 2001.

Loretan, M. and W. English. Evaluating changes in correlations during periods of high market volatility. *BIS Quarterly Review*, pages 29–36, June 2000.

Mahalanobis, P. C. On the generalized distance in statistics. National Institute of Science of India, 1936.

McNeil, A., R. Frey, and P. Embrechts. *Quantitative Risk Management*. Princeton University Press, Princeton, NJ, 2005.

McNeil, A., R. Frey, and P. Embrechts. *Quantitative Risk Management*. Princeton University Press, Princeton, NJ, 2nd edition, 2015.

Packham, N. Correlation parameterization and calibration for the LIBOR market model. Master Thesis, Frankfurt School of Finance & Management, March 2005.

Rebonato, R. *Modern Pricing of Interest-Rate Derivatives: The LIBOR Market Model and Beyond*. Princeton University Press, 2002.

References III

Schoenmakers, J. and B. Coffey. Stable implied calibration of a multi-factor libor model via a semi-parametric correlation structure. Weierstrass Institute for Applied Analysis and Stochastics, Preprints, No. 611, 2000.

Serfling, R. Quantile functions for multivariate analysis: approaches and applications. *Statistica Neerlandica*, 56(2):214–232, 2002.

United-States-Senate. JPMorgan Chase Whale Trades: A case history of derivatives risks and abuses. exhibits, 2013.

United-States-Senate. JPMorgan Chase Whale Trades: A case history of derivatives risks and abuses. report, 2013.

Thank you!

Prof. Dr. Natalie Packham
Professor of Mathematics and Statistics
Berlin School of Economics and Law
Badensche Str. 52
10825 Berlin
natalie.packham@hwr-berlin.de

Hochschule für
Wirtschaft und Recht Berlin
Berlin School of Economics and Law