جامعـــة خليفـــة
Khalifa University

# Benchmarking stochastic optimization approaches for pension fund managemen

**University of Cambridge**
**In honor of M.A.H. Dempster**

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# Outline

# Research project on PF ALM

Following early results based on multistage stochastic programming (MSP) in Consigli et al (2017), I will consider other approaches we have studied recently for similar PF-ALM problems:

- A distributionally robust optimization (DRO) approach developed in cooperation with A. Kleywegt (ISyE, Georgia Tech), A. Hitaj (UniMiB, Milan) and R.Gao (Univ of Texas).
- Dynamic stochastic control (DSC) based on a research project involving D.Lauria (former PhD and PostDoc at UniBG), Francesca Maggioni and myself published on *OR Spectrum* (2022)
- A combination of MSP and DSC a project in collaboration with B.Ji, Z.Chen and Z.Yan, published on *Quantitative Finance* (2022).

# Fundamentals of PF economics

- We consider an asset-liability management (ALM) model for a defined-benefit pension fund (PF)
- The PF liability is generated by current and future benefits to pay to passive members net of the contributions from the sponsor and the active members.
- In a DB scheme benefits are determined according to a mathematical formula and the PF thus carries portfolio risk. While they depend on the fund performance in a defined contribution (DC) scheme (in which contribution rates are given).
- Modern PF ALM is rooted on the evaluation of market-based solvency conditions through the funding (FR) and the solvency ratios (SR).
- At any point in time the PF should hold an asset portfolio and liquidity sufficient to cover its liabilities. According to International Accounting standards (IAS) these need to be evaluated consistently with market practice.

# Agents

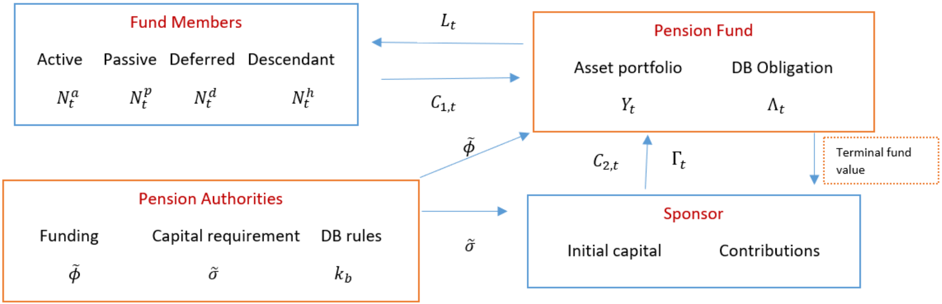


Figure: Relevant information flows in PF management

# Modern approach to PF ALM

Let $t \in \mathcal{T} := \{t_0, t_1, ..., T_\lambda\}$ possibly spanning several decades, and consider the following core variables in the problem specification. We may later on want to distinguish an investment horizon $T \neq T_\lambda$.

| | | |
|---|---|---|
| $X(t) := \sum_k x_t^k$ | | plan portfolio |
| $\Lambda(t) := \mathbb{E}_{\tilde{\mathbb{P}}}\left[\sum_{j=t}^{T_\lambda} \mathbf{L}_j \cdot \delta^j \mid \mathcal{F}_t\right]$ | (1) | liability (DBO) of the PF |
| $\phi(t) := \frac{X(t)}{\Lambda(t)}$ | (2) | funding ratio |
| $\mathrm{L(t)} := \sum_{g,b,s,x} N_t^p(g,x) \cdot L(b,s)$ | (3) | pension benefits |
| $\mathrm{C(t)} := \gamma\left[\sum_{g,b,s,x} N_t^a(g,s,x) \cdot w_b\right]$ | (4) | contributions |

The ALM problem may be formulated in real or in nominal terms. In the former case all relevant quantities are in constant monetary values (cmv): this is a key assumption in this project.

# Actuarial principles and liability-driven-investment

Actuarial standard principles (ASP) have and still play a fundamental role in DB pension fund management. In recent years they have been exposed to a rapidly evolving regulatory framework:

- The discount factors $\delta^t$ in (1) play a relevant role in policy debate between large insurers, pension funds and authorities.
- Members' projected survival rates recently updated to account for longevity risk
- The adoption in internal models of a DBO pricing principle under measure $\tilde{\mathbb{P}}$ in place of the forecasting method based on $\mathbb{E}(L_t)$
- The adoption of the projected liability valuation method to determine benefits $L_t$ and contributions $C_t$.
- Liability-driven-investment (LDI) provides an ALM approach in which the portfolio $X_t$ is expected to replicate the PF liability profile and generate a surplus. In this work we assume that no surplus is expected at PF termination. LDI almost by definition also provides a risk control approach.

# Risk models

- Lee-Carter (1992) model for $\ln(m_{x,t})$, where $m_{x,t} := D_{x,t}/E_{x,t}$ is the mortality rate of individuals of age $x$ at time $t$:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon^m_{x,t}. \tag{5}$$

  Here $a_x$ denotes the time average of $\ln(m_{x,t})$, $k_t$ a common time-dependent mortality factor with coefficient $b_x$, and $\varepsilon^m_{x,t}$ are residuals.

- Consider a real yield curve at time $t$ over the term $\tau$: $r_{t,\tau} = i_{t,\tau} - \pi_{t,\tau}$. The Nelson-Siegel model is

$$r_{t,\tau} = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{6}$$

Both models, once calibrated, can be used for forecasting: $\hat{m}_{x,T+h} = \exp\left(\hat{a}_x + \hat{b}_x \hat{k}_{T+h}\right)$ and for given constant $\lambda$, $\beta_{i,t+h}$ will determine for increasing $h$ an average evolution of the PF variables.

# Part II

# DR LDI

Joint with Anton Kleywegt, Asmerilda Hitaj and Rui Gao.

# Motivation and assumptions

We consider a closed US PF problem spanning the entire life of its members up to 110 years.

- The PF liability evaluation is required to span several decades, up to $T_\lambda$: a hedging strategy against an uncertain worst case distribution is strongly suggested.
- Over such a long horizon, hardly ever one has enough data to estimate a probability distribution with sufficient accuracy.
- This is true for both liability projection and discount rates estimation: this latter propagates into asset pricing models.
- Asset returns are also exposed to significant distributional uncertainty in the long term.
- Of specific relevance here to motivate a DRO approach, the natural adoption of the actuarial model to derive the PF projected liabilities and the associated nominal distribution.

# DRO formulation

We allow for distributional ambiguity over the probability law governing the future PF members' dynamics and pricing kernel.

$$\min_{z \in \mathcal{Z}} \sup_{\mathbb{Q} \in \mathfrak{M}} \mathbb{E}_{\mathbb{Q}}[\Psi(z, \xi)]. \tag{7}$$

The set $\mathfrak{M}$ can be determined by specifying:

- Shape properties as in Calafiore and El Gahoui (2006), Chen et al (2017), or
- Moment characteristics of the distribution as in Delage and Ye (2010), Goh and Sim (2010), Wieseman, Kuhn and Sim (2014) or
- by choosing a nominal distribution and by specifying $\mathfrak{M}$ to be the set of all distributions within a chosen statistical distance. See Ben-Tal et al (2013), Wang, Glynn and Ye (2016), Esfahani and Kuhn (2015), Gao and Kleywegt (2016).

Problem (7) looks for a solution such that, if the distribution in the future does not deviate from the past distribution (empirical distribution) too much in terms of Wasserstein distance, then the solution should perform well in terms of the objective of (7).

# Wasserstein distance

In this setting, we choose a metric $d$ such that $(\Xi, d)$ is a complete separable metric (Polish) space. For $p \in [1, \infty)$, let $\mathcal{P}_p(\Xi)$ denote the set of Borel measures on $\Xi$ with finite $p$-th moment.

$$\mathfrak{M} := \{\mu \in \mathcal{P}_p(\Xi) \ : \ W_p(\mu, \nu) \leq \theta\} . \tag{8}$$

Given the nominal distribution $\nu$, the Wasserstein ball of radius $\theta > 0$ contains **distributions of time series** $\mu \in \mathcal{P}(\Xi)$ close to $\nu$ in the Wasserstein distance

$$W_p(\mu, \nu) := \min_{\gamma \in (\Xi \times \Xi)} \left\{ \int_{\Xi \times \Xi} d^p(\xi, \zeta) \gamma(d\xi, d\zeta) \ : \ \gamma \text{ has marginals } \mu \text{ and } \nu \right\},$$

where $d$ measures how close two time series $\xi = (\xi_t)$ and $\zeta = (\zeta_t)$ are over a selected calibration period as clarified below.

# Solution approach

Following Kleywegt and Gao (2016), we apply to (7) the strong duality result:

$$\sup_{\mu \in \mathcal{P}(\Xi)} \{ \int_{\Xi} \Psi(z,\xi)\mu(d\xi) | W_p(\mu,\nu) \leq \theta \} = \inf_{\lambda \geq 0} \{ \lambda\theta^p + \int_{\Xi} sup_{\xi} [\Psi(z,\xi) - \lambda d^p(\mu,\nu)] d\nu \} \quad (9)$$

Assume a decision strategy **z** parametrized through a generic rule $\beta \in \mathcal{D}$: $\{z(\beta)\}_{\beta}$. With Wasserstein DRO, the worst-case cost $\mathbb{E}_{\mu}[\Psi(\beta,\xi)]$ for a given $\beta$, $p = 2$ and realized process $\hat{\xi}$ can then be evaluated by solving

$$\min_{\lambda \geq 0} \left\{ \lambda\theta^2 + \mathbb{E}_{\hat{\xi}\sim\nu} \left[ \sup_{\xi} \left\{ \Psi(\beta,\xi) - \lambda d(\xi,\hat{\xi})^2 \right\} \right] \right\},$$

which suggests to find the worst-case cost $\sup_{\xi} \left\{ \Psi(\beta,\xi) - \lambda d(\xi,\hat{\xi})^2 \right\}$ for *each sample path* $\hat{\xi}$ from $\nu$.

# Solution approach: more details

Solving $\sup_\xi \left\{\Psi(\beta,\xi) - \lambda d(\xi,\hat{\xi})^2\right\}$ requires the gradient $\nabla_\xi \Psi(\beta,\hat{\xi})$, which is often hard to obtain in closed-from expression. Thus we approximate it via gaussian smoothing

$$\nabla_\xi \Psi(\beta,\hat{\xi}) \simeq \frac{1}{\delta}\mathbb{E}_{e\sim\mathcal{N}(0,I)}[(\Psi(\beta,\hat{\xi}+\delta e) - \Psi(\beta,\hat{\xi}))e].$$

If $p=2$ and $\Psi(\beta,\xi)$ is twice continuously differentiable, we can then solve the following gradient-norm regularization problem

$$\min_{\beta\in\mathcal{D}} \quad \mathbb{E}_\nu[\Psi(\beta,\hat{\xi})] + \theta\cdot(\mathbb{E}_\nu[||\nabla_\xi\Psi(\beta,\hat{\xi})||^2])^{1/2}.$$

The solution depends on $\theta$, whose calibration is key to convergence to an optimum.

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# Sample average approximation

Problem (7) can be tackled through Sample Average Approximation (SAA) (Kleywegt, Shapiro and Homem-de-Mello, 2002) in the form

$$\min_{z} \left[ \hat{\Psi}_N(z) = \frac{1}{N} \sum_{n=1,..,N} \Psi(z, \hat{\xi}_n) \right] \tag{10}$$

where we assume that the sample corresponds to the realized sequence $\hat{\xi}_n$ for $n = 1, 2, .., N$. The expectation is taken with respect to the empirical measure $\nu$: $\hat{\Psi}_N(z) = \mathbb{E}_\nu [\Psi(z, \xi)]$.

Then the nominal distribution in the DRO problem corresponds to $\nu \in \mathcal{P}_p(\Xi) = N^{-1} \sum_{n=1}^{N} \delta_{\hat{\xi}_n}$ where $\delta_\xi$ denotes the unit point mass on $\xi$ and $\hat{\xi}_n$ is a sample from the data history.

# PF problem formulation

Consider the following time partition: $T > T_2 > T_1 > t_0$ with $T - T_2$ planning horizon, $T_2 - T_1$ for error forecasts calibration and $T_1 - t_0$ for statistical model estimation. The following objective function is considered in the DR LDI problem specification:

$$\Psi(z, \xi) := \sum_{t=T_2}^{T-1} \left( C_t + e_1 |\Gamma_t| \right) + e_1 |Y(z_T, \xi_T)| \mathbf{1}_{Y(z,\xi)<0} + e_2 Y(z_T, \xi_T) \mathbf{1}_{Y(z,\xi)>0} \quad (11)$$

To be minimized, for given input funding tolerance $\tilde{\phi}$ and $t = 0, 1, ..., T - T_2$, under the constraints (2), (3), (4) and:

$$Y_t^0 = C_t - L_t + \sum_{\tau} y_{t-\tau,t,\tau} e^{r_{t,\tau}\tau} + \Gamma_t \mathbf{1}_{\phi_{t-1}<\tilde{\phi}} \quad (12)$$

$$\Gamma_t = (\Lambda_t - Y_t - Y_t^0) \quad (13)$$

$$y_{t',t+1,\tau} = (y_{t',t,\tau} - u_{t',t,\tau}) \mathbf{1}_{Y_t^0<0} + (y_{t',t,\tau} + \min\{Y_t^0, -\Delta Y_t^{t'}\}) \mathbf{1}_{Y_t^0>0} \quad \forall t' < t, \forall \tau \quad (14)$$

# Model update and decision process

The set of constraints (14) reflects the LDI approach adopted in this project: when selling the discount bonds with earliest maturity are selected first and when buying the cash surplus is compared with $\Delta Y_t^{t'}$ which depends on the conditional expectation under the nominal measure of future cash inflows.
Then as $t = 0, 1, , \ldots T - T2$:

- Contributions $C_t$ and benefits $L_t$ evaluated according to the actuarial principles will determine:
  - the net benefits $l_t$ and the associated cash balance $Y_t^0$
  - The PF DBO $\Lambda_t$ and
  - the discounted asset value $Y_t$
- $Y_t$, $Y_t^0$ and $\Lambda_t$ will determine the FR $\phi_t$, which determines
- the extraordinary contributions $\Gamma_t$, that together with $Y_t^0$ will determine
- the cash surplus or deficit leading to buying or selling discount bonds of different maturity according to a pre-specified policy.

# Nominal distribution

From models (5) and (6) we construct the nominal distributions as the distribution of the time series of forecasting errors. Let $\hat{m}^t_{x,t+h} = \exp\left(\hat{a}^t_x + \hat{b}^t_x \hat{k}^t_{t+h}\right)$ denote the resulting mortality rate forecasts, and let

$$\varepsilon^t_{x,t+h} \quad = \quad m_{x,t+h} - \hat{m}^t_{x,t+h}$$

denote the empirical mortality rate forecasting errors.

$$\epsilon^t_{t+h,\tau} \quad = \quad r_{t+h,\tau} - \hat{r}^t_{t+h,\tau}$$

denote the empirical interest rate forecasting errors. Then, for each $t \in \{T_1, \ldots, T_2 - 1\}$, the observed value of the random variable is $\hat{\xi}^t = \left(\left(\varepsilon^t_{x,t+h}, x \in \{x_{\min}, \ldots, x_{\max}\}, h \in \{1, \ldots, T - T_2\}\right), \left(\epsilon^t_{t+h,\tau}, \tau \in \mathcal{T}, h \in \{1, \ldots, T - T_2\}\right)\right)$. The distributionally robust approach does not require $\hat{\xi}^t$, $t \in \{T_1, \ldots, T_2 - 1\}$, to be i.i.d. The resulting nominal distribution $\nu$ consists of $T_2 - T_1$ equally weighted times series:

$$\nu \quad = \quad \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2-1} \delta_{\hat{\xi}^t}.$$

# Computational results

We apply the above framework to a US pension fund problem under the following assumptions:

- The yield curves, inflation curves and mortality statistics span the 1933-2020 period.
- Residuals' time series and the nominal distribution are specified according to given problem specific time partition, in particular for: $h = 1, 2, .., 25$, $T_2 = 1996$, $T = 2020$, $T_2 - T_1 > T - T_2$.
- The PF ALM problem is formulated with annual portfolio revision, investment horizon of 25 years and a liability projection spanning the lifetime of every member.
- At the end of the planning horizon a PF termination value is computed amd the fund's operation ends.
- The investment universe is based on discount bonds with maturity $\tau = \{0.25, 1, 2, 3, 4, 5, 7, 10, 15, 20, 30\}$ years.
- An extraordinary contribution occurs upon violation of a regulatory FR threshold $\phi \in \left[\tilde{\phi}, 1\right]$.
- The problem solution will generate the minimal sponsor's contribution rate $\gamma$ and, for given parametric portfolio policy, extraordinary contributions needed to preserve the PF liquidity and funding conditions over several decades.

# US inflation and real interest rates' history

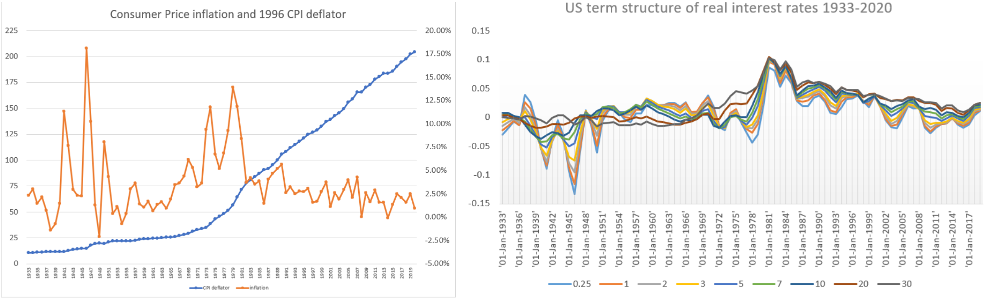


Figure: US CPI and Real yield curve 1933-2021

# Data process, DRO calibration

For every $t \in [T_1, T_2]$, $\tau \in \{0.25, ..., 30\}$ and $h \leq (T - T_2)$ the forecasting errors are estimated in-sample and characterize the nominal distribution and the resulting distributional uncertainty based on Wasserstein radius calibration.

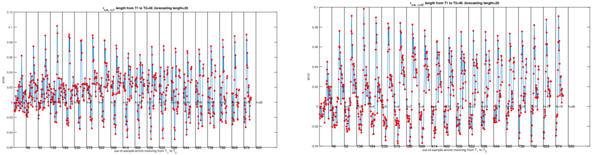

Figure: Real interest rates forecasting errors, $\tau = \{1, 10\}$

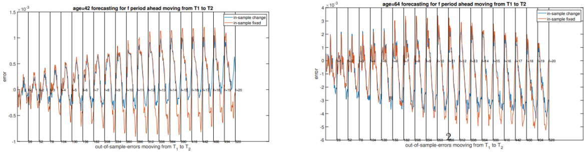


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Figure: 42y and 64y mortality rates forecasting errors

# PF initial conditions

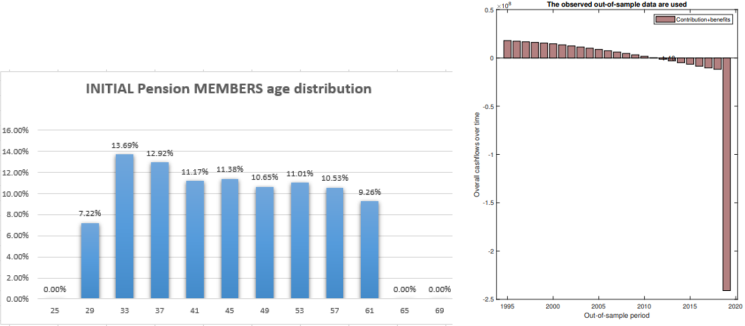
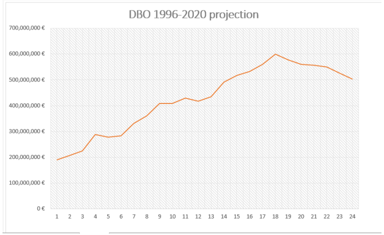



Figure: Pension fund age composition in t=0 and Net benefits evolution

# 25 year liability projection



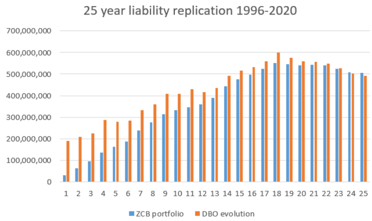




Figure: Liability projections and portfolio replication over 25 years

# 2019 OECD contributions rates

Pensions at a Glance 2019 - © OECD 2019
Table 8.1. Mandatory pension contribution rates for an average worker in 2018
Version 1 - Last updated: 18-Nov-2019
Disclaimer: http://oe.cd/disclaimer

**Table 8.1. Mandatory pension contribution rates in 2018 (% of gross earnings)**

For old-age and survivor pension schemes

| | Nominal rate | | | | Effective rate on average earnings |
|---|---|---|---|---|---|
| | Employee, public | Employer, public | Employee, private | Employer, private | |
| Australia | | | 0.0 | 9.5 | 9.5 |
| Austria* | 10.3 | 12.6 | | | 22.8 |
| Belgium | 7.5 | 8.9 | | | 16.4 |
| Chile | | | 11.2 | 1.2 | 12.4 |
| Czech Republic* | 6.5 | 21.5 | | | 28.0 |
| Denmark* | | | 4.0 | 8.0 | 12.8 |
| Estonia | 0.0 | 16.0 | 2.0 | 4.0 | 22.0 |
| Finland* | 6.7 [a] | 17.7 | | | 24.4 [a] |
| France | 11.2 [w] | 16.3 [w] | | | 27.5 |
| Germany* | 9.3 | 9.3 | | | 18.6 |
| Italy* | 9.2 | 23.8 | | | 33.0 |
| Japan | 9.15 | 9.15 | | | 18.3 |
| Netherlands | 18.0 | 0.0 | 7.7 [w] | 14.8 [w] | 25.6 |
| New Zealand | | | | | 0.0 |
| Norway | 7.6 | 10.5 | 0.0 | 2.0 | 20.1 |
| Sweden | 7.0 | 10.2 | 0.0 | 4.5 [w] | 21.7 |
| United States* | 6.2 | 6.2 | | | 12.4 |
| **OECD33** | | | | | **18.4** |

Figure: Official estimates of 2019 contribution rates in selected OECD countries

# DRO value function – 1995-2020 $\theta = \{0.005, 0.003\}$

We present a set of DRO results for alternative radius estimates, benchmarked against the SAA solution.

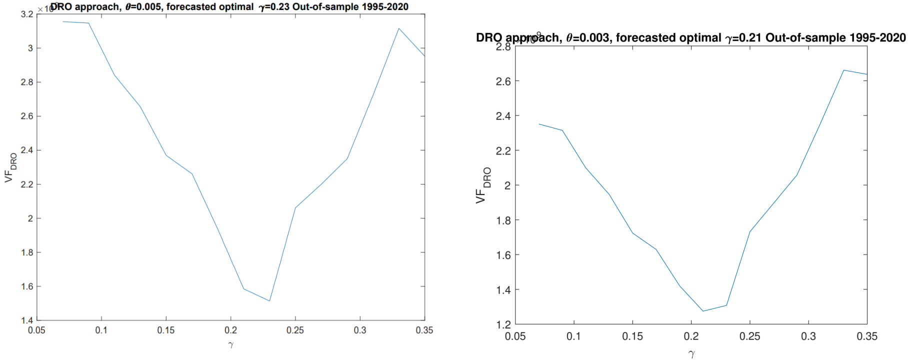



Figure: DRO solutions, 25 year problem, $\theta = 0.005 - 0.003$, 1995-2020

# DRO value function – 1995-2020 $\theta = \{0.001, 0.0005\}$

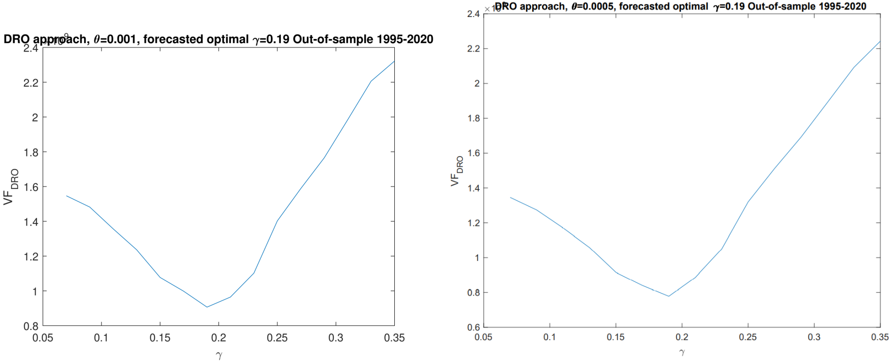



Figure: DRO solutions, 25 year problem, $\theta = 0.001 - 0.0005$, 1995-2020

# SAA – 1995-2020

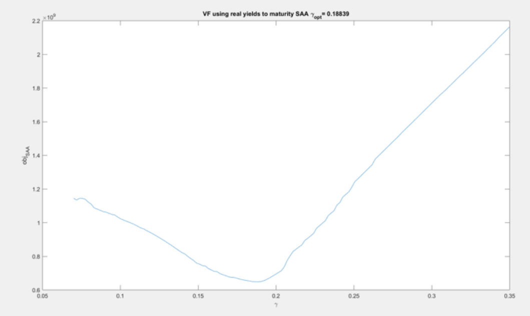



Figure: Sample average approximation, 25 year problem, 1996-2020

# Summary evidence

- As we reduce the radius from $\theta = 0.005$ to $\theta = 0.0005$, the optimal contribution rate $\gamma$ decreases to 0.19: this is the minimal ordinary contribution rate in the face of distributional uncertainty.
- At the optimum:
  - The extraordinary contributions are null,
  - The terminal ZCB portfolio plus the residual cash at $T = 25$ are equal to the terminal value of the fund.
- The DRO solution required 52.408 secs of CPU time for error forecast estimation and 6505.4003 secs (108 minutes) for the value fn estimation with $\theta = 0.001$ on a Lenovo PC with 16.0G RAM and 3.4 GHz Dual Core CPU.
- These preliminary results are in line with the in-sample tests and take fully into account regulatory requirement and the complex PF liability structure.

# Part III

# PFM METHODS BENCHMARKING

# Models and methods

Following the 2015-2016 R&D project above, in parallel with the DRO project, thanks to two research grants in Italy and in China we have been working on the following PFM problems:

- A dynamic stochastic control (DSC) method for an open occupational PF problem formulated as a chance-constrained semidefinite program (Lauria, GC and FM) with:
  - a 30-year liability evaluation horizon, and a shorter $3-4$ investment horizon with quarterly rebalancing,
  - a quadratic target-based objective function with a chance constraint on the funding ratio,
  - a continuous probability space with LC model of mortality and stochastic inflation, asset returns and yield curve. A rich investment universe.
- A combined MSP-DSC method for a Tier 1+3 open pension fund problem (Ji, Chen, GC, Yan) for the Chinese urban areas:
  - A 20-year investment horizon partitioned in two sub-periods with linking dynamic programming value function,
  - a hierarchical stochastic model for assets and liabilities with arbitrage-free scenario generation,
  - a target-based quadratic objective with respect to funding and welfare targets.

# Models and methods

### MSP approach

- $(\Omega, \Sigma, P)$ discrete, tree processes
- *T=20 years*, $T_\lambda = 30$ *years*, annual rebalancing
- Liability pricing based on replicating portfolio
- Stochastic model of the yield curve, inflation, mortality
- Investment portfolio (13 asset classes)
- LP with target path and duration risk control
- Feasibility: a.s. feasibility
- Decision support with stress testing and sensitivity analysis

### DSC approach

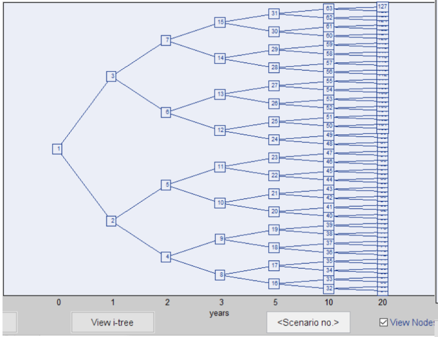
- $(\Omega, \Sigma, P)$, continuous
- *T=3-4 years*, $T_\lambda = 30$ *years*, quarterly rebalancing
- Stochastic model of the yield curve, inflation, mortality
- Investment portfolio (7 assets plus cash)
- Exogenous liability model
- Semidefinite QP with target path
- Chance constraint on the FR
- Back-testing and market validation

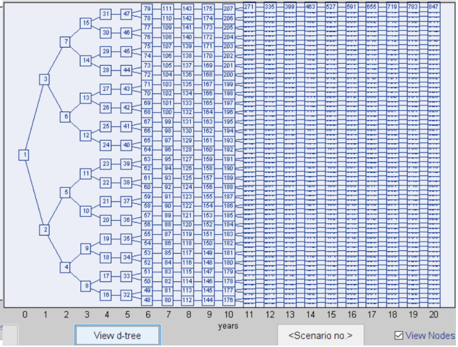
### MSP-DSC approach

- $(\Omega, \Sigma, P)$ tree processes + continuous beyond T
- *T=5 years*, $\tau = 20, T_\lambda = 6$ *years*, annual rebalancing plus policy
- Liability pricing based on statistical model
- Stochastic model of the yield curve, inflation, mortality, asset returns, liability costs
- Arbitrage free scenarios
- Investment portfolio 4 assets plus cash
- SQP target-based quadratic objective
- Open fund: Tier 1+3

# MSP (GC et al 2017)

Pension liabilities and asset returns tree processes, decision and non-decision nodes.



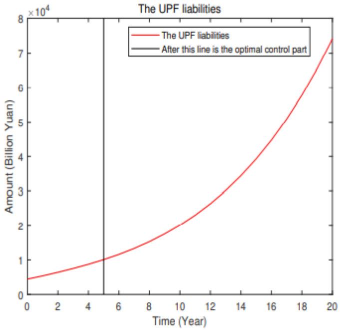



# A generic problem formulation

$$
\begin{aligned}
&\min_{x\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum\nolimits_{t\in\mathcal{T}} \rho(x_t, \xi_t(\omega))\right] \\
&s.t.\ \ for\ \mathrm{t} \in \mathcal{T} \\
&\mathrm{x}_t = x_{t-1}(1+r_t) + x_t^+ - x_t^- \\
&\mathrm{x}_t^+ - x_t^- = K_t(x_t, \bar{x}) \\
&\mathrm{x}_t^0 = l_t - x_t^+ + x_t^- + x_{t-1}^0(1+r_{t-1}^0) + \Gamma_t \mathbf{1}_{\phi_t \le \tilde{\phi}} \\
&\mathrm{x}_t \ge \tilde{\phi}\Lambda_t \quad w.p.\ \alpha \in [\tilde{\alpha}, 1] \\
&\mathrm{x}_t^0 + \Gamma_t \ge l_t
\end{aligned}
\tag{15}
$$

where the characterization of $\xi(\omega)$ leads to different problem formulations. $\Lambda_t$ and $l_t = L_t - C_t$ are exogenous, as the return process $r_t$. A given policy in the form of a linear feedback rule can be considered (Lauria et al, 2022) through the updating $K_t$.

# Open fund liability projection



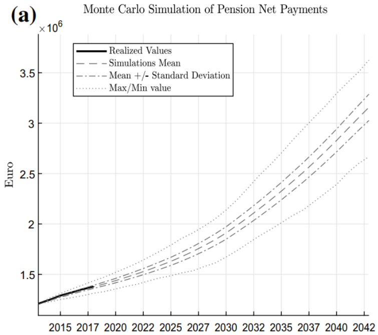
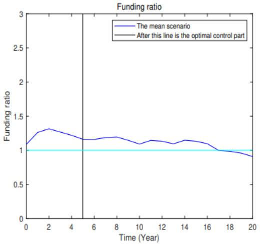




Figure: Two open pension funds liability projection

# Funding



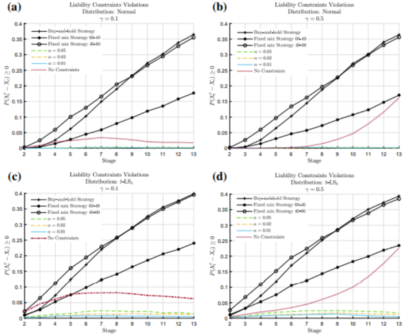




Figure: Evolution of funding conditions: almost sure versus chance-constrained feasibility

# A mixed approach (Ji, Chen, GC, Yan, 2022)

This is the type of underlying stochastic dynamics we are considering.

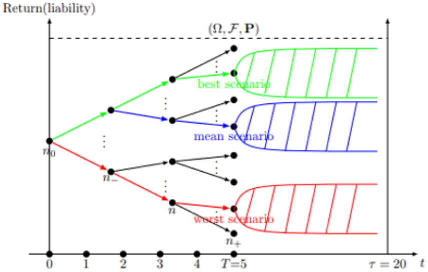
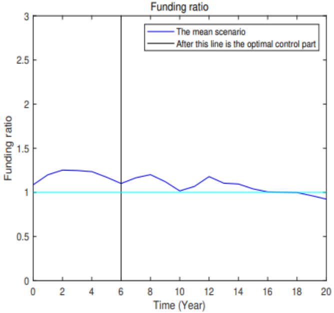



Figure: Combining MSP and DSC

See also Barro, D. and Canestrelli, E., (2016). Combining stochastic programming and optimal control to decompose multistage stochastic optimization problems and in *OR Spectrum*, 38(3) and same issue of OR Spectrum, Konicz, A., Pisinger D and Rasmussen K.M.: A combined SP and optimal control approach to personal finance and pensions

# Funding – mixed approach



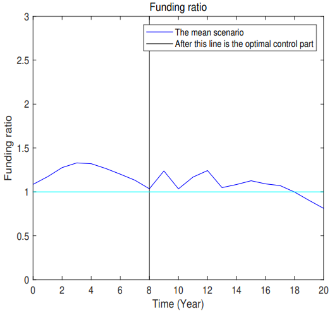




Figure: Further evidence on funding condition as the MSP horizon increases. Computational tests

# Computational evidence

Table: SP problem computational details

1.25

| Method | Branching structure | Number of scenarios | Scenario tree generation time | Size of technology matrix | Nonzeros elements before presolution | Total CPU solution time |
|---|---|---|---|---|---|---|
| MSP | $[10, 8, 4, 2^7]$ | 40960 | 50.53(s) | $2109053 \times 1622746$ | 6083999 | 113.20 |
| DSC | | 10000 | | 22227 | 1296548 | 2555 |
| MSP-DSC | $[15, 10, 5^3]$ | 18750 | 295.75(s) | $149473 \times 121476$ | 471914 | 5.28 |
| | $[6^6]$ | 46656 | 355.65(s) | $317245 \times 261258$ | 1017047 | 11.38 |
| | $[5^7]$ | 78125 | 898.37(s) | $624983 \times 507796$ | 1972599 | 25.23 |
| | $[4^8]$ | 65536 | 2044.15 (s) | $655341 \times 524270$ | 2031553 | 20.67 |

The DSC results are also collected on a 3.4 GHz Intel Core i7 processor, with 16.0G RAM running and adopting MOSEK Semidefnite program solver or MOSEK P/D IPQP solver.

# Modelling and methodological challenges in ALM

I would like to conclude by pointing out few recent contributions related to modelling and methodological challenges likely to attract new research in SO and DRO.

- defined contribution with guarantee: the transition towards DC pension schemes comes together with a protection which limits the risk transfer onto the pension members: then liability then depends on the fund performance (see Li, C. and Grossmann, I.E. (2021) review, Kopa, M. and T. Rusy (2021) on Annals of O.R.)
- sponsor credit risk: separation between PF managers and sponsors and these latter obligations to the fund require the evaluation of the credit risk faced by the PF. (Broeders D (2010) J of Risk and Insurance, Sun et al. (2017) EJOR).
- derivatives: Institutional investors such as PFs do use derivatives for performance protection and hedging under incomplete market assumptions (e.g.longevity bonds and swaps). See Moriggia, Vitali and Kopa (2021) work.
- machine learning: in the last years we have witnessed an increasing stream of contributions employing machine learning approaches in finance and specifically in ALM and PFM problems (Mulvey J.M. (2017, 2021), Fontoura et al (2021), Dixon, M. F. & Halperin, I. (2019), Kolm and Ritter (2020))