CFM
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14 April 2023 – Cambridge

Price Impact, Inelastic Markets and the Order-Driven View of “Anomalies”

Presented by

Jean-Philippe Bouchaud to honour Michael Dempster and “Quantitative Finance”

➢ Why do prices move?

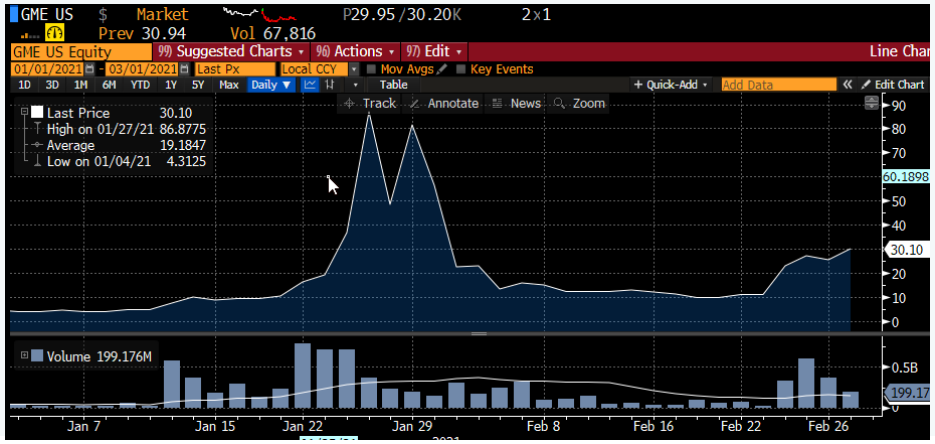
- EMH says: because of unanticipated exogenous news
- Most financial economists speak about the “fundamental” price
- Is this the way we should think about markets?

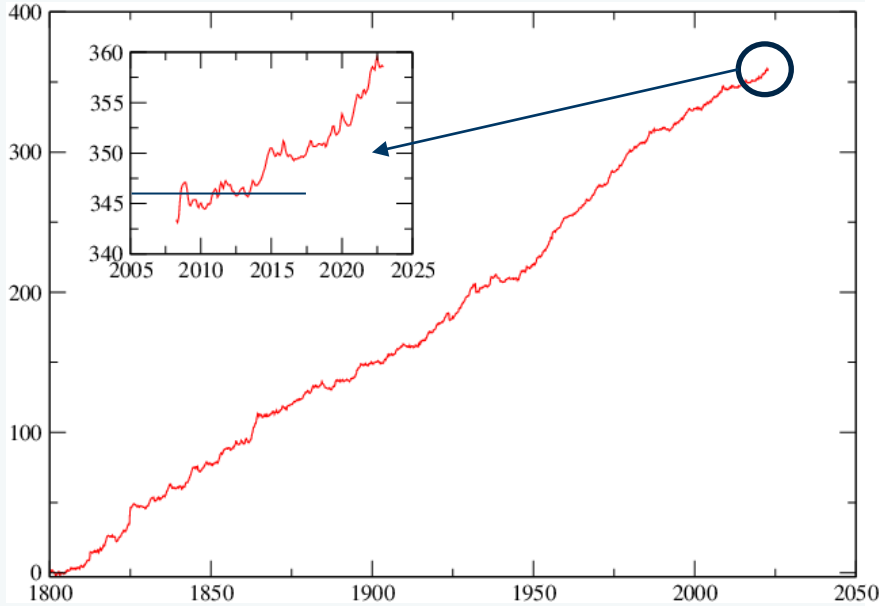
A slew of (old & new) market “anomalies” at odds with EMH

- The excess volatility puzzle
- The trend following puzzle + (many) other profitable “anomalies”:

Prices do not reflect all available information

- Most “jumps” are not due to news (Cutler-Poterba-Summers, CFM)
- Several microstructure stylized facts are at odds with EMH (e.g. persistence)
- Bitcoin and other fancy tulips – but “bubbles do not exist” (Fama)
- The recent “Reddit stocks” episodes, etc. etc.



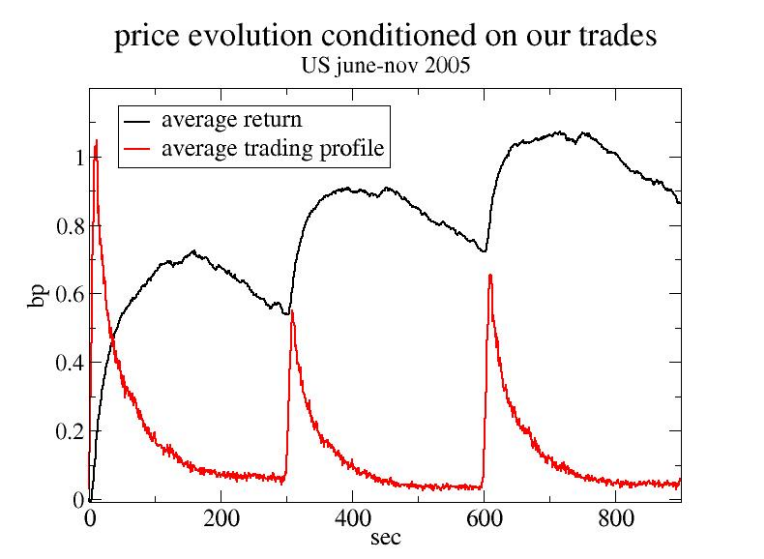


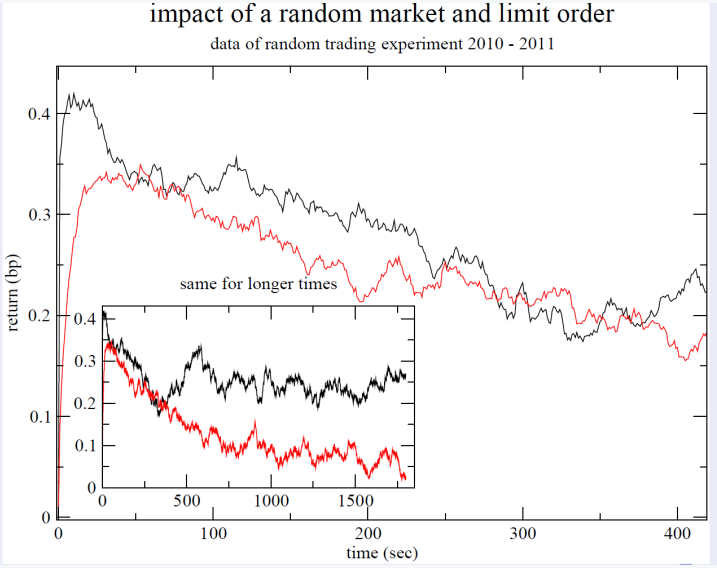
5 month trends: Some (obvious) public information is not included in the current price!
(Nb: no long bias here, but no costs either)

Flows do impact prices!

A (not so trivial) truism

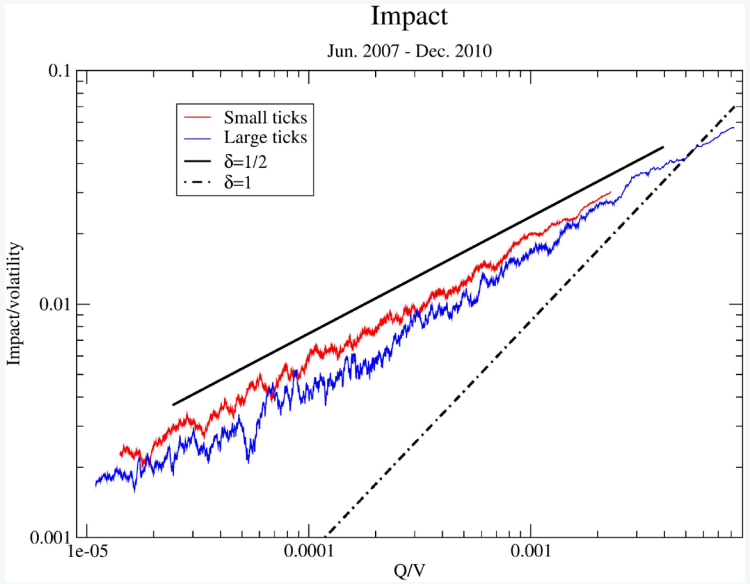
- Even though for each buyer there is a seller, trading impacts prices (“active” vs. “passive” flows breaks the symmetry)
- The EMH view: it is because informed traders predict prices!
- The quant industry view: it is a “mechanical”, order flow effect even when trades are not informed
- Impact is actually a major source of trading costs
- OK, say EMH diehards, but impact decays “quickly” anyway and has no long-term consequences – really?



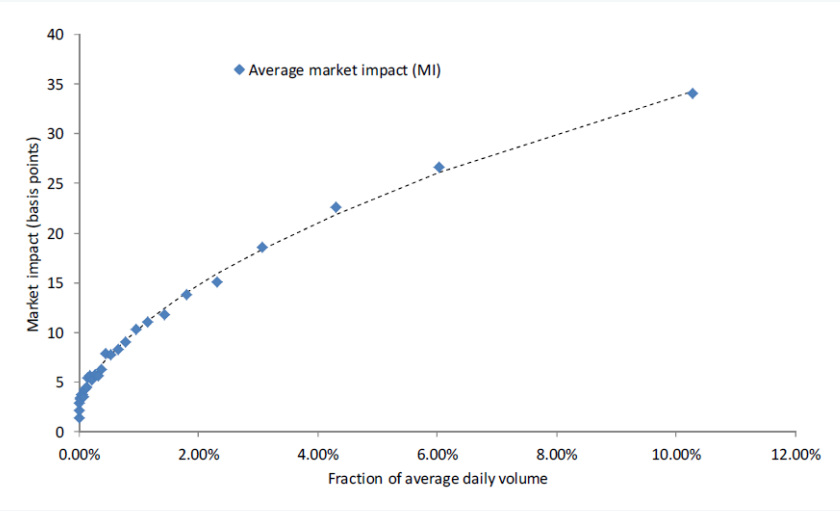


Impact: a rather non-trivial phenomenon

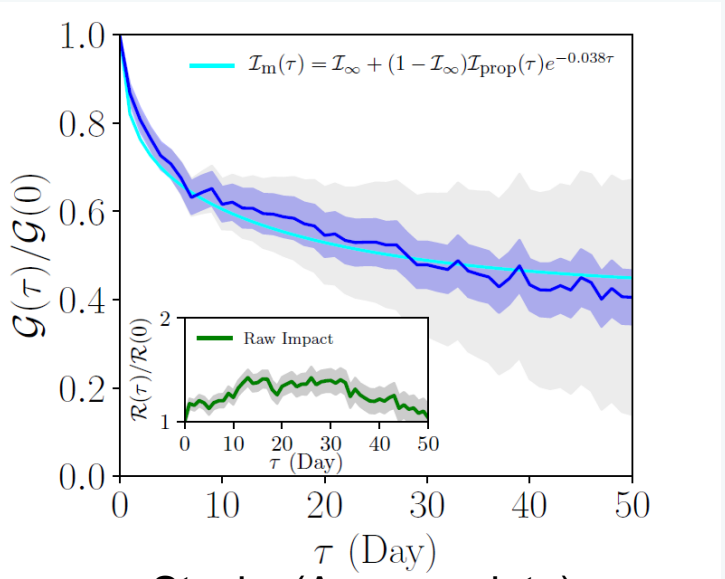
- Impact grows as the *square-root* of volume – a universal law! (markets, periods, trader types, execution style, etc.)
- Impact decays extremely slowly, so clearly has at least medium-term consequences
- Flows, even uninformed, must contribute to (excess) volatility
- Does impact of uninformed trades go all the way down to zero or actually saturate?
- Long-term impact: a difficult empirical question – Impact vs. “alpha” vs. noise?



Futures (CFM data)



Stocks (AQR data, limit orders)



Stocks (Ancerno data)

The “Inelastic Market Hypothesis” (Gabaix – Koijen)

The Gabaix – Koijen “multiplier” (2021)

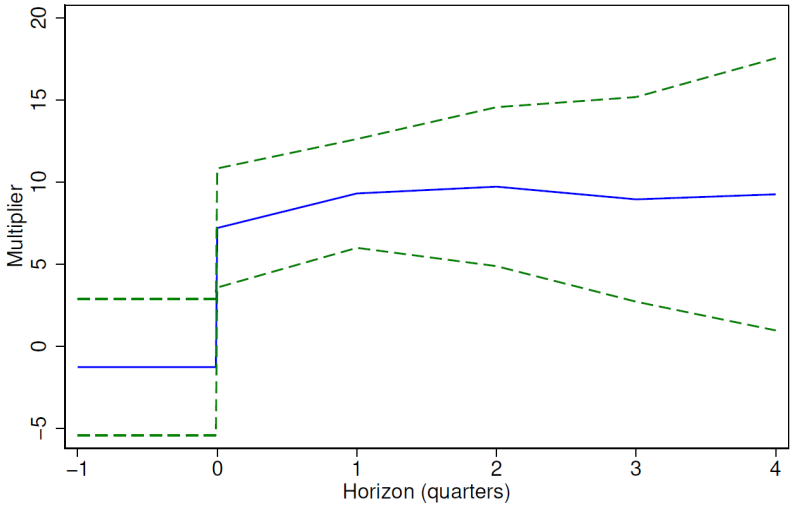
- Smart regression on holdings of mutual funds → Buying (/selling) 1$ of the whole market increases (/decreases) the long-term market cap. by 5$!
- For single stocks, the multiplier is M ≈ 1
- Mechanism: funds are inelastic due to e.g. mandate constraints

→ Holding a constant ratio of 80% stocks and 20% cash means that fund will only agree to sell 1$ of stocks if prices went up by 1$/(1-0.8)=5$

- A spectacular proposal – the median of economists’ guess is M=0 (and the average is M=0.01)….

→ A big splash: already 185 citations + FT, Bloomberg, The Economist, etc..

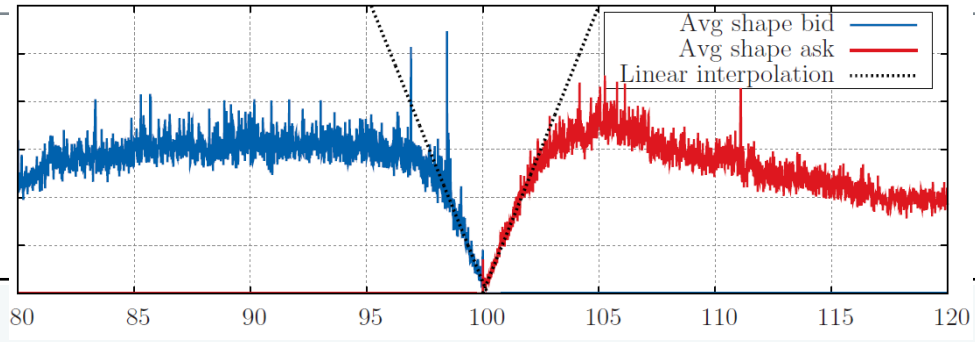
- Ballpark order of magnitude explanation of the 2009-2020 US market rally in terms of inflows



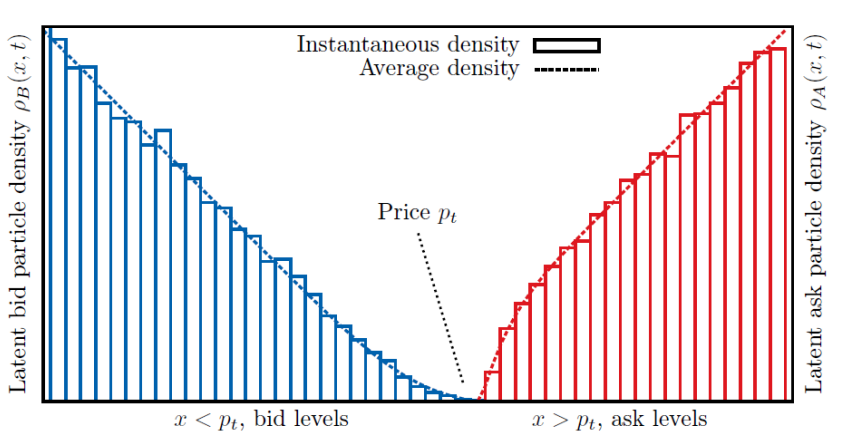
A unifying framework: The Latent Liquidity Theory

The Latent Liquidity Theory

(with J. Bonart, J. Donier, I. Mastromatteo, QF 2015)



- The Latent Liquidity Theory was developed to understand the square-root law: $I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$



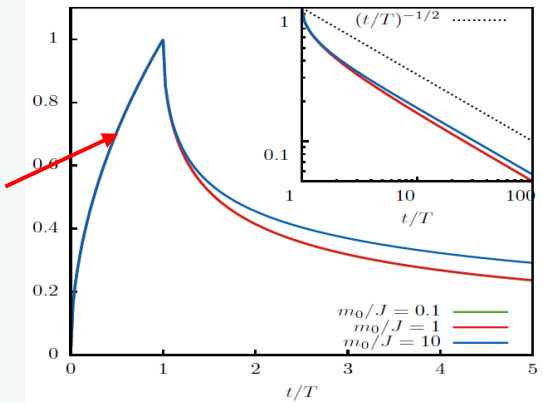
- A stylized, generic dynamical model for the "macro" (latent) liquidity around the current price x_t:

$$\partial_t \varphi_b = D\partial_{xx}\varphi_b - \nu\varphi_b + \lambda\Theta(x_t - x) - R_{ab}(x)$$
$$\partial_t \varphi_a = D\partial_{xx}\varphi_a - \nu\varphi_a + \lambda\Theta(x - x_t) - R_{ab}(x)$$

with cancellation rate ν and replenishment rate λ

- Equilibrium latent liquidity is V-shaped around the current price → square-root impact!
- The short-term inertia of liquidity explains initial impact decay (for $\nu T << 1$)
- Note: any round-trip is costly (on average) – no arbitrage

Square-root



From transient square-root to permanent linear impact

(with M. Benzaquen, QF 2018)

- But on long term νT >> 1, market participants take stock of recent price moves and update their reservation price accordingly

→ Order flow re-centers around the current price

→ Permanent impact

- Technically $x_t = \alpha \left[z_t^0 + \sqrt{\nu} z_t^1 + O(\nu) \right]$

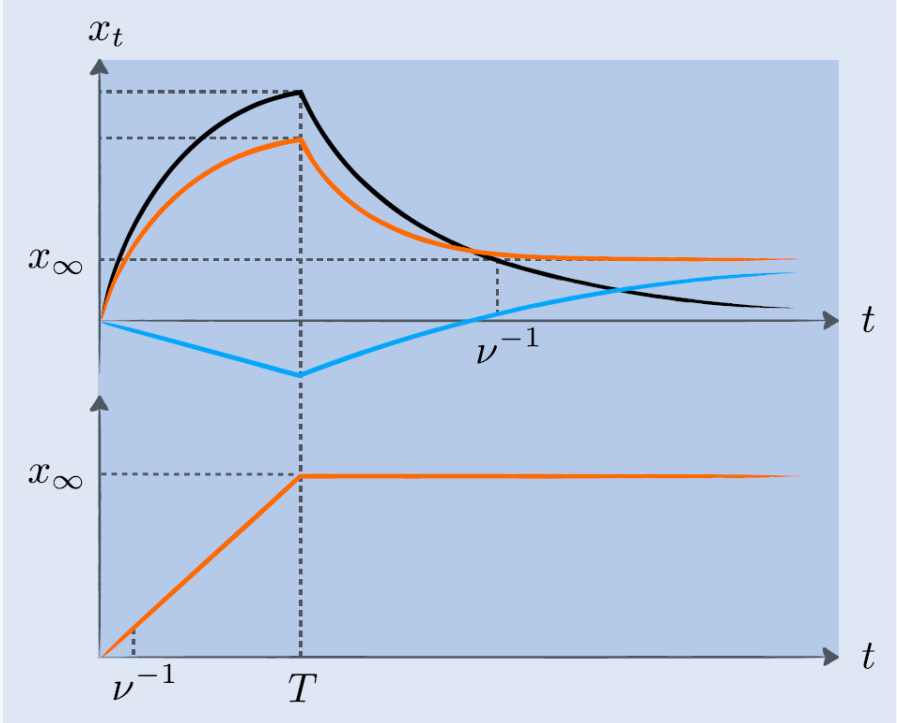
→ I_∞ = a Q : long term impact becomes linear in Q

- Which translates into a Gabaix-Koijen multiplier M:

$$M = \frac{1}{2} \sigma_1 \frac{\mathcal{M}}{V_1} \times \sqrt{\frac{1}{T_m}}.$$

« Memory time » = 1/ν

Market cap.



- Note: in the square-root regime, M ≈ 25 for Q=1% ADV

From transient square-root to permanent linear impact: analytical details

(with M. Benzaquen, QF 2018)

The general solution of Eq. (8) is given by:

$$\phi(x,t) = (\mathcal{G}_\nu * \phi_0)(x,t) + \int \mathrm{d}y \int_0^\infty \mathrm{d}\tau\, \mathcal{G}_\nu(x-y, t-\tau) s(y,\tau)\,, \tag{9}$$

where $\phi_0(x) = \phi(x,0)$ denotes the initial condition, and

$$\mathcal{G}_\nu(x,t) = \max(t,0) \frac{\exp\left[-\frac{x^2}{4\sigma_1^2 t} - \nu t\right]}{\sqrt{4\pi\sigma_1^2 t}}\,. \tag{10}$$

Following Donier *et al.* [44], we introduce a buy (sell) meta-order as an extra point-like source of buy (sell) particles with intensity rate $m = Q/T$, where Q is the volume of the metaorder and T the execution time, such that the source term in Eq. (8) becomes: $s(x,t) = m\delta(x - x_t)\cdot \mathbb{1}_{[0,T]} + \lambda\ \mathrm{sign}(x_t - x)$.

Performing the integral over space in Eq. (9) and setting $\phi_0(x) = \phi^{\mathrm{st}}(x)$ yields:

$$\phi(x,t) = \phi^{\mathrm{st}}(x) e^{-\nu t} + m\int_0^{t\wedge T} \mathrm{d}\tau\, \mathcal{G}_\nu(x - x_\tau, t-\tau) - \lambda \int_0^t \mathrm{d}\tau\, \mathrm{erf}\left[\frac{x - x_\tau}{\sqrt{4D(t-\tau)}}\right] e^{-\nu(t-\tau)}\,. \tag{11}$$

The price x_t solves the integral equation:

$$\phi(x_t, t) = 0\,. \tag{12}$$

For $\lambda, \nu \to 0$ and for $t > T$, one immediately recovers Eq. (16) of [44]:

$$x_t = x_t^0 = \frac{m}{\mathscr{L}} \int_0^T \mathrm{d}\tau\, \mathcal{G}_0(x_t - x_\tau, t-\tau), \tag{13}$$

which boils down, at large t, to

$$x_t^0 \approx \frac{Q}{\mathscr{L}} \frac{1}{\sqrt{4\pi\sigma_1^2 t}} = \frac{\sigma_1}{\sqrt{4\pi t}} \frac{Q}{V_1}. \tag{14}$$

Setting $t = T_{\mathrm{m}}$ in this equation immediately leads to Eq. (2), up to a numerical prefactor.

In order to compute the long term impact exactly, the main idea of the calculation is to expand the price trajectory x_t in powers of $\sqrt{\nu}$, i.e.

$$x_t = x_t^0 + \sqrt{\nu} x_t^1 + O(\nu), \tag{15}$$

where x_t^0 and x_t^1 respectively denote the 0th order and 1st order contributions. In the limit of short execution times ($T \ll T_{\mathrm{m}}$) and small meta-order volumes $Q \ll V_{\mathrm{m}}$, where $V_{\mathrm{m}} = V_1 T_{\mathrm{m}}$ is the total volume traded during the memory time T_{m}, one can look for a solution of the form $x_t^1 = F(\nu t)$. In the long time limit $t \gg T$, using the zero-th order solution Eq. 14 and setting $u = \nu t$, Eq. (11) boils down to

$$0 = F(u) + \beta \int_0^u \mathrm{d}v\, \frac{\sqrt{v} - \sqrt{u}}{\sqrt{\pi u v (u-v)}} e^v + \int_0^u \mathrm{d}v\, \frac{F(u) - F(v)}{\sqrt{\pi(u-v)}} e^v\,, \tag{16}$$

where β depends on the fast/slow nature of the execution (see [46] for more details). The solution of this equation for $u \gg 1$ can is found to be

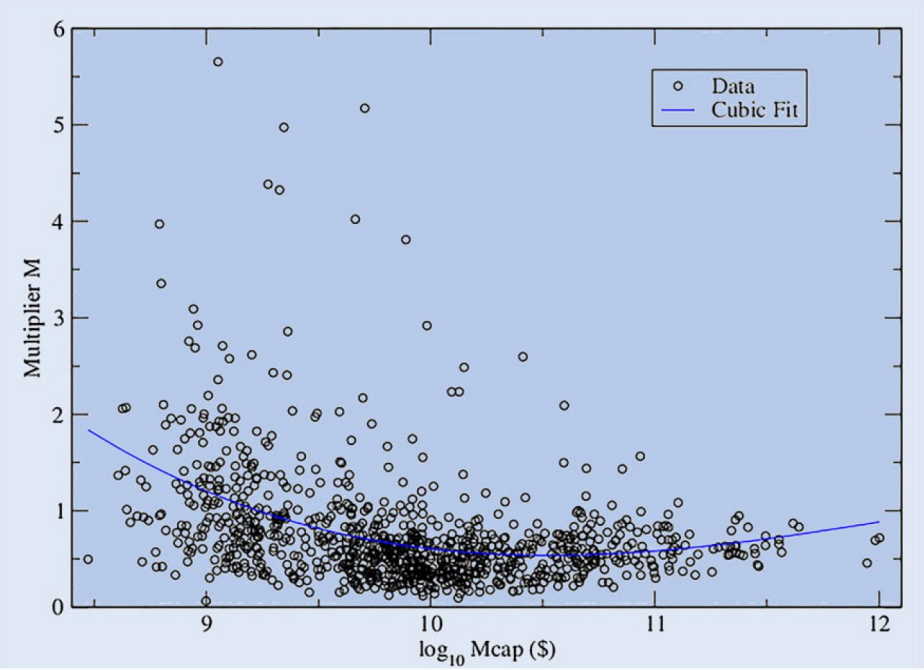
$$F(u) = F_\infty - \frac{\beta}{\sqrt{u}}\left[1 - e^{-u}\right], \tag{17}$$

The microstructural interpretation of GK's multiplier
(JPB, QF 2022)

- On long term, market participants take stock of recent price moves and update their reservation price → permanent impact and a prediction for M

$$M = \frac{1}{2}\sigma_1 \frac{\mathcal{M}}{V_1} \times \sqrt{\frac{1}{T_m}}.$$

- M = ration of two small quantities ~1%
- Guesstimate: T_m ~ 10 – 20 trading days
- Cross-impact explains why the market wide M is 5 times larger than the single stock M (Benzaquen, Eisler, Mastromatteo, JPB)



Consequences

Why do prices move?
Because people trade!

- The GK multiplier is *large* but can be rationalized by LLT without an explicit “equilibrium” mechanism
- On short to medium (months → years), flows are an important determinant of price moves – independently of why people trade!
- Predicting prices mean mostly predicting flows (only partly motivated by “fundamental” information)
- Excess trading and excess volatility are intimately linked

IMH explains much more than EMH

- The theory may explain (most?) markets anomalies
- → If people trend on prices, prices will trend
- → If people pile up on meme stocks, this will wreak havoc
- → If money flows into some funds or into the market as a whole, prices will increase – for no other reason (orders of magnitude match for the 2009-2021 market run)
- On the very long run (5 years), fundamentals may start playing a role and (weakly) anchor price on value
- We should stop speaking about the “fundamental price” which is an empty concept (see TQP)

Trades, Quotes and Prices