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Financial markets. The joy of volatility

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Financial markets. The joy of volatility

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Can there be any profitable investment when all assets in a market destroy, rather than create, value? Thanks to volatility, the answer is yes—even if one does not wish to risk bankruptcy by going short (i.e. by selling what one does not own). This result can be illustrated with the aid of a simple model of a financial market with only two risky assets, whose returns in each period are determined by flipping a fair coin (figure 1).

Placing one's money in Asset 1 will, on average, reduce one's investment by one-tenth within 10 periods. Buying and holding Asset 2 will result in losing as much as one-third of one's investment within the same time span. Since growth rates depend on the logarithm of the (gross) return, investing in the apparently profitable Asset 1, with net returns of +40% and –30%, actually results in a loss of money. Any buy-and-hold investment, which purchases both assets, but does not update the positions, can only do as well, in the long term, as the best-performing asset. Poverty is the inevitable fate of the passive investor.

Consider making an investment according to a simple active management style: buying or selling assets so as to always maintain an equal investment in both. On average, wealth will double in 80 periods and grow without limit. This investment style rebalances wealth according to a constant proportions strategy. It succeeds, where buy-and-hold fails, because of the volatility of asset returns.

It has recently been proved mathematically (Evstigneev and Schenk-Hoppé 2002, Dempster *et al.* 2003, 2007) that, with stationary asset returns, *every*

constant proportions rebalancing strategy beats the corresponding market index (defined as the weighted average of individual asset growth rates). In particular, if one only invests in the assets growing at maximum rate, or if the market is volatile enough, *any* such strategy will beat the best buy-and-hold portfolio. Though examples of this phenomenon have been reported for quite some time (Luenberger 1998), and the best rebalancing strategy is well-known for performing at least as well as any buy-and-hold portfolio (Algoet and Cover 1988), it is surprising to learn that no conjecture has ever been made as to the validity of a general growth–volatility link.

The power of rebalancing strategies is often claimed to be the result of “buying low and selling high”. However, this is the gambler's fallacy, arguing that the longer the run of black numbers, the higher the odds of red numbers at the next spin of the roulette wheel. When returns are determined by the flip of a coin, an asset's upside and downside potential does not change over time. Such an asset is not cheap or expensive at any point in time. Nor does arbitrage (the opportunity to get ‘something for nothing’) drive this phenomenon—the market in the example is free of arbitrage. Finally, all investors have equal opportunities, unlike in Parrondo's paradox (Harmer and Abbott 1999) where some investors (depending on their wealth) are given favourable odds, and this excludes the dynamics of the wealth distribution as an explanation.

The engine that generates growth from volatility is in fact an elementary mathematical relation, the Jensen inequality. It describes the effect of interchanging concave (or convex) functions and weighted averages,

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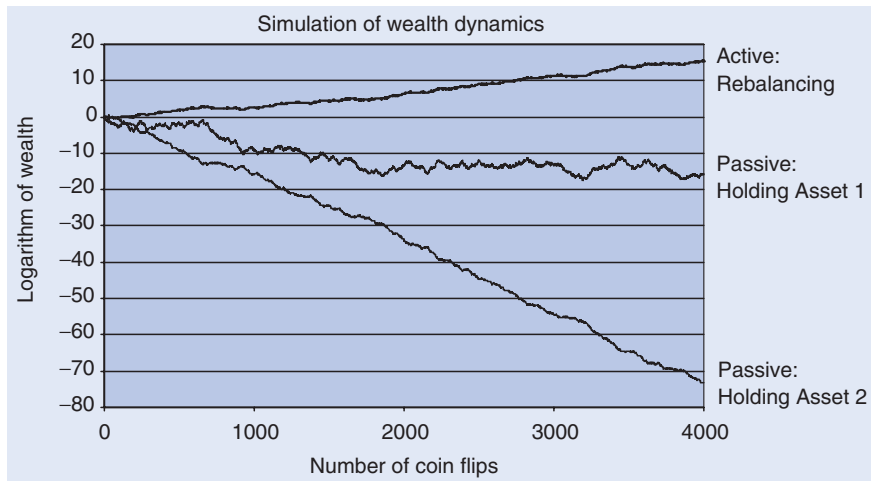


Figure 1. Performance of passive and active investment styles.

	Asset 1		Asset 2	
	Heads	Tails	Heads	Tails
Net return on assets	+40%	-30%	-20%	+15%
Growth rate of assets	$(1/2)\ln(1.4) + (1/2)\ln(0.7) \approx -0.010$		$(1/2)\ln(0.8) + (1/2)\ln(1.15) \approx -0.042$	
Buy-and-hold strategies cannot do better than Asset 1 in the long term. All passive styles lose				
	Heads		Tails	
Net return on active investment (1:1 mix)	$(1/2)40\% + (1/2)(-20\%) = +10\%$		$(1/2)(-30\%) + (1/2)15\% = -7.5\%$	
Growth rate of investment	$(1/2)\ln(1.10) + (1/2)\ln(0.925) \approx +0.0087$			
The constant proportions strategy generates positive growth. It beats all passive styles				

here portfolio asset proportions. Constant proportion rebalancing strategies combine random returns in *fixed* proportions at each portfolio rebalance. The expected logarithm, i.e. growth rate, of this financially engineered return is higher than the combination of the assets' individual growth rates in the same proportions, because the logarithm is a strictly concave function. For fixed, deterministic returns, both growth rates are equal, and no excess growth can be achieved.

This financial market phenomenon closely resembles observations on stochastic resonance (McClintock 1999), a theory in physics that has various applications, e.g. in biology and neurophysiology. Similar to amplifying a weak signal by adding noise to a nonlinear system, constant proportions strategies combine random processes to achieve an increase in the growth rate. The required nonlinearity is provided by the compound return on the investment and the logarithmic function that appears in the growth rate of wealth.

While the implications of the growth–volatility link for asset pricing and portfolio theory are still being explored, active investors should enjoy the bumpy ride of volatility. However, as with any investment advice, a word of caution is in order: constant proportions strategies do well in the long term but, over short time horizons, their superior performance cannot be guaranteed. The expected logarithm of per cent (gross) returns determines the long run growth rate of an asset or portfolio. For the short term investor, absolute returns may be of more interest!

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