

14 April 2023 – Cambridge

# Price Impact, Inelastic Markets and the Order-Driven View of “Anomalies”

Presented by

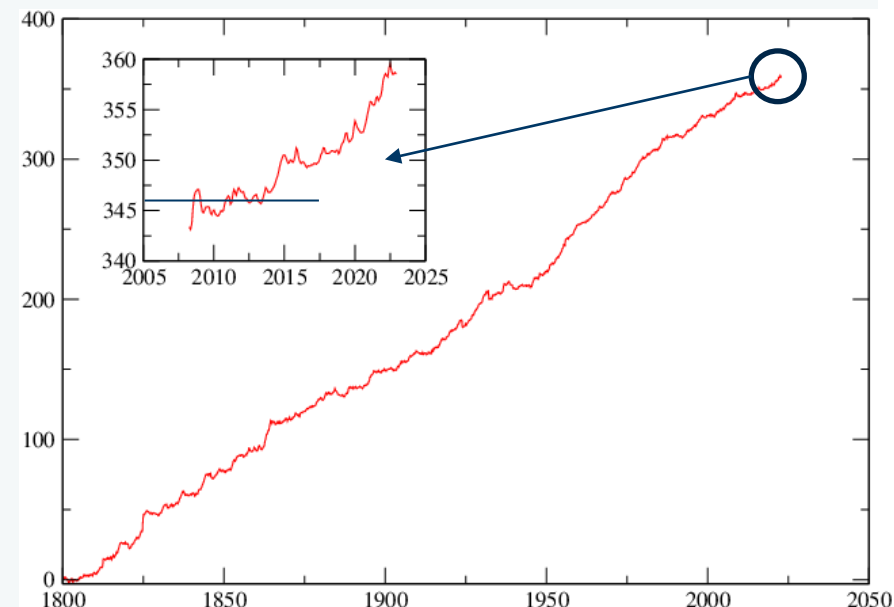
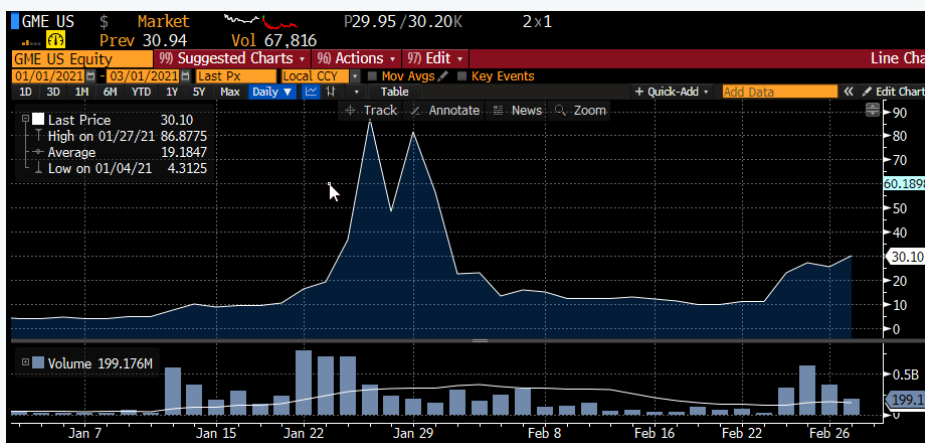
Jean-Philippe Bouchaud to honour Michael Dempster and “Quantitative Finance”

➤ **Why do prices move?**

- **EMH says: because of unanticipated exogenous news**
- **Most financial economists speak about the “fundamental” price**
- **Is this the way we should think about markets?**

## A slew of (old & new) market “anomalies” at odds with EMH

- ▶ The excess volatility puzzle
- ▶ The trend following puzzle + (many) other profitable “anomalies”:  
Prices do not reflect all available information
- ▶ Most “jumps” are not due to news (Cutler-Poterba-Summers, CFM)
- ▶ Several microstructure stylized facts are at odds with EMH (e.g. persistence)
- ▶ Bitcoin and other fancy tulips – but “bubbles do not exist” (Fama)
- ▶ The recent “Reddit stocks” episodes, etc. etc.

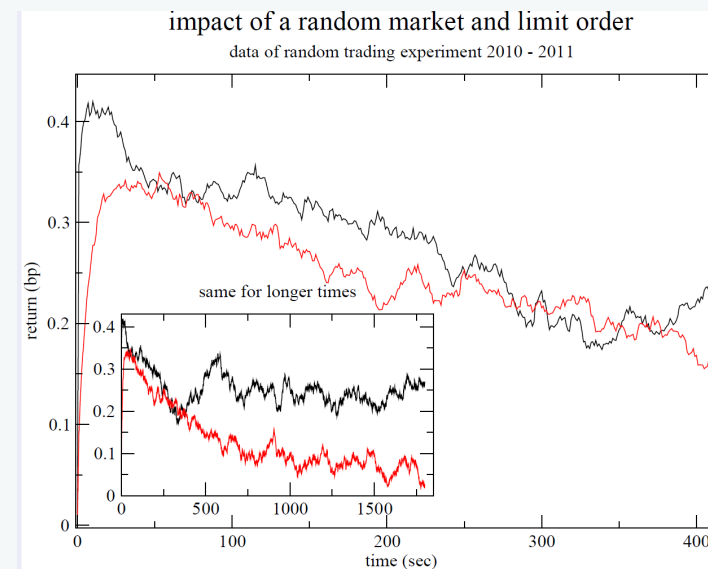
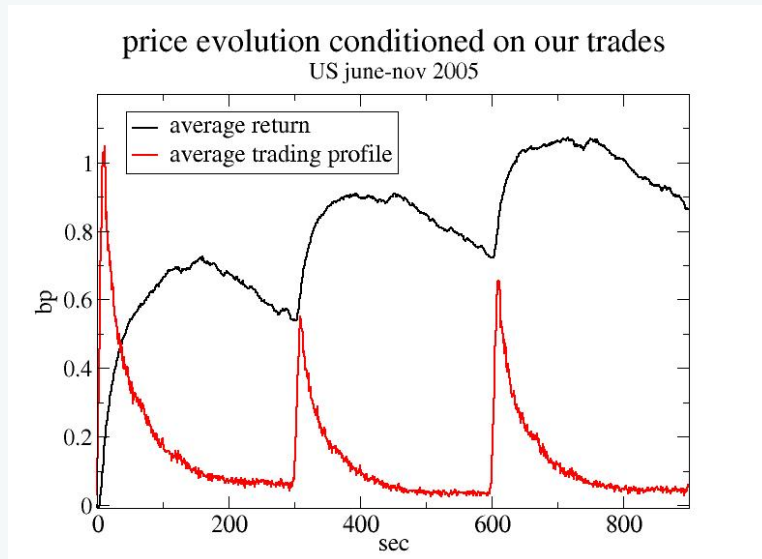


5 month trends: Some (obvious) public information is not included in the current price!  
(Nb: no long bias here, but no costs either)

# Flows do impact prices!

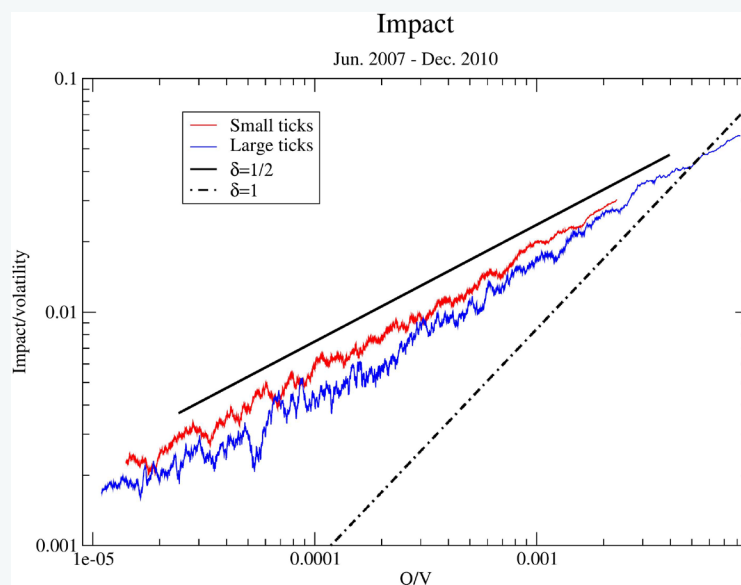
## A (not so trivial) truism

- ▶ Even though for each buyer there is a seller, trading impacts prices (“active” vs. “passive” flows breaks the symmetry)
- ▶ The EMH view: it is because informed traders predict prices!
- ▶ The quant industry view: it is a “mechanical”, order flow effect even when trades are not informed
- ▶ Impact is actually a major source of trading costs
- ▶ OK, say EMH diehards, but impact decays “quickly” anyway and has no long-term consequences – really?

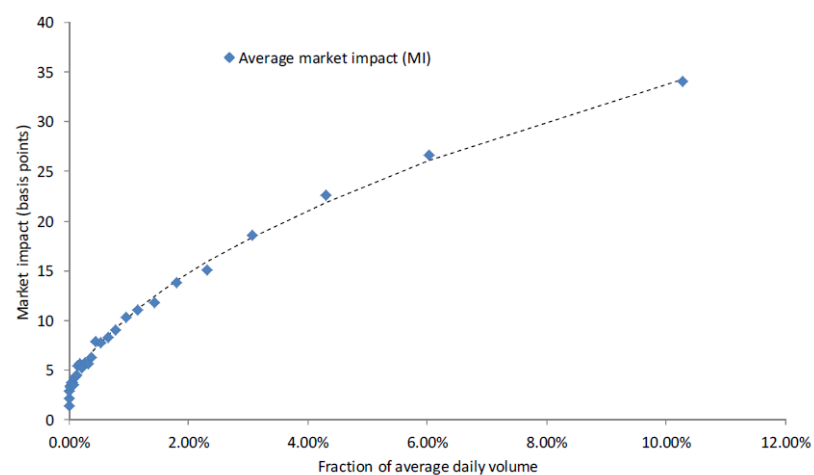


## Impact: a rather non-trivial phenomenon

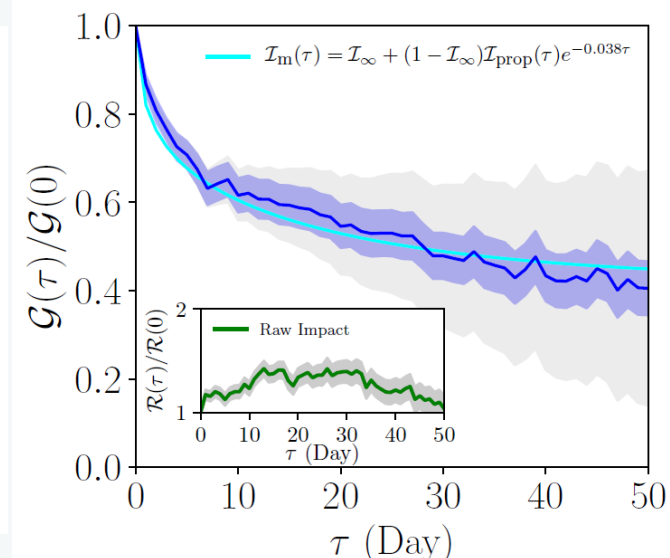
- ▶ Impact grows as the *square-root* of volume – a universal law! (markets, periods, trader types, execution style, etc.)
- ▶ Impact decays extremely slowly, so clearly has at least medium-term consequences
- ▶ Flows, even uninformed, must contribute to (excess) volatility
- ▶ Does impact of uninformed trades go all the way down to zero or actually saturate?
- ▶ Long-term impact: a difficult empirical question – Impact vs. “alpha” vs. noise?



Futures (CFM data)



Stocks (AQR data, limit orders)

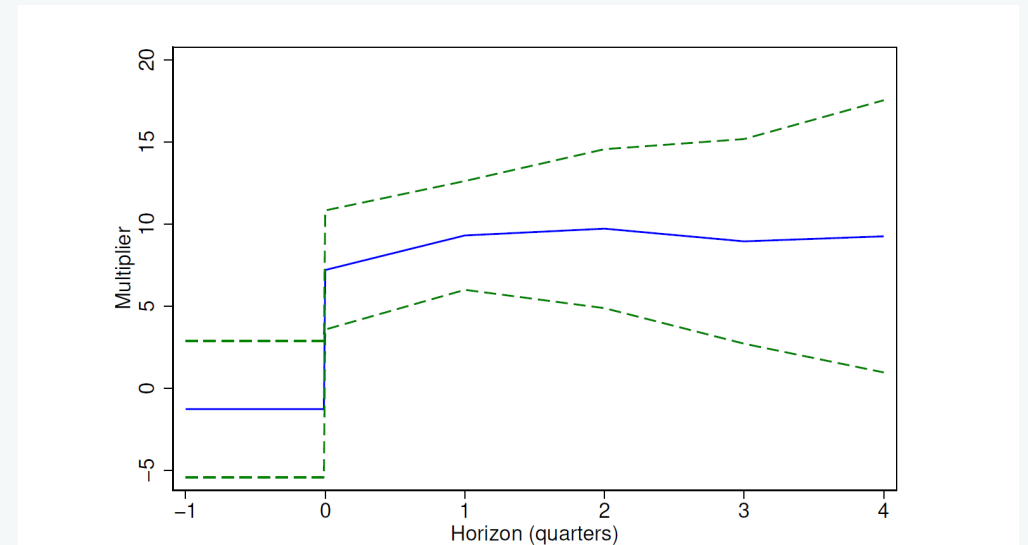


Stocks (Ancerno data)

# The “Inelastic Market Hypothesis” (Gabaix – Koijen)

## The Gabaix – Koijen “multiplier” (2021)

- ▶ Smart regression on holdings of mutual funds → Buying (/selling) 1\$ of the whole market increases (/decreases) the long-term market cap. by 5\$!
- ▶ For single stocks, the multiplier is  $M \approx 1$
- ▶ Mechanism: funds are inelastic due to e.g. mandate constraints
- Holding a constant ratio of 80% stocks and 20% cash means that fund will only agree to sell 1\$ of stocks if prices went up by  $1\$/ (1-0.8)=5\%$
- ▶ A spectacular proposal – the median of economists’ guess is  $M=0$  (and the average is  $M=0.01$ )....
- A big splash: already 185 citations + FT, Bloomberg, The Economist, etc..
- ▶ Ballpark order of magnitude explanation of the 2009-2020 US market rally in terms of inflows





# A unifying framework: The Latent Liquidity Theory

# The Latent Liquidity Theory

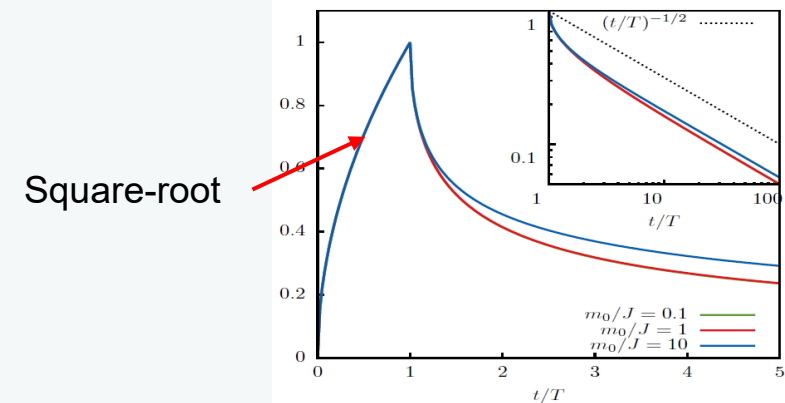
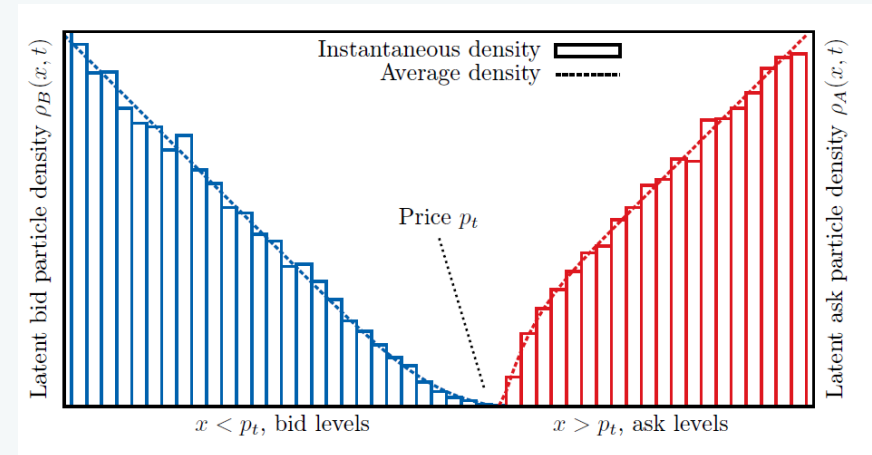
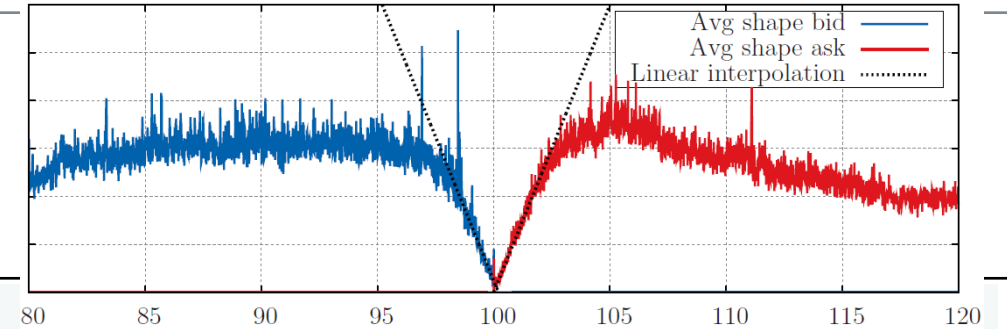
(with J. Bonart, J. Donier, I. Mastromatteo, QF 2015)

- ▶ The Latent Liquidity Theory was developed to understand the square-root law:  $I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$
- ▶ A stylized, generic dynamical model for the “macro” (latent) liquidity around the current price  $x_t$ :

$$\begin{aligned}\partial_t \varphi_b &= D \partial_{xx} \varphi_b - v \varphi_b + \lambda \Theta(x_t - x) - R_{ab}(x) \\ \partial_t \varphi_a &= D \partial_{xx} \varphi_a - v \varphi_a + \lambda \Theta(x - x_t) - R_{ab}(x)\end{aligned}$$

with cancellation rate  $v$  and replenishment rate  $\lambda$

- ▶ Equilibrium latent liquidity is V-shaped around the current price → square-root impact!
- ▶ The short-term inertia of liquidity explains initial impact decay (for  $vT \ll 1$ )
- ▶ Note: any round-trip is costly (on average) – no arbitrage



# From transient square-root to permanent linear impact

(with M. Benzaquen, QF 2018)

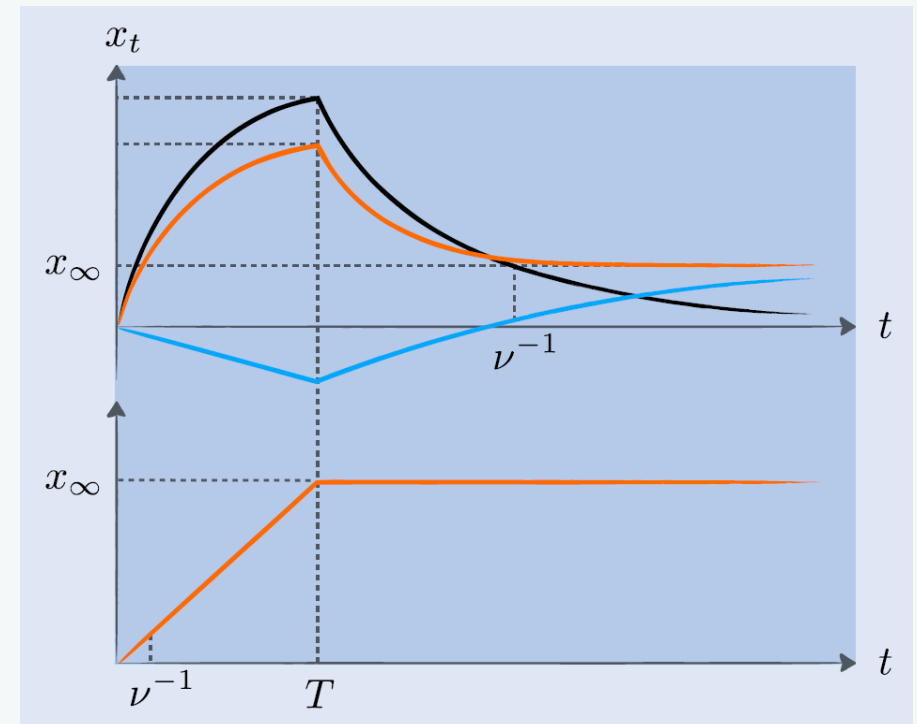
- ▶ But on long term  $\nu T \gg 1$ , market participants take stock of recent price moves and update their reservation price accordingly
- Order flow re-centers around the current price
- Permanent impact
- ▶ Technically  $x_t = \alpha \left[ z_t^0 + \sqrt{\nu} z_t^1 + O(\nu) \right]$
- $I_\infty = a Q$  : long term impact becomes linear in  $Q$
- ▶ Which translates into a Gabaix-Koijen multiplier  $M$ :

$$M = \frac{1}{2} \sigma_1 \frac{\mathcal{M}}{V_1} \times \sqrt{\frac{1}{T_m}}$$

Market cap.

« Memory time » =  $1/\nu$

- ▶ Note: in the square-root regime,  $M \approx 25$  for  $Q=1\%$  ADV



# From transient square-root to permanent linear impact: analytical details

(with M. Benzaquen, QF 2018)

The general solution of Eq. (8) is given by:

$$\phi(x, t) = (\mathcal{G}_\nu * \phi_0)(x, t) + \int dy \int_0^\infty d\tau \mathcal{G}_\nu(x - y, t - \tau) s(y, \tau), \quad (9)$$

where  $\phi_0(x) = \phi(x, 0)$  denotes the initial condition, and

$$\mathcal{G}_\nu(x, t) = \max(t, 0) \frac{\exp\left[-\frac{x^2}{4\sigma_1^2 t} - \nu t\right]}{\sqrt{4\pi\sigma_1^2 t}}. \quad (10)$$

Following Donier *et al.* [44], we introduce a buy (sell) meta-order as an extra point-like source of buy (sell) particles with intensity rate  $m = Q/T$ , where  $Q$  is the volume of the metaorder and  $T$  the execution time, such that the source term in Eq. (8) becomes:  $s(x, t) = m\delta(x - x_t) \cdot \mathbb{1}_{[0, T]} + \lambda \text{sign}(x_t - x)$ .

Performing the integral over space in Eq. (9) and setting  $\phi_0(x) = \phi^{\text{st}}(x)$  yields:

$$\phi(x, t) = \phi^{\text{st}}(x) e^{-\nu t} + m \int_0^{t \wedge T} d\tau \mathcal{G}_\nu(x - x_\tau, t - \tau) - \lambda \int_0^t d\tau \text{erf}\left[\frac{x - x_\tau}{\sqrt{4D(t - \tau)}}\right] e^{-\nu(t - \tau)}. \quad (11)$$

The price  $x_t$  solves the integral equation:

$$\phi(x_t, t) = 0. \quad (12)$$

For  $\lambda, \nu \rightarrow 0$  and for  $t > T$ , one immediately recovers Eq. (16) of [44]:

$$x_t = x_t^0 = \frac{m}{\mathcal{L}} \int_0^T d\tau \mathcal{G}_0(x_t - x_\tau, t - \tau), \quad (13)$$

which boils down, at large  $t$ , to

$$x_t^0 \approx \frac{Q}{\mathcal{L}} \frac{1}{\sqrt{4\pi\sigma_1^2 t}} = \frac{\sigma_1}{\sqrt{4\pi t}} \frac{Q}{V_1}. \quad (14)$$

Setting  $t = T_m$  in this equation immediately leads to Eq. (2), up to a numerical prefactor.

In order to compute the long term impact exactly, the main idea of the calculation is to expand the price trajectory  $x_t$  in powers of  $\sqrt{\nu}$ , i.e.

$$x_t = x_t^0 + \sqrt{\nu} x_t^1 + O(\nu), \quad (15)$$

where  $x_t^0$  and  $x_t^1$  respectively denote the 0th order and 1st order contributions. In the limit of short execution times ( $T \ll T_m$ ) and small meta-order volumes  $Q \ll V_m$ , where  $V_m = V_1 T_m$  is the total volume traded during the memory time  $T_m$ , one can look for a solution of the form  $x_t^1 = F(\nu t)$ . In the long time limit  $t \gg T$ , using the zero-th order solution Eq. 14 and setting  $u = \nu t$ , Eq. (11) boils down to

$$0 = F(u) + \beta \int_0^u dv \frac{\sqrt{\nu} - \sqrt{u}}{\sqrt{\pi u v (u - v)}} e^v + \int_0^u dv \frac{F(u) - F(v)}{\sqrt{\pi(u - v)}} e^v, \quad (16)$$

where  $\beta$  depends on the fast/slow nature of the execution (see [46] for more details). The solution of this equation for  $u \gg 1$  can be found to be

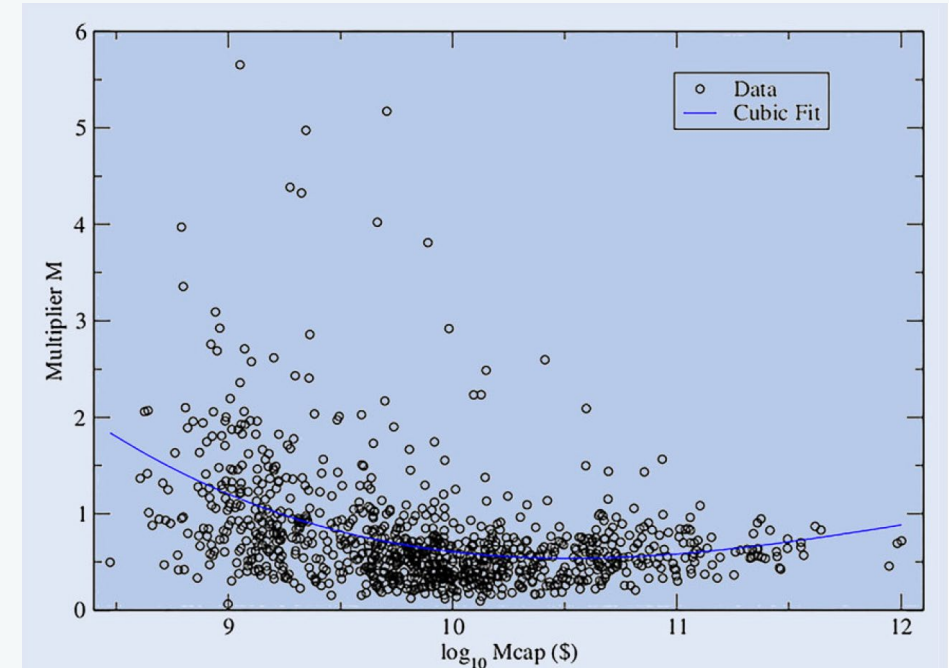
$$F(u) = F_\infty - \frac{\beta}{\sqrt{u}} [1 - e^{-u}], \quad (17)$$

# The microstructural interpretation of GK's multiplier (JPB, QF 2022)

- ▶ On long term, market participants take stock of recent price moves and update their reservation price → permanent impact and a prediction for M

$$M = \frac{1}{2} \sigma_1 \frac{\mathcal{M}}{V_1} \times \sqrt{\frac{1}{T_m}}$$

- ▶ M = ration of two small quantities ~1%
- ▶ Guesstimate:  $T_m \sim 10 - 20$  trading days
- ▶ Cross-impact explains why the market wide M is 5 times larger than the single stock M (Benzaquen, Eisler, Mastromatteo, JPB)



# Consequences

## Why do prices move?

Because people trade!

- ▶ The GK multiplier is *large* but can be rationalized by LLT without an explicit “equilibrium” mechanism
- ▶ On short to medium (months → years ), flows are an important determinant of price moves
  - independently of why people trade!
- ▶ Predicting prices mean mostly predicting flows (only partly motivated by “fundamental” information)
- ▶ Excess trading and excess volatility are intimately linked

## IMH explains much more than EMH

- ▶ The theory may explain (most?) markets anomalies
  - If people trend on prices, prices will trend
  - If people pile up on meme stocks, this will wreak havoc
  - If money flows into some funds or into the market as a whole, prices will increase – for no other reason (orders of magnitude match for the 2009-2021 market run)
- ▶ On the very long run (5 years), fundamentals may start playing a role and (weakly) anchor price on value
- ▶ We should stop speaking about the “fundamental price” which is an empty concept (see TQP)





# Trades, Quotes and Prices

"An impressive book that no serious student of market microstructure can afford to be without. Simultaneously quantitative and highly readable."

Jim Gatheral, Baruch College, CUNY

"I highly recommend this to anyone who wants to see how physics has benefitted economics, or for that matter, to anyone who wants to see a stellar example of a theory grounded in data."

Doyne Farmer, University of Oxford

"This is a masterful overview of the modern and rapidly developing field of market microstructure, from several of its creators. This book will be an essential resource for practitioners, academics, and regulators alike."

Robert Almgren, New York University and Quantitative Brokers

The widespread availability of high-quality, high-frequency data has revolutionised the study of financial markets. By describing not only asset prices, but also market participants' actions and interactions, this wealth of information offers a new window into the inner workings of the financial ecosystem. In this original text, the authors discuss empirical facts of financial markets and introduce a wide range of models, from the micro-scale mechanics of individual order arrivals to the emergent, macro-scale issues of market stability. Throughout this journey, data is king. All discussions are firmly rooted in the empirical behaviour of real stocks, and all models are calibrated and evaluated using recent data from NASDAQ. By confronting theory with empirical facts, this book for practitioners, researchers and advanced students provides a fresh, new and often surprising perspective on topics as diverse as optimal trading, price impact, the fragile nature of liquidity, and even the reasons why people trade at all.

**Jean-Philippe Bouchaud** is a pioneer in Econophysics. He co-founded the company Science & Finance in 1994, which merged with Capital Fund Management (CFM) in 2000. He was awarded the CNRS Silver Medal in 1995 and the Risk Quant of the Year Award in 2017.

**Julius Bonart** is a lecturer at University College London, where his research focuses on market microstructure and market design.

**Jonathan Donier** completed a PhD at University Paris 6 with the support of the Capital Fund Management Research Foundation and currently works in the technology sector.

**Martin Gould** currently works in the technology sector. Previously, he was a James S. McDonnell Postdoctoral Fellow in the CFM–Imperial Institute of Quantitative Finance at Imperial College London.

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Bouchaud, Bonart, Donier and Gould  
TRADES, QUOTES AND PRICES

## TRADES, QUOTES AND PRICES

Financial Markets Under the Microscope

Jean-Philippe Bouchaud, Julius Bonart,  
Jonathan Donier and Martin Gould

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