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Individual asset liability management

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1. Introduction

A recent discussion of the future of life-cycle saving and investment posed the question: "Can computer-based personal financial planning models that conform to the principles of economics be both helpful and commercially viable?" (Bodie 2007, p. xvii).

The heterogeneity of personal financial plans and the interplay between economic considerations and individual aspirations make the problem of personal finance one of the most challenging in economics. At the heart of personal finance problems lies the fundamental consumption/investment problem which has been studied by some of the best minds in economics and finance. Samuelson (1948) devoted much of his early work to communicating the practical implications of economics for household decision making. Modigliani and Brumberg (1954) proposed the life-cycle hypothesis based on the relationship between saving and consumption over a lifetime. Then Samuelson (1969) and Merton (1969) formulated relationships between consumption and portfolio allocation in terms of expected returns and volatilities in order to maximize total lifetime utility. Kahneman and Tversky (1979) introduced a utility function which applies to gains and losses from financial assets and emphasized the qualitative aspects of decisions made by individuals. In spite of the importance of the life-cycle investment and saving problem for the rapidly

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growing aging populations of the developed nations, there is relatively little academic research focussed on personal finance. Advice to individuals is mainly based on the expertise of financial planning professionals which continues to be "a domain of common sense, which is not the same thing as good sense"† (Samuelson 2007, p. 2).

As computing power has advanced, solutions to personal financial planning problems have turned to new technologies. The two most notable examples, both within the asset liability framework, are the Home Account (Berger and Mulvey 1998), which is based on stochastic programming methodology, and ESPlanner (Kotlikoff 2008), which uses dynamic programming techniques.

In this article we describe a third example, the *individual* Asset Liability Management (*i*ALM) tool (Cambridge Systems Associates 2008a,b), which advances the dynamic stochastic programming approach further. The system handles many aspects of an individual investor's circumstances and generates an optimal lifelong financial plan.

Our focus is the functionality of *i*ALM. The basic concepts behind its implementation are given in section 2, but the technical details, which are vast, are omitted. In section 3, typical family data (household 'profiles') are used to illustrate how *i*ALM's solutions depend on the personal preferences of individuals such as retirement age, priorities of major consumption goals and numerous other factors. These inputs to the problem influence optimal investment and saving decisions, set the household's varying attitude to risk over time and define the feasibility of goal achievement. Experimenting with variations of preferences expressed in the data inputs shows that *i*ALM emulates behavioural patterns. We believe that the optimization results using *i*ALM support many empirical observations from behavioural finance. In the future, such systems can help families to identify sustainable spending levels for retirement, which is of paramount importance for the ever-expanding retired populations of the world.

2. *i*ALM formulation and implementation

There are enormous variations in age, family structure, initial wealth, income and investment objectives across individuals looking for financial advice. The following features are common in any specific household instance of the *i*ALM life-cycle financial planning problem.

The *time horizon* of the *i*ALM problem is household life span. This is given by the life span of the surviving partner. Therefore, the long-term investment problem is of *random length*. For an individual who has just started his/her professional career the duration of this life-cycle problem may be over 70 years.

With liabilities arising at any time, investment, saving and other financial decisions must change across household lifetime as a response to changing life and market conditions. This is therefore a *dynamic multi-stage problem*. The stages correspond to major changes in personal circumstances, e.g. retirement date, big purchases such as real estate, and many others.

The household decision maker must deal with mostly *stochastic cash flows*; both incomes and liabilities are linked to future economic fundamentals, which are uncertain.

Any solution procedure should accommodate 're-use' of the model as real time evolves, with the *ability to modify inputs and recalibrate* the life-cycle plan.

It should quantify the satisfaction gained from accumulating wealth as the ability to achieve the household's desired lifetime goals, which include specified annual living costs. Formally, this translates into the objective of *maximizing the real spending* on selected goals which the financial portfolio can sustain throughout the client's lifetime. This objective falls conceptually into the definition of wealth as 'sustainable spending' (Arnott 2006).

2.1. Principles of dynamic stochastic programming

The *i*ALM tool is implemented using *dynamic stochastic programming* (DSP) methodology and solution techniques.

There are many applications of DSP in industrial planning and management (Prekopa 1995, Dempster *et al.* 2000). Institutional funds, and particularly pension funds, use stochastic programming techniques for portfolio construction and for the formulation of optimal trading strategies (see, for example, Zenios and Ziemba 2007 and Dempster *et al.* 2008).

In what follows we briefly describe the major steps in the construction of a dynamic stochastic programme, with the aim of introducing this methodology to the novice reader.

Dynamic stochastic programming incorporates many alternative futures in the form of simulated scenarios from a discrete-time, continuous-state, multi-dimensional stochastic *data process*‡

$$\begin{aligned}\boldsymbol{\omega} &:= \{\boldsymbol{\omega}_t : t = t_{1,0}, \ldots, t_{T+1,0}\}\\ &= \{\underbrace{\boldsymbol{\omega}_{t_{1,0}}, \ldots, \boldsymbol{\omega}_{t_{1,u}}}_{\text{stage 1}}; \underbrace{\boldsymbol{\omega}_{t_{2,0}}, \ldots, \boldsymbol{\omega}_{t_{2,u}}}_{\text{stage 2}}; \ldots; \underbrace{\boldsymbol{\omega}_{t_{T,0}}, \ldots, \boldsymbol{\omega}_{t_{T,u}}; \boldsymbol{\omega}_{t_{T+1,0}}}_{\text{stage } T.}\}\end{aligned}$$

The *stages* correspond to the expected times of major changes for decisions in the future. In general, this is a discretization of time at a frequency different from that of the data process's simulation steps.

The evolution of the simulated data process across time is given by a *scenario tree*. For example, in figure 1 the 3–3–2 scenario tree shows *branches* three times at stage 1, then at stage 2 each scenario branches into three further scenarios, and again at stage 3 each scenario branches into two scenarios. This branching schematically represents the uncertainty regarding the

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†Keynote address at the above conference.

‡Throughout we use boldface to denote random entities.

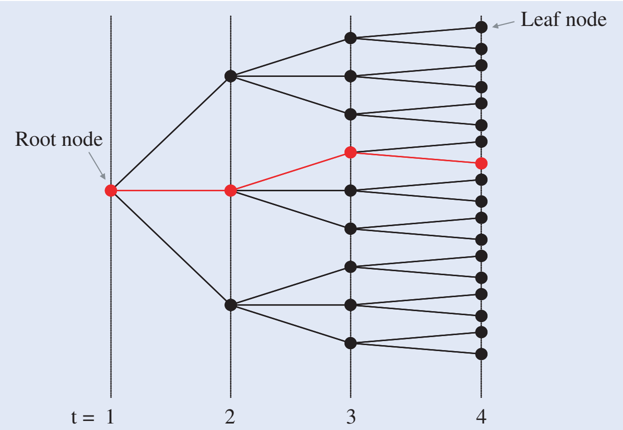

Figure 1. An example scenario tree schema.

state of the underlying simulated data process in 18 scenarios.

All decisions at intermediate nodes of the tree take into account the possible evolution of the stochastic data process from that point forward. The decision at the root node encompasses all uncertainty and, in this sense, it is a ‘robust’ solution of the DSP problem with respect to all generated states of the stochastic data process.

A generic dynamic stochastic programming problem (Dempster 1988, 2006) is given by

$$\min_{x_{t_{1,0}},\dots,x_{t_{1,u}}} f_1(x^{t_{1.u}}) + \mathbb{E}_{\boldsymbol{\omega}^{2,0}}\left\{ \min_{x_{t_{2,0}},\dots,x_{t_{2,u}}} f_2(\boldsymbol{\omega}^{t_{2,0}}, \mathbf{x}^{t_{2,u}}) + \cdots \right.$$
$$\left. + \mathbb{E}_{\boldsymbol{\omega}^{t_{T,0}}|\omega^{t_{T-1,0}}}\left[\min_{x_{t_{T,0}},\dots,x_{t_{T,u}}} f_T(\boldsymbol{\omega}^{t_{T,0}}, \mathbf{x}^{t_{T,0}}) \right] \right\},$$

s.t.

$$A_{1,1}x_{t_{1,0}} = b_1,$$
$$A_{2,1}(\boldsymbol{\omega}^{t_{1,1}})x_{t_{1,0}} + A_{2,2}(\boldsymbol{\omega}^{t_{1,1}})x_{t_{1,1}}(\boldsymbol{\omega}^{t_{1,1}}) = b_2(\boldsymbol{\omega}^{t_{1,1}}) \quad \text{a.s.},$$
$$\vdots$$
$$A_{Tu+1,1}(\boldsymbol{\omega}^{t_{T+1,0}})x_{t_{1,0}} + \cdots + A_{Tu+1,Tu}(\boldsymbol{\omega}^{t_{T+1,0}})x_{t_{T,u}}(\boldsymbol{\omega}^{t_{T+1,0}})$$
$$= b_{Tu+1}(\boldsymbol{\omega}^{t_{T+1,0}}) \quad \text{a.s.},$$

where the constraints hold *almost surely* (a.s.), i.e. with probability one.

The idea of the multi-stage model is that, at each stage, an observation is made, which is then followed immediately by a decision, i.e. an observation is taken just before a decision is made. Decisions are *non-anticipative*, which means that decisions made at any stage are only dependent on the information available up to that time. This is achieved by fixing portfolio decisions to be the same across all scenarios originating from the same branch point. Subsequent decisions in periods between stages on scenarios in the tree take into account all possible scenarios in that stage.

The objective of the DSP problem is in the form of nested optimization problems given by the conditional expectation of the data and decision process

$$\mathbf{x} := \{x_{t_{1,0}}, x_{t_{1,1}}, \dots, x_{t_{1,u}}; x_{t_{2,0}}, \dots, x_{t_{2,u}}; x_{t_{T,0}}, \dots, x_{t_{T,u}}\}.$$

The constraints run across time and correspond to stages of the *decision process* with its first period deterministic decision $x_{t_{1,0}}$.

This conceptual dynamic representation is used to generate a *deterministic equivalent* of the DSP with the specific probabilistic structure (given by the scenario tree) for the solution (Dantzig and Madansky 1961) as

$$\min \left\{ f_1(x^{t_{1,u}}) + \sum_{\Omega_{t_{2,0}}} p_{t_{2,0}}(\omega_{t_{2,0}}) f_{t_{2,0}}(\omega_{t_{2,0}}, x_{t_{2,0}}(\omega_{t_{2,0}}), \dots, \right.$$
$$x_{t_{2,u}}(\omega_{t_{2,0}})) + \cdots + \sum_{\Omega_{t_{T,0}}} p_{t_{T,0}}(\omega_{t_{T,0}}) f_{t_{T,0}}(\omega_{t_{T,0}}, x_{t_{T,0}}(\omega_{t_{T,0}}), \dots,$$
$$\left. x_{t_{T,u}}(\omega_{t_{T,0}})) \right\},$$

s.t.

$$A_{1,1}x_{t_{1,0}} = b_1,$$
$$A_{2,1}(\omega_{t_{1,1}})x_{t_{1,0}} + A_{2,2}(\omega_{t_{1,1}})x_{t_{1,1}}(\omega_{t_{1,1}}) = b_2(\omega_{t_{1,1}}), \quad \omega_{t_{1,1}} \in \Omega_{1,1},$$
$$\vdots$$
$$A_{Tu+1,1}(\omega_{t_{T+1,0}})x_{t_{1,0}} + \cdots + A_{Tu+1,Tu}(\omega_{t_{T+1,0}})x_{t_{T,u}}(\omega_{t_{T+1,0}})$$
$$= b_{Tu+1}(\omega_{t_{T+1,0}}), \quad \omega_{t_{T+1,0}} \in \Omega_{t_{T+1,0}}.$$

Note that all previous values of both the data and decision processes are allowed here to influence the current decisions.†

In the deterministic equivalent problem, all random coefficients in the constraints of the DSP are *realizations* of the underlying stochastic process represented in the scenario. In the case of linear constraints and objective this is a very large *linear programming* (LP) problem which becomes very sparse when the problem is Markovian. We can therefore use standard solution techniques to solve this linear programme numerically.‡

2.2. Implementation of *i*ALM

In general, the solution of the household life-cycle DSP problem by *i*ALM is comprised of three stages: forward simulation of the stochastic data processes, solution of the optimization problem and analysis of the optimal decisions.

Figure 2 illustrates how different models and processes in *i*ALM are linked to form a stochastic optimization problem.

2.2.1. Stage 1. Simulation of stochastic processes. As mentioned above, the time span of an individual household’s lifetime is random. The event of death of the

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†This non-Markovian structure is required for *i*ALM when considering, for example, mortgaged house purchases.
‡Stochastics$^{\text{TM}}$ is CSA’s generic modular software for the formulation and solution of DSP models.

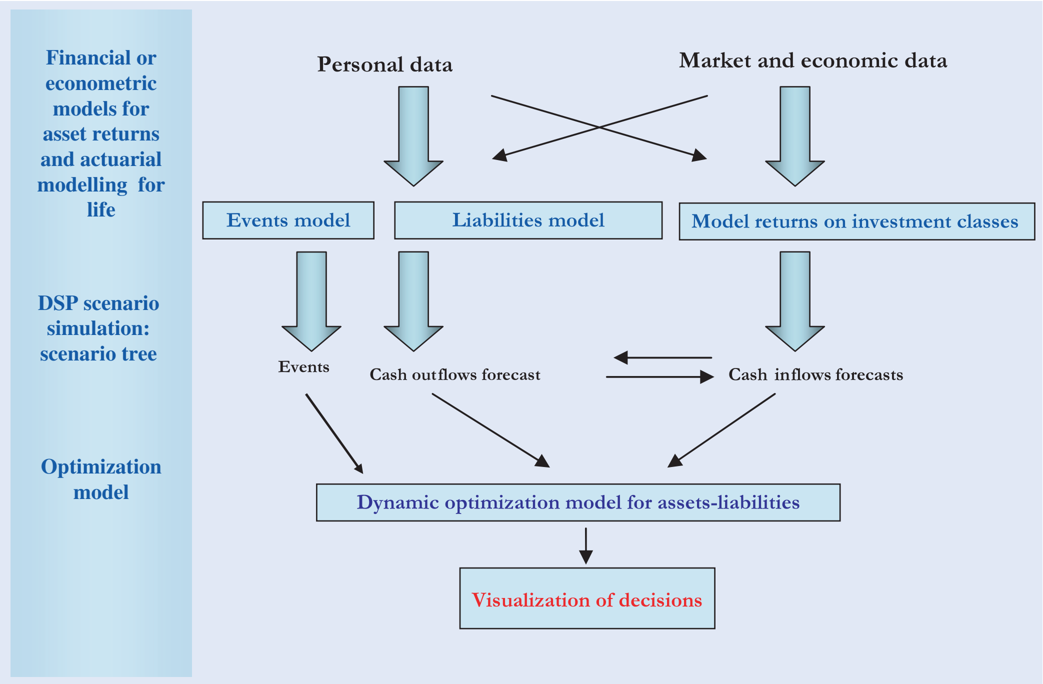


Figure 2. Overview of the *i*ALM system.

surviving partner ends its life, and hence the scenario. Therefore, lengths of generated scenarios correspond to the random durations of individuals' lives (events model). Similarly, occurrences of a variety of liability events, e.g. serious illness, and costs of associated liabilities, etc., are also incorporated in the model, based on actuarial data. The simulation time step is annual.

Models for asset returns are similar to those used in institutional ALM (Dempster *et al.* 2006). The link with the stochastic costs of liabilities is implemented through an inflation process over which appropriate spreads can be defined for different types of cash flows.

2.2.2. Stage 2. Optimization.

Wealth is generated through optimum portfolio allocation. We see wealth as generating 'sustainable spending' and the primary goal of *i*ALM is thus "to increase the real spending that a portfolio can sustain" (Arnott 2006, p. 6).

The overall *objective* of the *i*ALM optimization is to maximize the *expected utility of lifetime consumption*, taking into account total tax payments and excess borrowing, i.e.

$$\mathbb{E}\left[\int_{t=1}^{T} 1_{\{\text{any alive},t\}}\,\mathrm{u}_t(\boldsymbol{C}_t)\right],$$

where

$$\mathrm{u}_t(\boldsymbol{C}_t) = \sum_{g\in G} \mathrm{u}_{g,t}(\boldsymbol{y}_t) - \frac{1}{\boldsymbol{\varphi}_t}\left(\pi^{\mathrm{xs}}\mathbf{z}_t^{\mathrm{xs}} + \pi^{\tau i}\mathbf{I}_t^{\tau}\right).$$

Here, $1_{\{\text{any alive},t\}}$ is an indicator function to handle the *random length* of life *scenarios*, u_t is the *utility* at time t, G is the set of all *goals* with $\mathrm{u}_{g,t}$ being the utility for a specific goal g at time t, $\boldsymbol{\varphi}_t$ is the *inflation* index at time t, $\mathbf{z}_t^{\mathrm{xs}}$ is *excess borrowing*—an auxiliary variable introduced for dealing with possible *bankruptcy*, and $\mathbf{I}_t^{\tau}$ is the total tax payable with π^{xs} and $\pi^{\tau i}$ being the respective penalty coefficients.

Consumption $\mathbf{C}_t$ is defined as spending on chosen goals at time t. Spending will grow with goal-specific inflation rate $\boldsymbol{\varphi}_{g,t}$ and is distributed between equity (preserving) goals, like real estate, and non-capital goals. Thus

$$\mathbf{C}_t = \sum_{g\in G_m} \boldsymbol{\varphi}_{g,t_g^s}\left(\mathbf{F}^{\mathrm{d}}_{g,t} + \mathbf{F}^{\mathrm{m}}_{g,t}\right) + \sum_{g\in G\setminus G_m} \boldsymbol{\varphi}_{g,t}\hat{\mathbf{y}}_{g,t},$$

where the subset of goals G_m is the set of *real estate* goals, which may be mortgaged. Such goals with purchase price z_g^- require a *down payment* $\mathbf{F}^{\mathrm{d}}_{g,t}$ at t_g^s in the first year of the goal and an annual *mortgage payment* $\mathbf{F}^{\mathrm{m}}_{g,t}$ thereafter. Other 'non-capital' goals have no equity value but have *spending* $\hat{\mathbf{y}}_{g,t}$ on goal g at time t.

Net goal wealth consists of cash holdings (liquid wealth) and the value of equity in goals, e.g. equity in real estate (see figure 3). For example, home equity in any year is purchase price scaled up by inflation less the present value of future mortgage payments.

The *utility function for each individual goal* is constructed for a range of spending between *acceptable* (s) and *desirable* (g) values, subject to existing and foreseen liabilities, and a *minimum* required spending (h). The utility function for a specific goal is a piece-wise linear function as illustrated by figure 4. The slope of the (s, g) section can be thought of as the goal's *priority*. At times when multiple goals are present it has the effect of directing spending to goals with higher marginal utilities of consumption.

Objectives for investment are dependent on many factors, like personal priorities, aspirations, human

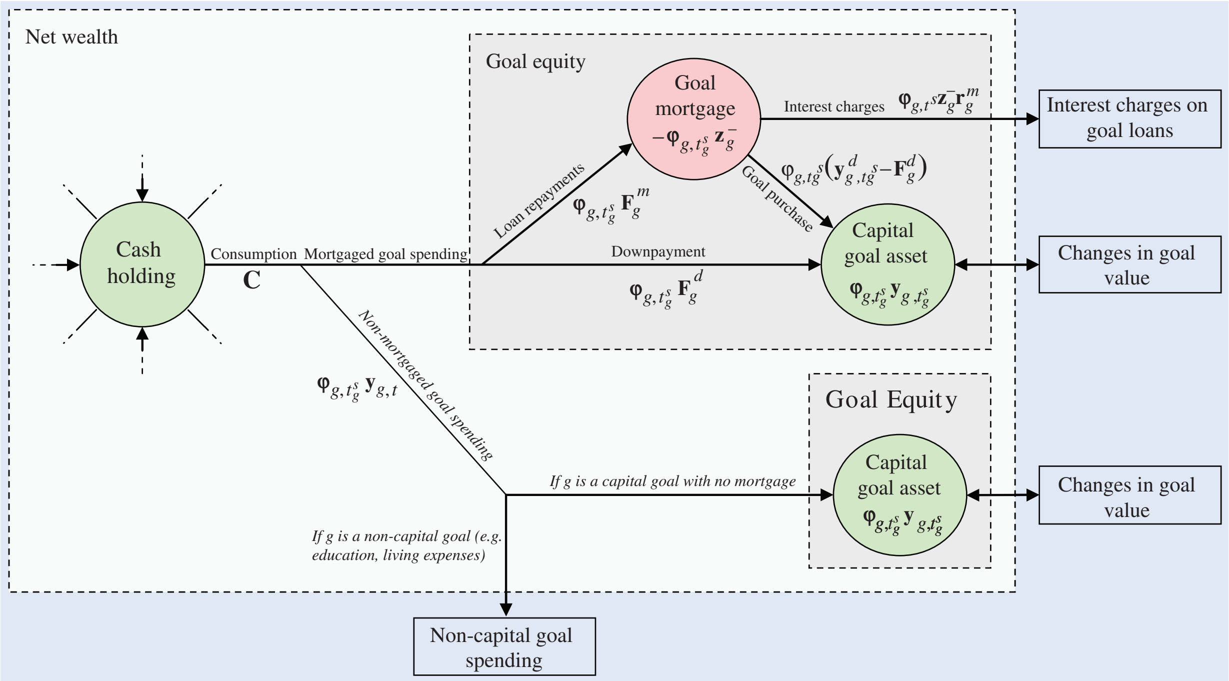


Figure 3. Goal spending cash flow diagram.

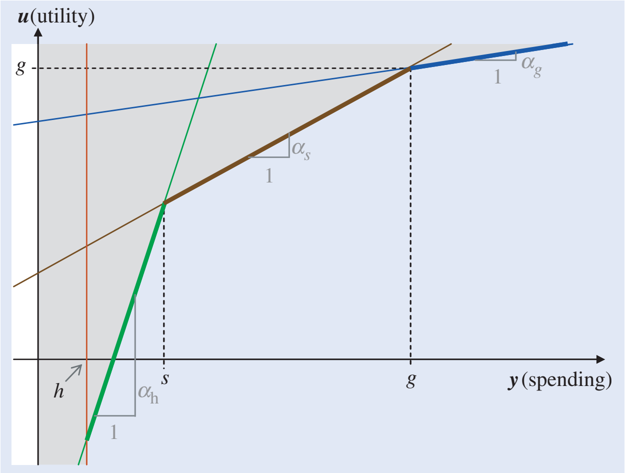


Figure 4. Piecewise utility function for goals.

capital, family status, and so on. In this context, *i*ALM may be interpreted as *an optimum resource allocation problem* over an individual household's life.

The *parameters* of the model used for portfolio construction are:

price $\mathbf{p}_{a,t}$ of asset *a* at time *t*
return $\mathbf{r}_{a,t}$ on asset *a* at time *t*
coupon return $\mathbf{r}_{a,t}^{\mathrm{c}}$ on asset *a* at time *t*
interest rate $\mathbf{r}_t^{\mathrm{cash}}$ on cash deposits at time *t*
spread r^{m} of interest rate *on margin* loans over cash rate
dividend return r_a^{d} on asset *a*
transaction cost $r_a^{\mathrm{tx}-}$ (proportional) of *purchase* of asset *a*
transaction cost $r_a^{\mathrm{tx}+}$ (proportional) of *sale* of asset *a*
lower position limit l_a^{lower} for asset *a* (proportion of portfolio)
upper position limit l_a^{upper} for asset *a*
turnover limit α^x (as proportion of portfolio value) for each *asset*
turnover limit α_0^x for *initial rebalance*
minimum acceptable portfolio *value* ρ^d (proportional to previous year).

The *decision variables* (those in bold are decisions at the nodes of the scenario tree) for implementing the optimum portfolio to generate wealth and control borrowing are:

value of holding $\mathbf{x}_{a,t}$ of asset *a* at time *t*
value of asset a sold at time *t*, $\mathbf{x}_{a,t}^-$
value of asset a bought at time *t*, $\mathbf{x}_{a,t}^+$
cash holding (*banked cash*) at time *t*, $\mathbf{z}_t^+$
quantity of asset a held at time *t*, $\mathbf{q}_{a,t}$
quantity of asset a sold at time *t*, $\mathbf{q}_{a,t}^-$
quantity of asset a bought at time *t*, $\mathbf{q}_{a,t}^+$
decrease in portfolio value at time *t*, $\mathbf{P}_t^-$
increase in portfolio value at time *t*, $\mathbf{P}_t^+$
portfolio value at time *t*, $\mathbf{P}_t$
portfolio losses in excess of maximum acceptable loss at time *t*, $\mathbf{P}_t^d$
income from coupon payments at time *t*, $\mathbf{I}_t^{\mathrm{C}}$
income from dividend payments at time *t*, $\mathbf{I}_t^{\mathrm{D}}$
margin borrowing at time *t*, $\mathbf{m}_t$
repayment of margin loans at time *t*, $\mathbf{m}_t^-$
additional margin borrowing at time *t*, $\mathbf{m}_t^+$
income borrowing, $\mathbf{z}_t^-$
excess borrowing (used to define bankruptcy), $\mathbf{z}_t^{xs}$.

Fundamental constraints used for portfolio construction are similar to the setting of an institutional ALM problem (Zenios and Ziemba 2007, Dempster *et al.* 2008). The main constraints, which are significantly

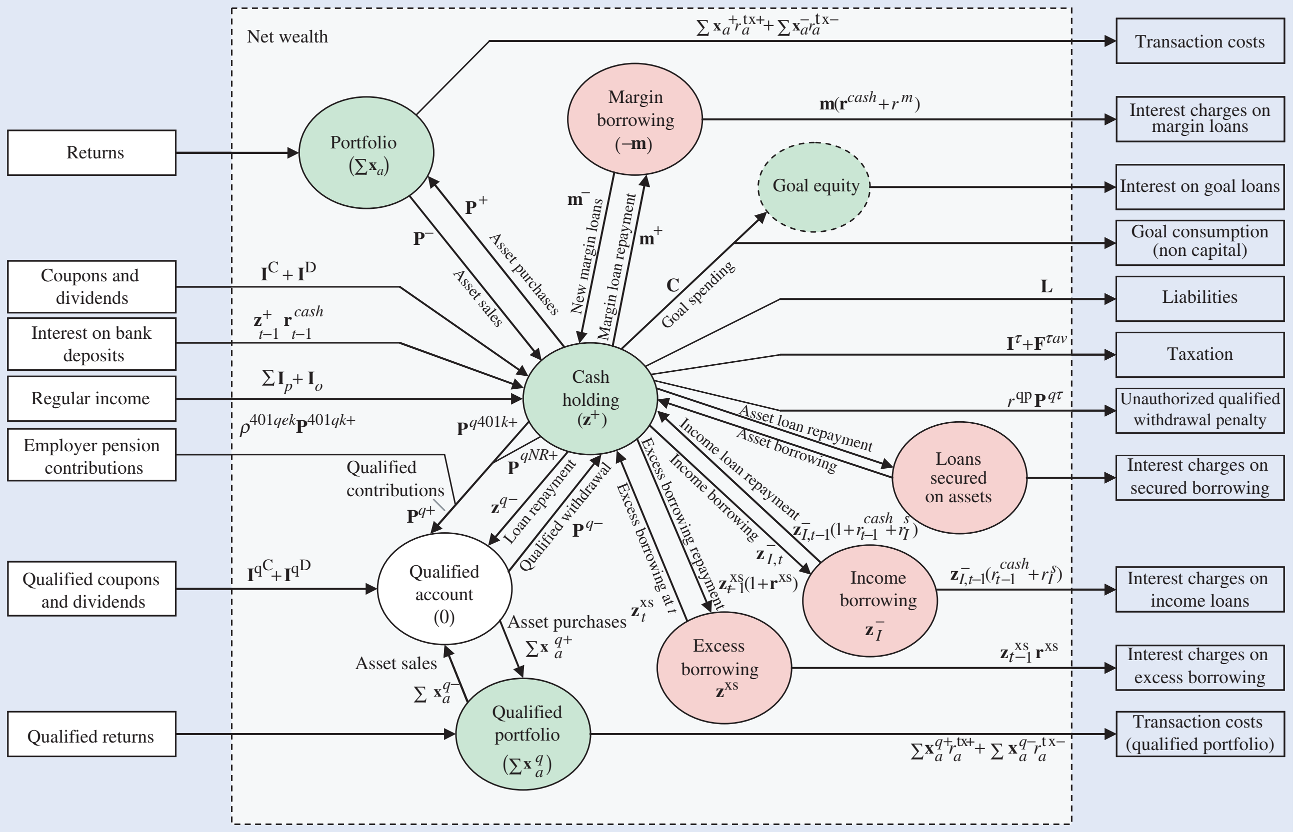


Figure 5. Cashflow diagram for the model.

more complex than in institutional ALM problems, are cash flow balance constraints. These take into account all cash flows at all nodes of the scenario tree. To obviate numerous mathematical expressions the cash-flow balance constraints are shown graphically in figure 5.

In summary, the solution of the *i*ALM optimization problem involves making decisions related to optimal spending on specified goals, optimal portfolio allocations, optimal saving, borrowing and many other decisions over an individual household's life-cycle. In a DSP problem, the set of optimal decisions at the root node of the scenario tree corresponds to the present moment and must be implemented, e.g. the recommended portfolio rebalancing for the coming year.

2.2.3. Stage 3. Visualization of results. The graphical user interface is designed to incorporate an extensive range of possible inputs. It allows for inclusion of various life insurance policies, different types of pension policies, a choice of mortgages, an assortment of loans and borrowing opportunities, social security and a large range of assets. By using *i*ALM in an interactive manner the relationship between inputs to the model and optimal results are critical for analysis of the proposed solution and examination of the household's alternatives.

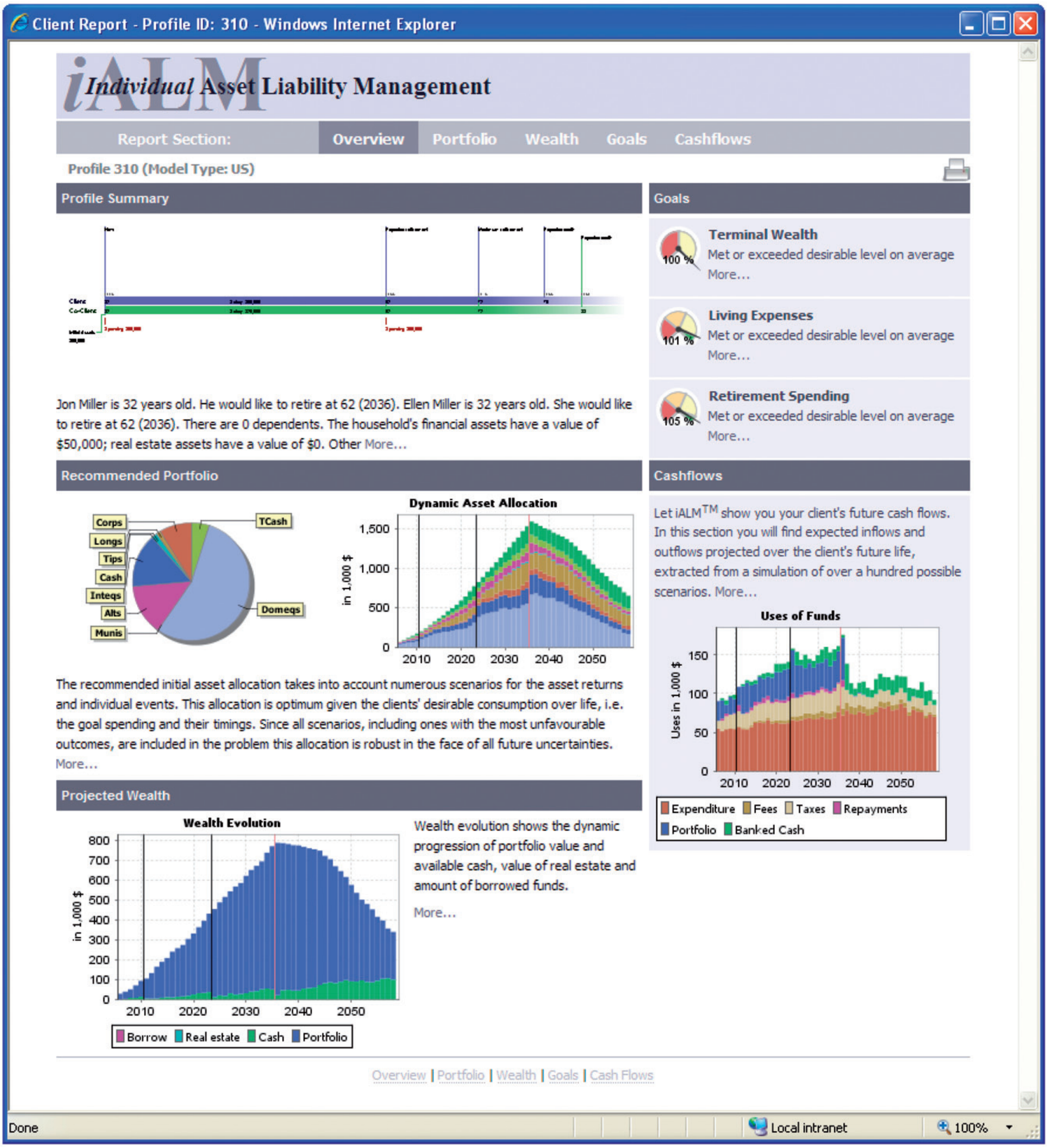
With the amount of information contained in the solution of *i*ALM, the representation of the inputs, and particularly the solution, is a challenging problem. The method of coping with the complexity of the solution is to present information at different levels of detail. At each level, *i*ALM presents an overview of the main results first and then allows drilling down to the part of the solution the user wishes to investigate further. The first level of the solution is a summary classified into categories of the financial plan such as portfolio, wealth, goals and cash flows, as shown in figure 6. The peaking of both portfolio and wealth evolution at retirement, which agrees with the Modigliani–Brumberg life-cycle hypothesis, is evident in the figure.

Before illustrating the nature of the lifetime financial plans producing by *i*ALM, it is worth noting other new features of *i*ALM not yet treated in the open literature on stochastic optimization. The most important of these in the context of asset-liability management problems (institutional or individual) are:

- automatic placement of major rebalancing times (stages of the DSP) based on problem data cashflows;
- random scenario lengths; and
- occurrence of other non-terminal events with random entry and exit times.

3. *i*ALM financial plans

The capabilities of *i*ALM can be illustrated on some examples of typical households. The model we use for these examples is the US model, formulated around the



Terminal Wealth: Met or exceeded desirable level on average More…

Living Expenses: Met or exceeded desirable level on average More…

Retirement Spending: Met or exceeded desirable level on average More…

Jon Miller is 32 years old. He would like to retire at 62 (2036). Ellen Miller is 32 years old. She would like to retire at 62 (2036). There are 0 dependents. The household's financial assets have a value of $50,000; real estate assets have a value of $0. Other More…

Let iALM^{TM} show you your client's future cash flows. In this section you will find expected inflows and outflows projected over the client's future life, extracted from a simulation of over a hundred possible scenarios. More…

The recommended initial asset allocation takes into account numerous scenarios for the asset returns and individual events. This allocation is optimum given the clients' desirable consumption over life, i.e. the goal spending and their timings. Since all scenarios, including ones with the most unfavourable outcomes, are included in the problem this allocation is robust in the face of all future uncertainties. More…

Wealth evolution shows the dynamic progression of portfolio value and available cash, value of real estate and amount of borrowed funds.

More…

Figure 6. Overview of solution output.

US tax code.† This includes modelling the 401 k retirement savings account as well as pensions, insurance and mortgage schemes available in the United States.

All examples are solved using a four-stage dynamic stochastic formulation of *i*ALM with a 15–2–2–2 scenario tree, resulting in 120 future scenarios. The optimum decisions in particular scenarios (what-if decisions) may vary significantly. Therefore, their distribution across scenarios and the expected values of decision variables are of particular interest in the analysis of probabilities of goal achievement (see, for example, figures 4 and 11).

Typically, a household would have many goals, such as buying a new home, paying school fees, holidays, and so on. In collecting individual data it is unrealistic to ask a user to specify a single value for their projected expenditure on any goal due to their subjective attitude to consumption. Therefore, for each individual goal their spending is associated with *minimum*, *acceptable* and

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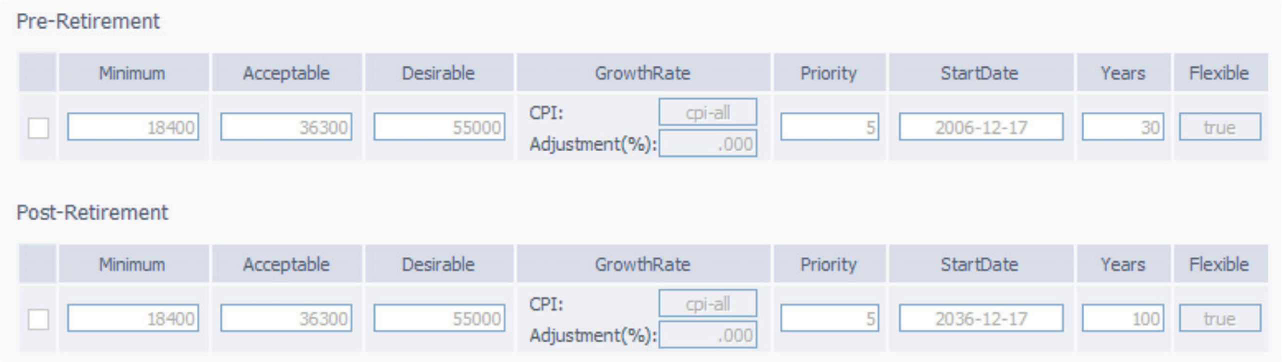
†A model for the UK has also been developed and is currently under test. The UK tax qualified accounts are individual ISA and SIPP saving accounts.



Pre-Retirement

	Minimum	Acceptable	Desirable	GrowthRate	Priority	StartDate	Years	Flexible
☐	18400	36300	55000	CPI: cpi-all Adjustment(%): .000	5	2006-12-17	30	true

Post-Retirement

	Minimum	Acceptable	Desirable	GrowthRate	Priority	StartDate	Years	Flexible
☐	18400	36300	55000	CPI: cpi-all Adjustment(%): .000	5	2036-12-17	100	true

Figure 7. Goal spending inputs.

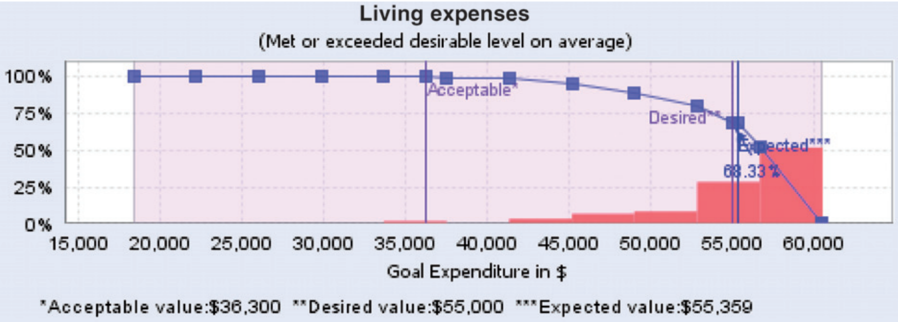


Figure 8. Distribution of average pre-retirement consumption across all scenarios.

desirable amounts. Each goal has also a *priority*, a *start date* and a *duration*. This information, together with general data about household members, their starting assets, salaries and other current and foreseen incomes and liabilities, constitute the *profile data input*.

3.1. Household profile 1. Young couple's retirement planning

In this first example we start with a household with only two goals: pre-retirement consumption ('living expenses' for short) and post-retirement consumption, as shown in figure 7 from *i*ALM inputs.

There are two individuals in the family—the *client* and the *co-client*. Both client and co-client are 32 years old and wish to retire at 62. The client currently has a salary of \$60k and the co-client \$25 k per annum. These incomes are modelled to grow at an age-dependent spread above inflation. The family's starting assets include a non-qualified account of \$30 k and a tax qualified 401 k account of \$20 k.

The household specifies a desirable living expenditure of \$55 k per annum and an acceptable living expenditure of \$36.3 k per annum. It wishes to maintain its real living standard throughout retirement.

Given the goal inputs detailed above, in conjunction with the other inputs used to describe the example household, a solution can be produced. The *i*ALM solution contains an optimal portfolio allocation for the current year, the expected spending on goals and their probability, a cash flow calendar, a portfolio projection, and a wealth projection. An overview of the solution output is shown in figure 6.

The first question *i*ALM can help answer for our household is

Is the goal to retire at 62 on \$55,000 achievable?

For this profile, *i*ALM calculates that across all scenarios there is a 94% likelihood of achieving the desired retirement lifestyle of \$55 k. Given that the client and co-client wish to retire at 62 we can now look at their distribution of pre-retirement living expenses to see if this retirement age is suitable. Figure 8 shows this pre-retirement distribution and how it varies over the full range scenarios. This is useful information for the household in determining what their probabilities are of reaching acceptable or desirable standards of living.

Although this is a simple profile, household members may still consider numerous trade-offs, such as when to retire, what level of pre- and post-retirement spending is feasible, or how much to save for retirement. It is possible to investigate the complex relationships of many inter-related choices for subjectively defined inputs. This can be done by using the *i*ALM tool interactively due to the reasonably short computing times needed to solve the optimization problem (3 to 5 minutes on a current laptop for most profiles). The following questions illustrate this type of analysis.

How does changing retirement age affect goal achievement?

In order to provide an answer to this question we generate variations of the above example with a range of

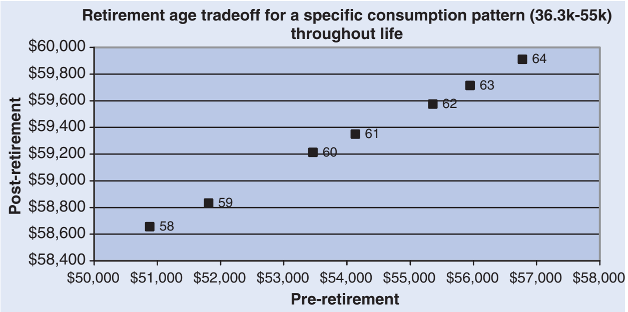


Figure 9. Varying retirement dates.

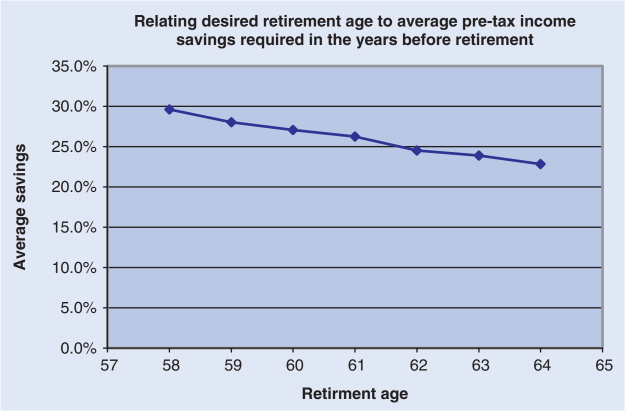


Figure 10. Saving for retirement.

retirement dates, keeping all other inputs constant.† Figure 9 shows the corresponding mean values over all scenarios for pre-retirement and post-retirement goal spending, which are indexed by the age of retirement, plotted against each other.

We see that retiring just one year earlier, at 61, will mean that the household fails to achieve the desirable *pre-retirement* spending. In this household the retirement date has a notably smaller effect on the average *post*-retirement consumption than on the average *pre*-retirement consumption.

How does retiring later affect lifestyle now?

Using the same set of problems, the relationship between the retirement age and saving before retirement is further investigated by looking at the expected average saving from income before retirement (pre tax) over all scenarios and years (figure 10).

For this example, if the household chooses to retire at 62 they need to save about 25% of their income.

What is the trade-off between lifestyle now versus lifestyle after retirement?

Now assume the couple retires at 62. To investigate how sensitive the pre/post-retirement optimal goal spending is to changes in user-specified inputs, we solve a number of problems whose inputs are summarized in table 1. The graph in figure 11 shows the results of these

Table 1. Input for sensitivity analysis.

Input				Output	
Pre-Retirement		Post-Retirement		Pre	Post
Accept.	Desir.	Accept.	Desir.		
Yellow					
$33,000	$50,000	$33,000	$50,000	$53,308	$54,603
$33,000	$50,000	$36,300	$55,000	$52,642	$60,014
$33,000	$50,000	$39,600	$60,000	$51,901	$65,332
$33,000	$50,000	$42,900	$65,000	$51,083	$70,588
$33,000	$50,000	$46,200	$70,000	$50,122	$75,728
Pink					
$36,300	$55,000	$33,000	$50,000	$56,362	$54,325
$36,300	$55,000	$36,300	$55,000	$55,359	$59,574
$36,300	$55,000	$39,600	$60,000	$54,349	$64,866
$36,300	$55,000	$42,900	$65,000	$53,343	$70, 046
$36,300	$55,000	$46,200	$70,000	$52,374	$75,112
Blue					
$39,600	$60,000	$33,000	$50,000	$58,528	$53,950
$39,600	$60,000	$36,300	$55,000	$57,444	$59,215
$39,600	$60,000	$39,600	$60,000	$56,332	$64,390
$39,600	$60,000	$42,900	$65,000	$55,238	$69,361
$39,600	$60,000	$46, 200	$70,000	$54,099	$74,380

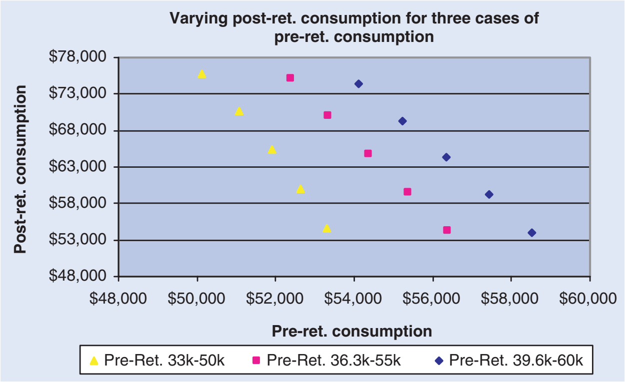


Figure 11. Varying post-retirement consumption.

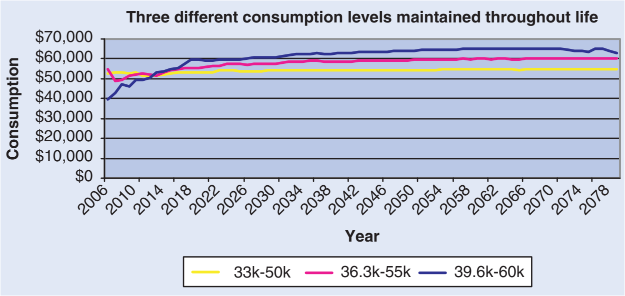


Figure 12. Spending strategy over life.

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†Household consumption is kept the same pre- and post-retirement with priority 10 for both goals: acceptable £36.3 k, desirable £55 k.

optimizations, colour-coded by user's acceptable and desirable inputs for pre-retirement spending. It shows, for example, that if the problems are in the mid-range of pre-retirement consumption, for approximately $1000 saved before retirement the household will have on average about $5000 to spend in retirement per annum.

The household's consumption pattern over time is shown in figure 12. The three plans of table 1 have a constant consumption over life: $33 k–$50 k (yellow), $36.3 k–$55 k (pink) and $39.6 k–$60 k (blue). We see that even though the 'blue' and 'pink' plans have a higher mean for most of the household's life, they involve cutting down expenditure early on in life.

3.1.1. Optimal dynamic portfolio allocation.

Figure 13 shows the optimal initial portfolio allocation satisfying the constraints for all generated scenarios for household profile 1. The recommended portfolio has a return of 8.49% and a volatility of 11.71%, which corresponds to profile liabilities and optimal goal spending.

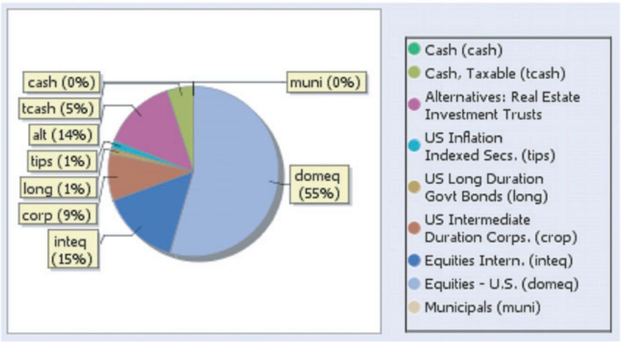

Figure 13. Initial recommended portfolio allocation as shown by *i*ALM.

The dynamic evolution of this portfolio over time (averaged across all scenarios) is shown in figure 14.

For example, four years after retirement the *i*ALM projection of the expected optimum allocation is

Year	muni	domeq	inteq	corp	long	tips	alt	tcash	cash
2040	$0	$623,631	$207,186	$75,638	$229,395	$10,826	$82,050	$81,396	$173,982

3.2. Household profile 2. Young couple's lifestyle planning

A key strength of *i*ALM is its ability to adapt to the requirements of specific households with an appropriate tax scenario, market environment, level of spending, etc. With more goals, the trade-offs that *i*ALM performs become more complex, with the results reflecting more stringent requirements for wealth and optimum resource allocation.

Our second example profile is an extension of the first. The client and co-client are the same age and have the same assets as previously. However, rather than just having living and retirement goals they now have some extra goals as summarized in table 2.

The information about how likely the household is to achieve each of their goals is summarized in figure 15. This is *i*ALM's 'goals' output screen which produces the images shown in the figure. These images include the expected values for goal spending (optimized decision variables) and the acceptable and desirable goal spend values (the household's input data).

Since this household has several different goals and expenditures, a more aggressive investment strategy is recommended as optimal. Figure 16 shows the initial portfolio with an increased proportion of equities and a decreased proportion of bonds relative to the previous initial portfolio.

The dynamic asset allocation over household lifetime is also more aggressive, as shown in figure 17. The *expected* wealth at retirement is also slightly higher for this household than for the previous household in order to ensure that goals and liabilities can be covered later in life.

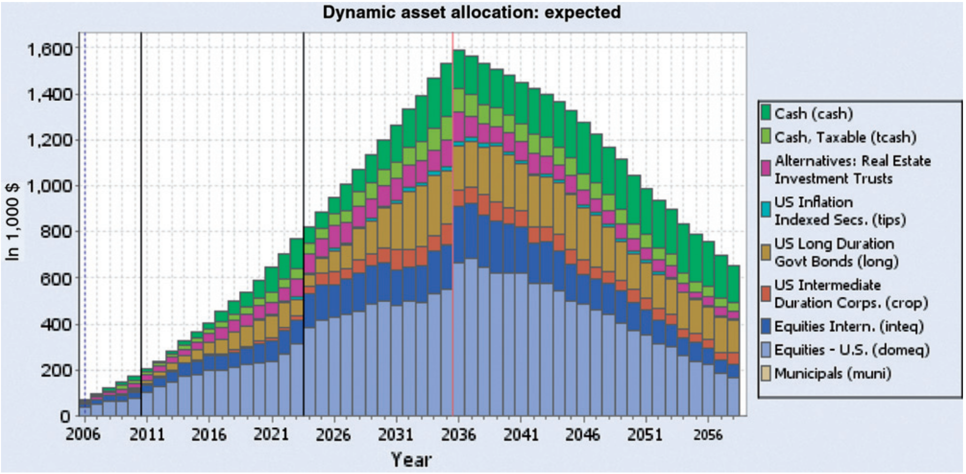

Figure 14. Portfolio projection over lifetime.

The optimum allocation four years after retirement in 2040 continues to have larger proportions of equities in order to assure the desired level of consumption and, particularly, the spending on retirement holidays starting in 2036.

Year	muni	domeq	inteq	corp	long	tips	alt	tcash	cash
2040	$0	$749,314	$254,108	$40,203	$208,090	$9,263	$112,264	$90,091	$134,793

Table 2. Summary of additional goals for household 2.

	Min ($)	Acc ($)	Des ($)	Start	Duration (in years)
Future childrens' education fund	1000	8000	12000	2006	15
Family cars	2000	5000	6000	2006	40
Charitable giving	500	4000	7000	2016	40
Retirement holidays	2500	5500	7000	2036	30

3.3. *Household profile 3. Lifestyle planning with inheritance*

The aim of this example is to investigate how a change in initial wealth affects portfolio allocation decisions. We extend our previous profile by assuming that the household has just had an inheritance of $1m from a relative. The new injection of wealth means that the household can afford a less aggressive approach to investing. The optimum initial portfolio and the dynamic asset allocation over the household's lifetime are shown respectively in figures 18 and 19.

As expected, the increase in holdings of bonds and decrease in equities reflects the fact that the household is concerned with wealth preservation after their $1 m inheritance.

The proportion of government bonds throughout the life of the household is much higher than in both previous cases. The portfolio allocation for 2040 is shown below for comparison with households 1 and 2.

Year	muni	domeq	inteq	corp	long	Tips	alt	tcash	cash
2040	$270	$336,753	$123,909	$478,808	$682,716	$68,822	$28,647	$176,440	$310,205

3.4. *Comparison with mean–variance optimization*

In the following we analyse the initial (current) year portfolios obtained by *i*ALM from the viewpoint of *mean–variance optimization* (MVO) (Markowitz 1952). It is important to note that this comparison can only be

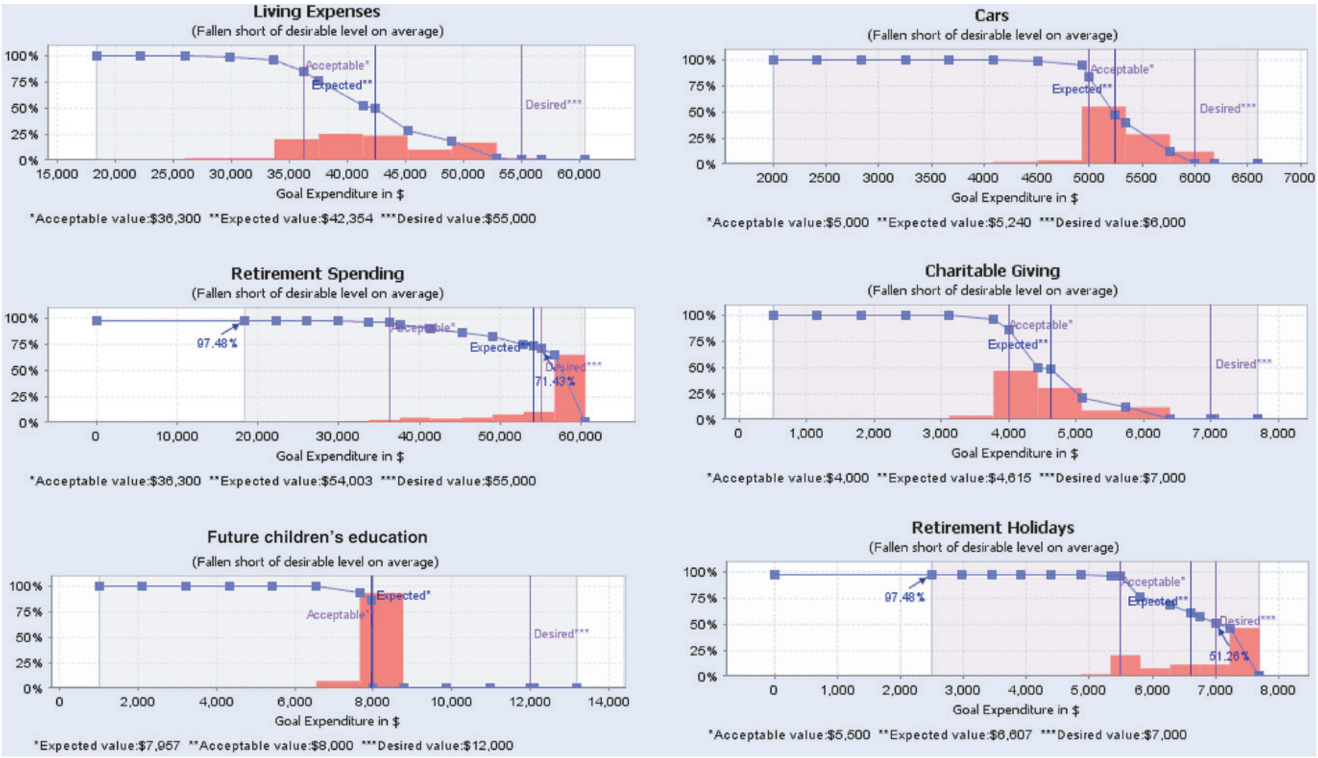

Figure 15. Goal spend distributions from the stochastic solution.

done in terms of the traditional MVO risk/return tradeoff and omits all other benefits provided by *i*ALM. Nevertheless, the optimum portfolios should be in close proximity to the Markowitz efficient frontier to assure users that *i*ALM does not contradict established economic concepts and practices.

The Markowitz efficient frontier is constructed from the nine assets listed in the legend of figure 19. The initial recommended portfolios for household profiles 1, 2 and 3 are shown in figure 20 with respect to the efficient frontier. As can be seen, these example portfolios are slightly below the Markowitz frontier due to various effects from liabilities, transaction costs, management fees and portfolio drawdown limitations as modelled in our multistage dynamic problem.

This result is however only the beginning of the insight into a household's sustainable risk/return trade-off provided by a dynamic model for life-cycle planning. Over time, *i*ALM shows the amount of risk a household must take in its investment portfolio to have a reasonable chance of meeting its future goals. It calibrates the 'right' *variable* risk-weighting relative to household goals dynamically over an entire life time rather than meeting a *prior fixed* risk attitude as with conventional single period MVO-based advice. In testing *i*ALM it was found that advisors tended to consistently *over-estimate* the risk tolerance of households. Moreover, the dynamic approach is adaptive to the importance of having sufficient liquidity to meet short-term goals—a topical issue in the present credit crunch.

4. Conclusion

The DSP approach to individual household lifestyle lifecycle planning uses a conceptually new liability-driven assessment of investors' risk attitudes. The dynamic *i*ALM portfolio allocations change over a lifetime to satisfy the specific needs of individual households, while the initial recommended portfolio allocations are efficient and robust with respect to all generated future scenarios.

Despite the long problem horizons the multi-stage DSP problem, formulated using a tree with high first stage branching factors and small branching factors at the following stages, generates a stable dynamic stochastic programming problem. Problems formulated with 120 scenarios are solved in 3–5 minutes (254 seconds for our initial example household profile). With computing time limited to a few minutes the effects of different trade-offs over life may be investigated to obtain *quantified* answers to the types of questions we posed in our illustrative examples.

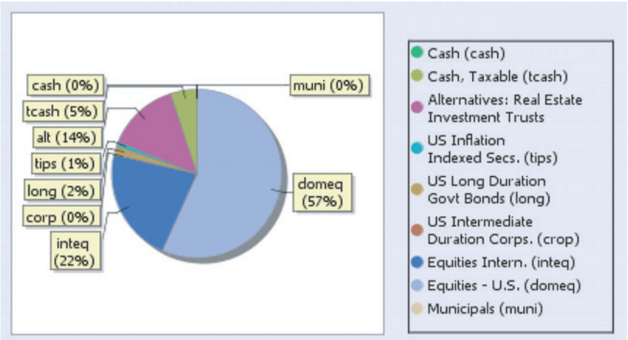

Figure 16. Initial recommended portfolio for household 2.

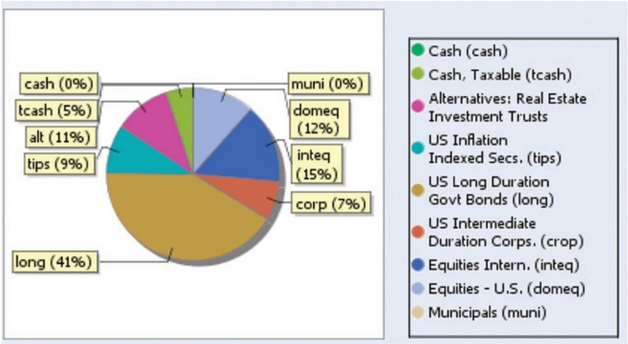

Figure 18. Initial recommended portfolio for household 3.

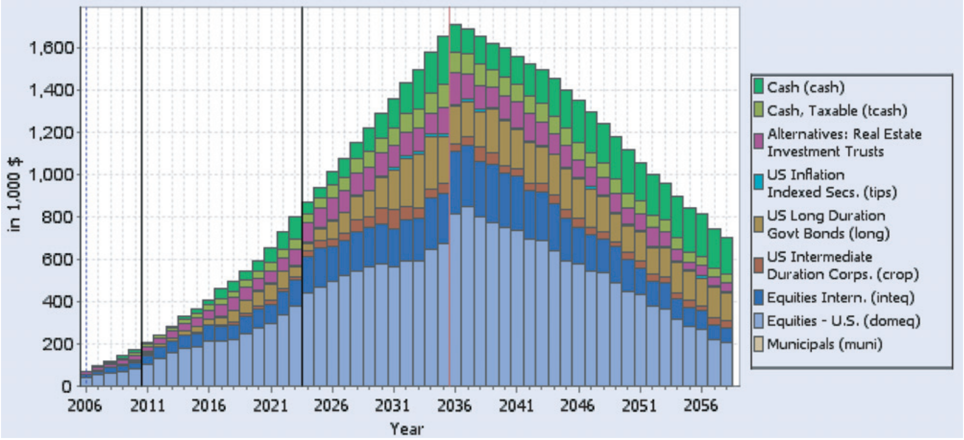

Figure 17. Portfolio projection over the lifetime of household 2.

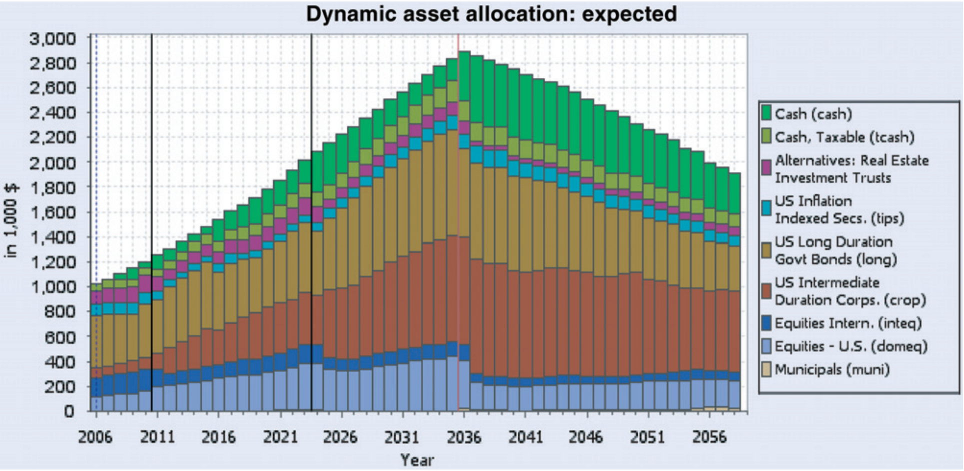

Figure 19. Portfolio projection over the lifetime of household 3.

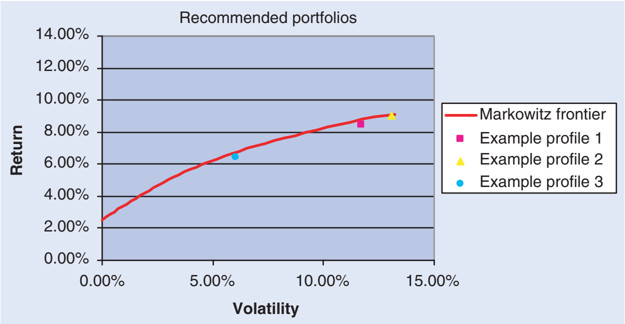

Figure 20. Initial recommended asset allocations chosen by *i*ALM for our three example households.

We have shown on constructed household examples the effects of different retirement ages on available spending throughout life, looked at required saving rates given different retirement ages, and investigated how pre-retirement lifestyle affects a household's spending in retirement. The results obtained generally support empirical studies from behavioural finance (e.g. Dynan *et al*. 2004). In line with conventional advice, *i*ALM's solutions confirm that it is optimal to reduce portfolio return volatility over the household's lifetime. Samuelson (1969) considered that as a household grows older it 'owns' less and less human capital, represented by the net present value of future salaries, and considered to be a low volatility asset. *i*ALM models salary increases as a non-stochastic age-dependent spread above long-run inflation and salary reductions as a result of unlikely long-term care events or early deaths, so that salary risk remains relatively low. With human capital reducing with age, a household with sufficient capital should optimally reduce the proportion of risky assets in its financial portfolio to compensate the effect discussed by Samuelson. Households with less capital may not have this option.

The distinguishing characteristics of *i*ALM are that it enables a household to understand the inter-dependence between their choices and associated risks. It helps them decide where they want to take risk: in goal achievements, in life events (e.g. by not buying insurance cover) or in their investment portfolio. The primary benefit of *i*ALM is thus that it encourages a very different thought process and helps to bridge the gap between "*I have enough money to meet this goal now, so I will spend on it*" and "*I have no real insight into the consequences for my future goals*".

The *i*ALM formulation can easily be adopted to different tax codes and legal jurisdictions. The versatility of the DSP modelling approach allows further extensions which can cover different pension schemes, insurance policies and investment asset classes. Such practical software tools can help households to make more informed investment decisions when planning for ever-increasing life spans in an environment in which governments and corporations have increasingly devolved financial risk to individuals.

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