

Banking capital and operational risks: comparative analysis of regulatory approaches for a bank¹

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Abstract

The nature of operational risk means that although it constitutes a small part of a bank's risk profile, it includes unexpected events that could potentially cause the collapse of the entire bank. To understand the relationship between economic and regulatory operational risk capital we first examine selected large internationally active banks' capital disclosures and review the Basel II approaches to allocation of regulatory capital for operational risk. We apply the 'extreme risk capital model' (ERCM) to calculate the operational risk capital of a specific bank using its internal operational loss data over a four-year period and the results are compared to the proposed alternatives. This comparison supports the argument that the extreme risk capital allocation model view point provides an integrated and holistic view of a bank's operational risk exposure which is especially suitable for risk management at the strategic level.

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The term operational risk began receiving widespread recognition in 1995 following the shocking failure of Barings Bank, one of the U.K.'s oldest financial institutions. A rogue trader had caused the bank to lose around U.S.\$1.3 billion and drove Barings into bankruptcy. In spite of much publicity, the lessons from this event have not yet been learned. The recent U.S.\$7 billion losses at Société Générale at the hands of a rogue trader shows the fallibility of banks' operational risk management. We have again been reminded that from time to time the inevitable extreme operational risk events occur with many billions of dollars lost. Landau, the Deputy Governor of the Bank of France, recently stated: "With slight exaggeration, a case can be made that modern finance has been built, in practice, if not in theory, on implicit tolerance and widespread ignorance of extreme events" [BCBS (2008a)]. He also acknowledged that models with 'fat tail' distributions are available, but seldom used due to lack of reliable data over a sufficient period of time. Wellink, the chairman of the Basel Committee, stated that banks will have to develop more rigorous approaches to measure and manage their operational risk exposures and hold commensurate capital [BCBS (2008b)]. It is clear that in practice operational risk is difficult to identify, measure, and control. Traditionally, banks have relied on internal processes, risk management and control functions, auditors, and insurance protection to manage operational risk. These methods remain of vital importance, but the growing complexity of the banking industry and the widely publicized extreme operational losses in recent years reveal the need for a more prudent and transparent regulatory regime.

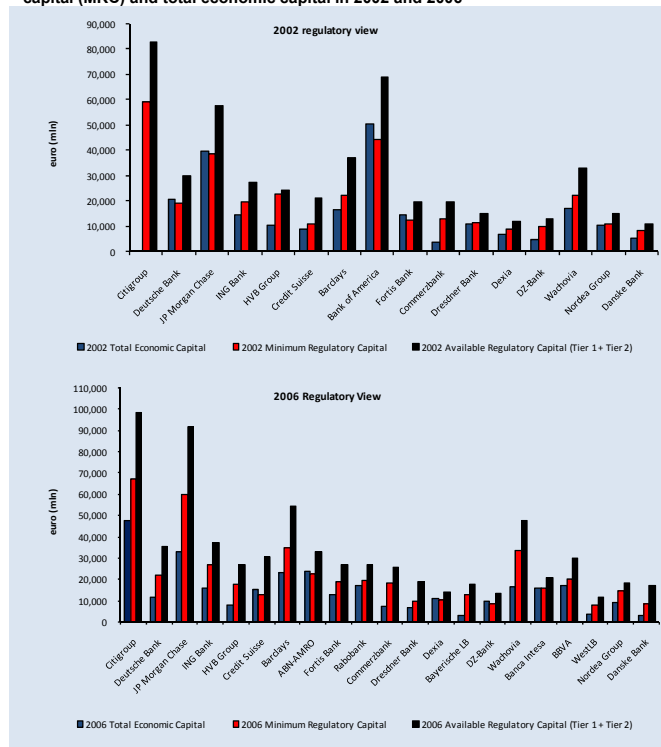
The Basel II definition of operational risk is "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events." This definition includes legal risk, but excludes strategic and reputational risk [BCBS (2006a)]. For estimation purposes, banks usually define an operational loss as the amount charged to the profit and loss (P&L) account net of recoveries, in accordance with Generally Accepted Accounting Practices [ITWG (2003)]. According to Basel II banks must explicitly hold equity capital against operational risks. It has become the bank's responsibility to add transparency about its operational risk profile by quantitative assessment of risks using internal loss data, external loss data, scenario analysis using expert judgment, and key risk indicators.

The Basel II framework for operational risk proposes three methods for calculating operational risk (OR) capital charges on a scale of increasing sophistication and risk sensitivity: (i) the basic indicator approach (BIA); (ii) the standardized approach, which is an extension of the BIA at a more detailed level; and (iii) the advanced measurement approach (AMA) [BCBS (2006a)]. Currently, under the AMA approach, the financial industry uses the following methods to determine OR capital: the loss distribution approach (LDA), the scenario based approach, and methods based on extreme value theory (EVT), or a hybrid of all three. Our proposed stochastic model for measuring operational risk was the first application of extreme value theory to operational risk modeling [Medova (2000, 2001), Medova and Kyriacou (2000, 2002)]. In such an extreme risk capital model (ERCM) operational risk is measured as an excess over levels for market and credit risks.

As banks are at different stages of systems development they show considerable dispersion in OR capital estimates [BCBS (2006b)]. Unfortunately, before an industry standard operational risk model has emerged it is highly likely that there could be possible regulatory arbitrage and more severe model risks.

The objective of this paper is to compare the operational risk models and capital estimates determined by the different methods: BIA, LDA, and the extreme risk capital model.

Figure 1 - Comparisons between available regulatory capital, minimum regulatory capital (MRC) and total economic capital in 2002 and 2006



Bank disclosure of risk capital

There is a requirement for all regulated banks to hold regulatory capital assessed according to their ability to withstand credit, market, and operational risks. From the regulatory perspective this capital is divided into three tiers:

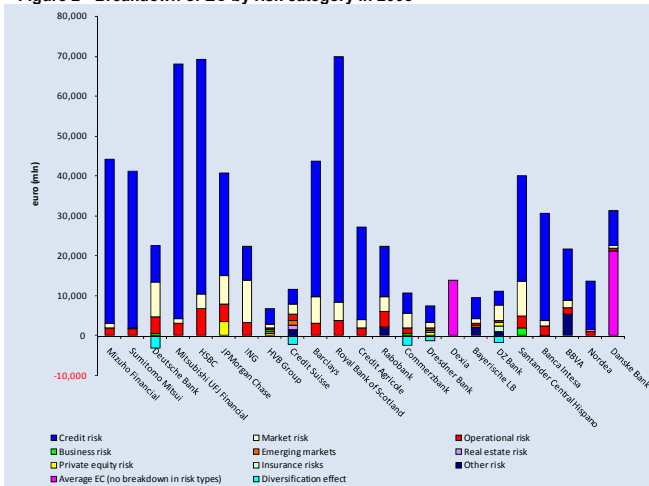
- Tier 1 capital** – the highest quality capital from a risk perspective, which consists of paid-up ordinary shares, general reserves, retained earnings, and certain preference shares, less specified reductions.
- Tier 2 capital** – includes general provisions for doubtful debts (subject to a limit), asset revaluation reserves, mandatory convertible notes, and similar capital instruments.
- Tier 3 capital** – short-term unsecured subordinated debt that can be used only for meeting market risk capital requirements.

Currently, the minimum regulatory capital (MRC) ratio requirement is eight percent of assets, of which four percent must be equity or reserves². This capital adequacy ratio is an important financial ratio that supervisors must examine thoroughly and must be maintained at all times since it is geared toward investor protection. The operational risk capital component of MRC is around 12-15% [BCBS (2004a,b, 2006a)] depending on the individual bank's risk profile. The calculation of operational risk capital (Pillar 1 of the Basel II recommendations) is left to the discretion of the bank, although procedures and the capital disclosed will be subject to regulatory scrutiny.

Economic capital is generally described as the amount of equity capital the bank needs to be able to absorb unexpected losses from its current exposures. The Board of Directors and senior management of the bank are actively involved in determining the level of economic capital, which is based on a chosen objective for overall risk level, the statutory capital adequacy requirement,

² Note that subordinated debt will not prevent a bank from failing although it may in part absorb losses after failure and therefore help depositors.

Figure 2 - Breakdown of EC by risk category in 2008



an internal assessment of required risk capital, and capital held against business objectives. Determining an appropriate level of economic capital by a bank's top management reflects two main perspectives: the shareholders' view of capital, who expect a risk-adjusted return on their investment, and the regulator's view of capital, who are promoting safety and soundness in the financial system. Unfortunately these two perspectives are often at odds.

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The Basel Committee makes the following distinction between economic capital and regulatory capital: "Economic capital is the capital that a bank holds and allocates internally as a result of its own assessment of risk. It differs from regulatory capital, which is determined by supervisors on the basis of the Capital Accord" [BCBS (2001b)]. According to the regulatory view, a bank is considered well-capitalized when Tier 1 capital plus the provisions for credit losses qualifying for Tier 2 capital compares favorably with the bank's estimated economic capital [BCBS (2003)]. Some considerable risks that are not taken into account by the regulatory framework, like liquidity risk, business and strategic risk, and external factors such as the business cycle, have revealed themselves in the current banking crisis.

Annual reports of 50 banks³ and Basel Committee documents [BCBS (2001a,b, 2003, 2006a)] provided data for a review and comparison of capital adequacy across banks. Our analysis of the selected 50 largest banks by total assets shows substantial differences in banks' economic capital figures [Berg-Yuen and Medova (2006)], which have changed significantly from year to year (Figure 1). Although the definition of economic capital is not consistent across banks, most banks seem to agree that it is a measure intended to cover unexpected losses during one year with 99.00-99.99 percent confidence or, equivalently,

3 Annual reports of selected banks are analyzed in detail from 2001 to 2005.

corresponding to an A to AAA Standard & Poor's debt rating. Although economic capital should take into account all risks faced by a banking group, our review indicates that this is not yet the case. All banks include credit and market risk in the economic capital calculation, and most banks also include operational risk and a variety of other risk categories (Figure 2). We found that business risk is either a stand-alone risk category or included in the operational risk category. Furthermore, most banks identify liquidity, reputational, and strategic risk categories but do not include them in their economic capital calculations. Our examination of economic capital disclosures also reveals that the disclosure of diversification effects is meager. For example in 2008 only 5 out of 23 banks disclosed the amount of diversification benefit.

Despite the paucity of economic capital level amounts disclosed in 2001-2007 annual reports, they reveal some interesting facts concerning the evolution of the reporting of economic capital in the global banking industry. We found as expected that all banks report available risk capital to be higher than the statutory MRC. A majority of banks, however, report that the total economic capital is lower than MRC, which has been interpreted as suggesting that the regulatory minimum is set too high or, with hindsight, that the accounting practices for defining a bank's capital has serious flaws.

Fundamental questions to be resolved are:

- Why was there such a significant difference between economic and regulatory capital valuations?
- Has this apparent difference between banks' internal valuation of risks and the regulatory capital measurement based on the disclosures in balance sheets lead to the current financial crisis?

Operational risk capital measurement

In this section we give an overview of three alternative approaches to operational risk capital allocation.

Basic indicator approach

The BIA is easy to implement and universally applicable across banks. However, the Basel Committee acknowledges that the downside of the BIA's simplicity is its lack of responsiveness to firm-specific needs and characteristics. Its prospective capital charge should be seen solely as a buffer for losses from unexpected exposures. This BIA capital charge may be expressed as

$$Capital_{BIA} = \frac{\alpha \sum_{i=1}^n GI_i 1_{\{GI_i > 0\}}}{n}$$

where GI_i is the annual gross income⁴ in year i , n is the number of positive annual gross income years with $n \leq 3$ ($1_{\{GI_i > 0\}}$ is an indicator function for this condition) and α is 12 or 15%.

Some controversy surrounds the alpha factor⁵. Basel II set it originally to achieve the target of 12 percent of the minimum regulatory capital [BCBS (2004a, 2006a)] or $\alpha = 0.12 * MRC / GI$.

4 Gross income is defined as net interest income plus net non-interest income. It is intended that this measure should be gross of any provisions (i.e., unpaid interest); be gross of operating expenses, including fees paid to outsourcing service providers; exclude realized profits/losses from the sale of securities in the banking book; and exclude extraordinary or irregular items as well as income derived from insurance [BCBS (2004a)].

5 To determine the appropriate value of α , i.e., contribution of operational risk to minimum required capital, the Basel Committee conducted quantitative impact studies (QIS1: April 2001; QIS2: May 2001; QIS2.5: June 2002; QIS3: October 2002; QIS5: September 2005).

There is an array of opinions regarding the impact of the BIA within the financial industry. One view is that the proposed BIA yields operational risk estimates that are grossly exaggerated relative to the industry's experienced losses [BCBS (2003)]. Most banks that commented on Basel II strongly objected to the assumptions underlying the BIA, particularly to calculations based on gross income. Utilizing a charge based on gross income puts all banks into the same category regardless of how they derive their income, the volatility of that income, or the level of exposure to various operational risks. Transactions within a banking group might increase gross income, which in turn would generate a higher operational risk capital requirement, whilst the underlying operational risk had not changed at all. Conversely, falling revenue, which would call for less capital, could actually be a reflection of poor business practices, increased errors, or reputational problems. As a result, a huge loss due to operational problems could paradoxically result in a lower capital requirement for operational risk.

Loss distribution approach

Under the loss distribution approach, the bank estimates the probability distribution functions for severity and frequency of losses for each business line/risk type cell using its internal or external data, and computes the probability distribution function of the aggregated operational losses across all risk types and business lines over the year [BCBS (2001a)]. The Basel Committee proposes to calculate the total OR capital charge by the simple addition⁶ of the capital charges for every business unit and risk type, viz.

$$Capital_{LDA_{\alpha,T}} = \sum_{b=1}^B \sum_{q=1}^Q x_{\alpha,T}(X(b, q))$$

where T=1 year and $x_{\alpha,T}[X(b,q)]$ is the 99.9th percentile of the estimated operational loss distribution function for the loss in each of the Q=7 risk types and B=8 business lines.

The LDA is based on models and techniques adopted from the insurance industry [Klugman et al. (1998), Mirzai (2001), Courage (2001)]. Generally all LDA implementations assume the independence of severity and frequency of losses. Since an expression for the aggregate loss distribution is not analytically derivable in closed form, all calculations are done numerically. Issues with the LDA methodology have been examined in Frachot et al. (2001, 2004) and Baud et al. (2002). Peters and Sisson (2006) advocate Bayesian approaches to operational risk modeling and explore how Bayesian inference and Monte Carlo sampling fits into an LDA setting. Most internationally active banks report the use of some version of LDA using models but without supplying implementation details. For example, Deutsche Bank's AMA model is given in Aue and Kalkbrener (2006).

Extreme risk capital model

Extreme operational losses represent the greatest risks for a bank. The extreme value risk capital model (ERCM) for measuring operational risk has been proposed and tested on a number of data sets [Medova (2000, 2001), Medova and Kyriacou (2002)]. It provides an integrated risk framework using the conceptual definition which matches the Black Swan metaphor [Taleb (2007)]. Extreme losses are rare, high impact events which contribute the most to operational risk. These losses are above a sufficiently high impact threshold and are explained statistically by the generalized Pareto distribution (GPD). Estimation of the parameters of the GPD is given by the peaks over threshold (POT) method [Leadbetter (1991)]. Although theory provides expressions for the relevant estimators, the difficulties of this extreme value theory (EVT) method lie in the interpretation of a

'sufficiently high' threshold for excesses and in the statistical reliability of GPD parameter estimates [Galambos et al. (1994), Embrechts et al. (1997)]. We estimate the posterior values of GPD parameter distributions in our Bayesian hierarchical approach using Markov chain Monte Carlo (MCMC) techniques [Smith (1985, 1998)]. The MCMC algorithm samples from probability distributions based on a Markov chain that has the desired posterior distribution as its stationary distribution. The distinguishing feature of MCMC is that the random samples of the generated posterior density are correlated, whereas in conventional Monte Carlo methods such samples are usually statistically independent. The extreme risk capital model (ERCM) is summarized below.

Severity of the extreme losses (i.e., excess loss) is modeled by the GPD with shape parameter ξ and scale parameter $\beta := \sigma + \xi(u-\mu)$.

For a sufficiently high threshold u which is much greater than expected loss, the occurrence of extreme loss events follows a Poisson process with intensity

$$\lambda = \left(1 + \xi \frac{(u - \hat{\mu})}{\hat{\sigma}} \right)^{-\frac{1}{\xi}}$$

The extreme risk capital $Capital_{ERCM}$ is defined as the sum of the threshold and expected excess loss. Given the shape parameter estimate $\hat{\xi} < 1$, the daily and annual ERCM capital estimates are respectively:

$$Capital_{ERCM_{T=1}^{\hat{\xi}<1}} = u + \lambda_u \frac{\hat{\beta}_u + \hat{\xi}u}{1 - \hat{\xi}}$$

$$Capital_{ERCM_{T=365}^{\hat{\xi}<1}} = u_T + \lambda_u \cdot T \frac{\hat{\beta}_u + \hat{\xi}u}{1 - \hat{\xi}}$$

where T is the one year.

For the case when the shape parameter estimate $\hat{\xi} > 1$, the median expected excess is used [Beirlant et al. (1996), Reiss and Cormann (2006), Rootzén and Tajvidi (1997), Reiss and Thomas (2001)]. Thus, the daily and annual ERCM capital estimates are respectively:

$$Capital_{ERCM_{T=1}^{\hat{\xi}>1}} = u + \frac{\hat{\beta}_u}{\hat{\xi}} \left(2^{\hat{\xi}} - 1 \right) \cdot \lambda_u$$

$$Capital_{ERCM_{T=365}^{\hat{\xi}>1}} = u_T + \lambda_u \cdot T \cdot \frac{\hat{\beta}_u}{\hat{\xi}} \left(2^{\hat{\xi}} - 1 \right)$$

The choice of the threshold is crucial because our derivations are based on asymptotic limit theorems [Fisher and Tippett (1928), Gnedenko (1941), Pickands (1975)]. For each calculation of extreme capital, we adopt a pragmatic approach in which a range of diagnostic techniques are used to assess the threshold selection and the resulting estimate of the associated GPD shape parameter within the limit of MCMC simulations. Detailed descriptions of the procedures employed are given in Medova and Kyriacou (2002) and Berg-Yuen (2008).

Operational risk case study

We illustrate here the calculation of operational risk capital on an example of a large internationally active bank, referred to as 'the Bank.' The 'basic indicator approach' calculations are based on a three year average of gross income as reported in the Bank's

⁶ This assumes that statistically all unexpected losses occur simultaneously, i.e., are perfectly correlated.

Figure 3 - Histogram of aggregated total log-loss data above the €10,000 threshold

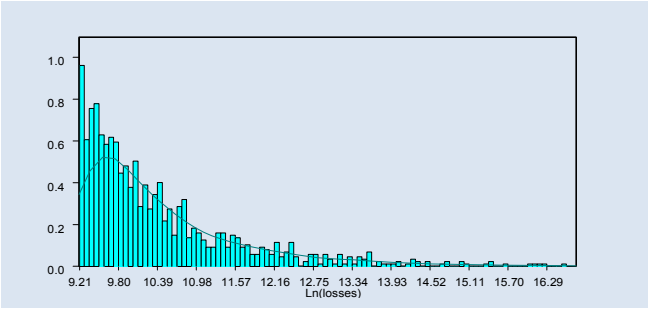
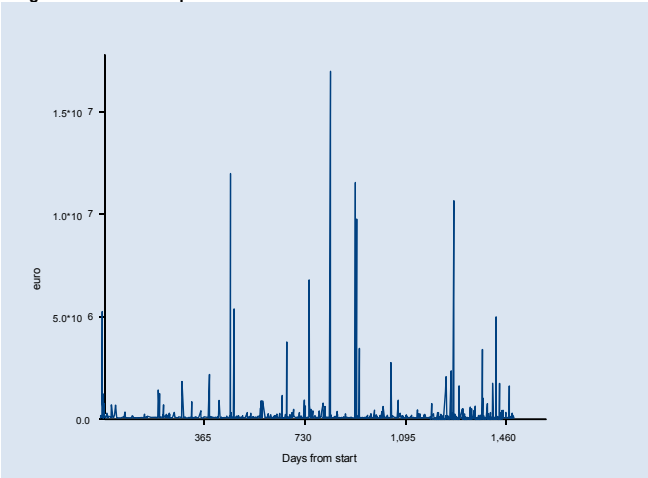


Figure 4 - Time series plot of total losses



annual reports. The LDA and ERCM calculations are implemented on the Bank's internal loss data.

Loss data

Losses are defined as the realized losses of the Bank in financial statements, i.e., in the P&L accounts, in accordance with GAAP. Operational loss data for all business lines at different geographic locations over four years correspond to four major risk types given by:

- Risks A** - risks related to processes, policy and infrastructure.
- Risks B** - risks due to people, human errors, inadequate procedures and controls, etc.
- Risks C** - risks caused by systems, technological shortcomings, breakdowns, etc.
- Risks D** - risks caused by external factors such as natural disasters, fraudulent activity, etc.

These daily data are analyzed separately and in the aggregate (total data). To understand the characteristics of the underlying loss distributions we plotted the losses, calculated summary statistics, and tested whether specific operational losses are independent identically distributed (iid) as is commonly assumed in operational risk modeling. Berg-Yuen (2008) gives a detailed analysis of correlograms, scatter plots, and runs tests which together support the applicability of the iid assumption for the aggregated total and four risk types loss datasets.

The losses were ranked according to size using the following ranges: below threshold if the loss is less than €10,000; low if it is less than €49,999; rather low if it is between €50,000 and €249,999; rather high if it is between €250,000 and €999,999; high if the loss is more than €1,000,000. The impacts of small and large losses on the overall loss process in terms of number of

losses and loss severity are as follows. Contributions to the total number of losses are such that about 85.29 percent of losses are below threshold, 11.04 percent are low losses, 2.60 percent are rather low losses, 0.73 percent are rather high losses, and 0.35 percent are high losses. In other words, small losses are the most frequent. On the other hand, loss severity below threshold account for only 6.66 percent of total losses, low losses account for 9.27 percent, rather low losses account for 11.29 percent, rather high losses account for 14.70 percent, and high losses account for 58.08 percent of the total loss, with the largest loss itself accounting for 8 percent of the total. There are many high frequency small losses and a few low frequency large losses, with the total loss to the bank dominated by a few very large losses. Although the Bank collects/records all losses, it is therefore appropriate to left-truncate the Bank's loss data at €10,000 or 9.21 on the natural log loss scale (Figure 3). Figure 4 shows a time series plot of total losses. There are a few extreme losses but there does not appear to be any evidence of a clustering pattern for large losses.

Loss distribution approach

Using the standard assumption that the frequency and severity distributions of losses are independent we implement the LDA by

Figure 5 - Comparison between theoretical cdfs and the empirical distribution function for total losses

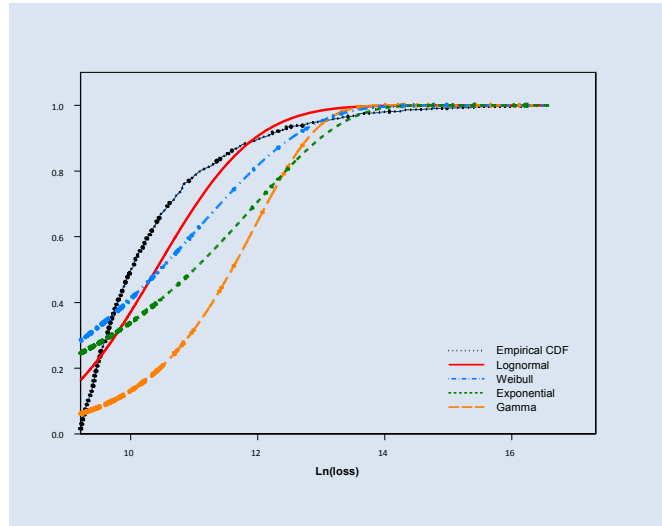


Figure 6 - Comparison of Poisson fit ($\lambda=0.81$) and negative binomial fit ($\beta=0.68, r=1$) against the observed number of losses per day

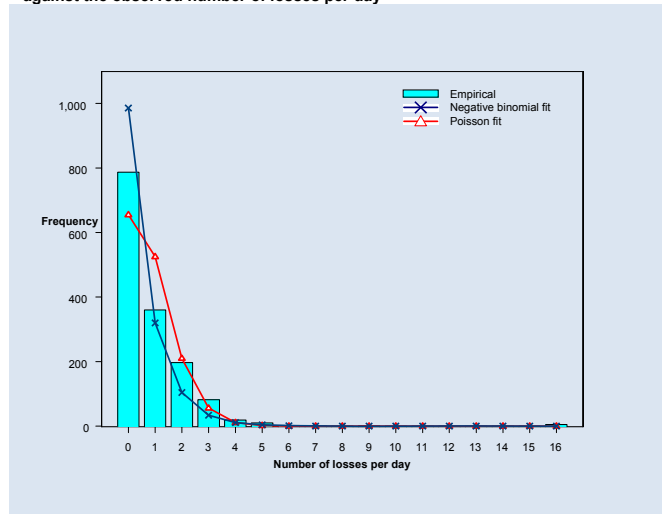


Table 1 - Annual LDA percentile estimates of OR capital using data for total losses and four risk type losses data expressed in millions of euros

Annual LDA percentile estimates based on Poisson/lognormal distributions with MLE using 2001-2004 data and 100,000 simulations

Total loss:	Poisson(297), Logn(10.399, 1.214)					
Risk A:	Poisson(52), Logn(10.400, 1.201)					
Risk B:	Poisson(16), Logn(11.072, 1.769)					
Risk C:	Poisson(217), Logn(10.350, 1.152)					
Risk D:	Poisson(13), Logn(10.401, 1.224)					
Median percentile estimates	Total loss	A Process losses	B People losses	C System s losses	D External losses	Sum
99.90%	30.5	62.3	20.2	4.5	8.7	95.6
99.97%	32.8	98.8	21.6	5.8	10.3	136.6

simulating losses from the appropriate distributions and calculating the aggregate loss distribution. To model severity of losses four theoretical distributions were considered: lognormal, exponential, Weibull, and gamma. Although no fit is fully satisfactory, the lognormal and Weibull cumulative distribution functions (cdfs) seem to best represent our loss data (Figure 5).

For our frequency data we study the times between loss events and fit a frequency distribution to the set of observed loss event interarrival times. Initially the number of loss occurrences is assumed to be a homogeneous Poisson distribution, where the interarrival times between successive losses are iid and exponentially distributed with finite mean. The Poisson distribution parameter estimates reveal that on average 0.81 occurrences per day of total losses above the €10,000 threshold should be expected or about 297 losses per year. Figure 6 shows graphically that both the Poisson and negative binomial distribution functions seem to fit our loss interarrival data well enough to use for modeling the frequency distribution of total losses.

The Basel Committee allows a bank's internally determined dependency measures for losses across individual operational risks [BCBS (2006a)]. In order to analyze dependence between loss severities and frequencies at the daily frequency we would need to consult the Bank's incidence reports and research the complex dependency relationships between losses, which were not available. Cope and Antonini (2008) surveyed a range of correlations and dependence measures among operational losses from an international data consortium of banks. They found little evidence of strong correlations, but some slight evidence of tail dependencies for quarterly aggregate loss values among business line, event type, and Basel business line/risk type cell units.

Results

Given that Basel II requires that the confidence level be set to 99.9% over a one-year time period, we set our LDA operational risk capital estimate, termed $Capital_{LDA0.999,1}$, at this level. In Table 1 a range of high percentile estimates are displayed from the LDA's annual aggregate loss distributions using data on four risk type losses and total losses from 2001 to 2004. To produce reasonably stable LDA estimates for operational risk capital we used a Monte Carlo simulation with 100,000 paths. The numbers of losses are drawn from an annual Poisson distribution and the severities of losses are drawn from a daily lognormal distribution. Maximum likelihood estimation (MLE) was used to estimate the parameters of these distributions. At the 99.9% level prescribed by the regulators, the OR capital for total losses is equal to €30.50 million (confidence interval €30.18 to €30.87) using the LDA (Poisson/lognormal). At the 99.97% level, often chosen by banks to define a capital level which reflects the AA+ target rating level, the OR capital for total losses is equal to €32.76 million, or an increase of 7.42 percent over that of the 99.9% level prescribed by the regulators. We find that the difference in capital is about 2.15 percent if we use the negative binomial distribution instead of

the Poisson as the frequency distribution. This suggests only small differences in estimated capital at group level whether the Poisson or negative binomial distribution is used for the LDA.

Figure 7 shows the generated aggregate annual loss distribution for total losses in 2005 with the 99.9 percentile risk OR capital estimate of €30.50 million. If the number of simulations paths is increased from 100,000 to a million the capital estimate changes only marginally.

At the 99.9% level, and taking the conservative view of perfect positive dependence between risk types, the sum of OR capital for the four risk type losses (termed 'simple sum' in Figure 8) is equal to €95.62 million (confidence interval €90.60 and €101.41). At the simple sum 99.97% level, the OR capital for losses is equal to €136.62 million, a 42.87% increase over the 99.9% level. Figure 8 also shows that the total capital (Poisson/lognormal) estimate is substantially less dispersed over confidence levels due to the averaging effect than those of the simple sum estimate.

Backtests

Verifying the accuracy of any internal risk model used in setting capital requirements requires backtesting. The aim is to ensure that the capital estimate generated accurately reflects the loss level that can be expected to be exceeded only 0.01 percent of the time. We perform a historical backtest on the LDA by

Figure 7 - The aggregate annual loss distribution with LDA capital measure ($\alpha=99.9$ percentile)

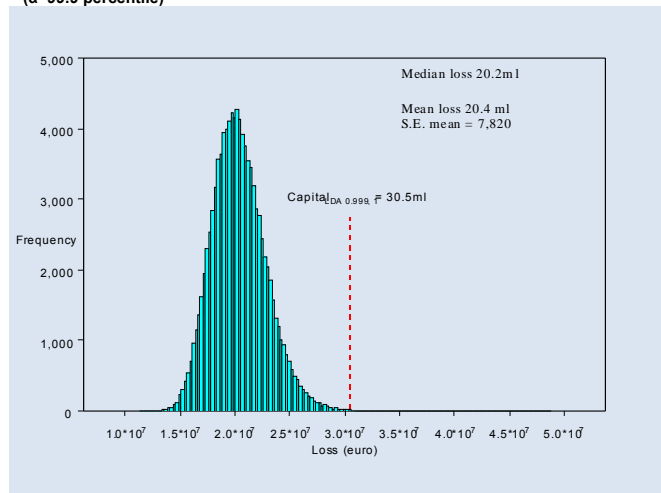


Figure 8 - Annual LDA OR capital using total loss data and OR capital obtained by adding the capital allocated for the four risk types

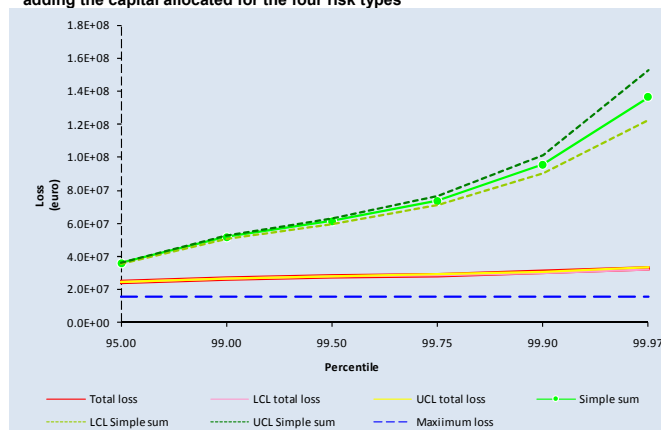


Figure 9 - Backtesting projected annual LDA OR capital for total losses against accumulated losses

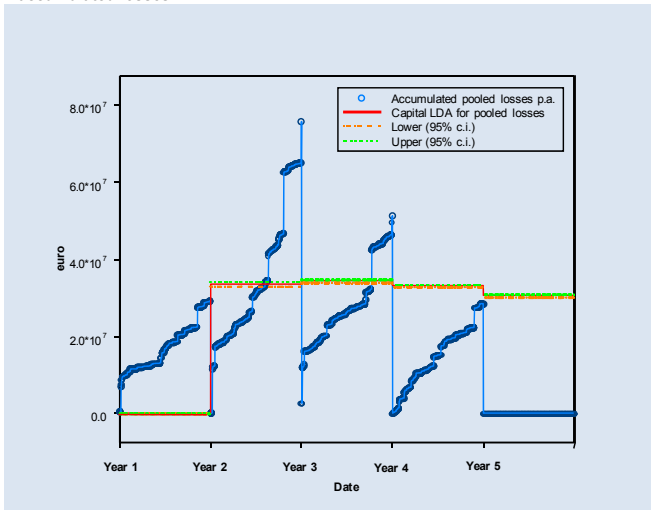
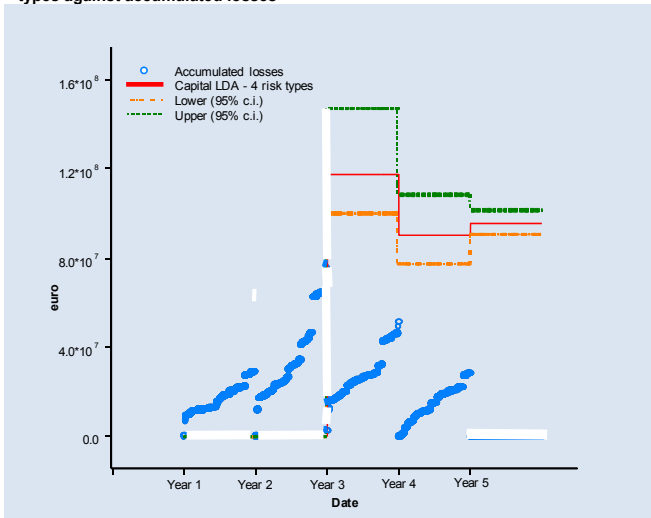


Figure 10 - Backtesting projected annual LDA OR capital sum for the four risk types against accumulated losses



matching the estimated capital projections for 2002, 2003, and 2004 with the Bank's actual loss experience. The projected LDA capital estimates are based on a 99.9% confidence level using the total loss data and on simply adding the OR capital estimates for the four risk types. OR capital calculated using the aggregated total data is insufficient to cover the actual accumulated losses in 2002 and 2003 (Figure 9), whereas the annual capital estimate based on the four risk types does cover the actual losses for all years (Figure 10). We observe a clear decrease in the projected capital level in 2005, corresponding to the actual decrease in realized losses in 2004.

Stress tests

Stress testing is a commonly used tool to enhance and validate models [BCBS (2006b)]. We perform a worst case scenario analysis by adding external loss data to our loss dataset. An historically high loss data point from the last 20 years is added to the existing loss dataset, the capital estimate is re-calculated, and the impact of the increased capital value measured. Naturally, the Bank should be choosing the relevant historical loss events with regard to their own organization. We selected the four historical

Table 2 - Impact on LDA OR capital when adding a data point representing an extreme loss from a relevant historical event

Worst case analysis adding an extreme loss data point from the industry					
Assessment of impact on CapitalLDA based on the Poisson/lognormal with MLE in millions of euros					
	Benchmark	Deutsche Bank	AXA	Citigroup	Sumitomo
99.90%	30.4	32.4	32.7	33.4	33.4
change in %	0.0%	6.4%	7.4%	9.9%	10.0%
99.97%	32.5	35.2	35.5	36.5	36.5
change in %	0.0%	8.2%	9.3%	12.2%	12.3%

events⁷ based on similar institutions. The recalculated capital estimate using the extreme losses experienced at Deutsche Bank, AXA, Citigroup, and Sumitomo increases our capital estimate by between 6 and 11% over the benchmark (Table 2).

Apparently, the capital estimate obtained using LDA is moderately sensitive to an additional extreme loss data point being added to the existing loss data. We infer that when the underlying loss distribution is heavy tailed, the probability of infrequent and large loss is higher than practitioners typically assume. Thus, the next step is to explore techniques based on extreme value theory and compare the results.

Extreme risk capital model

As we saw previously, the tail losses of the empirical loss distribution should mainly determine the size of the OR capital estimate. It is, therefore, important that we accurately fit the tail of the empirical loss distribution to the GPD theoretical distribution proposed above. To select a threshold, in addition to the common estimation methods - Pickands estimator [Pickands (1975)], Hill estimator [Hill (1975)] and the GPD MLE with 95 percent asymptotic confidence interval - we use a novel graphical technique based on the analysis of the MCMC Bayesian estimation outputs. They suggest that the threshold should be set at $u = 782,377$ corresponding to the 97th percentile of the

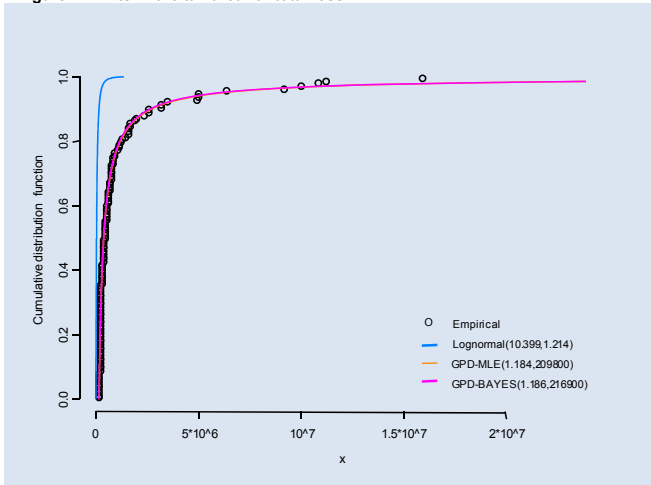
Table 3 - Comparison between Bayesian and MLE GPD parameter estimates for total losses

Threshold u	Number of exceedances v	% Tail fit $P(X>u)$	Bayesian shape parameter ξ	MLE shape parameter ξ	Bayesian scale parameter β	MLE scale parameter β
172,514	118	10%	1.186 (0.000)	1.184 (0.169)	216,900 (73)	209,800 (25,331)
198,160	106	9%	1.180 (0.000)	1.175 (0.161)	246,400 (89)	238,400 (30,067)
224,792	94	8%	1.196 (0.000)	1.195 (0.178)	284,600 (118)	273,400 (35,244)
252,853	83	7%	1.214 (0.001)	1.229 (0.206)	333,500 (191)	314,500 (41,222)
350,000	67	6%	1.156 (0.001)	NA	426,800 (216)	NA NA
426,958	59	5%	1.130 (0.001)	NA	504,300 (255)	NA NA
594,702	47	4%	1.067 (0.001)	NA	651,400 (346)	NA NA
782,377	35	3%	1.115 (0.001)	NA	898,500 (604)	NA NA

Standard error of the estimate expressed in brackets

7 (i) Deutsche Bank Group (Deutsche Bank) agreed to settle a WorldCom shareholder lawsuit on 10 March, 2005 for U.S.\$325 million regarding its role as an underwriter of WorldCom bonds; (ii) AXA Group's (AXA) Money Manager, Alliance Capital, announced on 18 December, 2003 that it had agreed to a U.S.\$600 million settlement related to allegations that it permitted improper trading of its mutual funds; (iii) Citigroup agreed to pay U.S.\$2.58 billion to a class of shareholders who bought WorldCom bonds before the telecom company filed for bankruptcy in 2002. Citigroup's end occurrence date was 30 June 2002; and (iv) Sumitomo Corporation (Sumitomo) lost U.S.\$2.60 billion in 1996 in the world's then biggest unauthorised trading scandal. A former chief copper trader, Yasuo Hamanaka, faked the signatures of two supervisors on documents, giving him full authority for copper trading and the transfer of funds.

Figure 11 - Fits in the tail area for total loss



cumulative distribution function and a MCMC posterior median shape parameter estimate of 1.12.

The stability of GPD shape and scale parameters for varying thresholds is shown for total losses in Table 3. Note that the precision of LDA parameter estimates are described as asymptotic standard errors of a point estimate, whereas the precisions of the MCMC parameter estimates are described by the full posterior distributions of the parameters. Clearly the Bayesian posterior median shape parameter estimate is stable with a moderate standard error across the threshold range examined. This estimate remains stable even using very few observations, whereas the maximum likelihood estimator (MLE) cannot be calculated beyond the 94th percentile of the empirical distribution. In addition to their instability, the MLEs are increasingly imprecise, i.e., they have significantly larger standard errors as the threshold increases [Medova and Kyriacou (2002)]. We estimate the ERCM both as an individual risk type model and a multi risk type model where the losses are those of the four basic risk types described earlier and all are considered together (using the hierarchical structure) to examine dependencies between risk type extremes. The shape parameter estimates of the four losses data are similarly robust to changes in threshold for high percentiles. These results taken together adequately demonstrate that the ERCM parameter estimates are approximately constant over different thresholds so that the stability property which is required to hold under GPD assumptions is valid.

Figure 11 shows graphically the lognormal and GPD tail fits for total losses when the threshold is set at 170,0008. It appears that

Table 4 - Summary of capital values for the total losses and four risk type losses

Median Capital _{ERCM} millions of euros	Total losses	External losses	Process losses	People losses	System losses	Total
Total risk: 4-year series	196	70	205	149	50	475
Four risk types: 2-year series		98	345	132	107	681
Four risk types: 3-year series		193	333	189	179	895
Four risk types: 4-year series		195	316	183	181	875
Median Capital _{ERM} millions of euros	2001 loss	2002 loss	2003 loss	2004 loss	2001-02 loss	2001-03 loss
Total risk: 1-year series	81	105	73	66	141	202

8 To be able to incorporate both severity distributions in the same plot.

Figure 12 – Backtesting annual mean and median OR capital levels using the ERCM for total losses

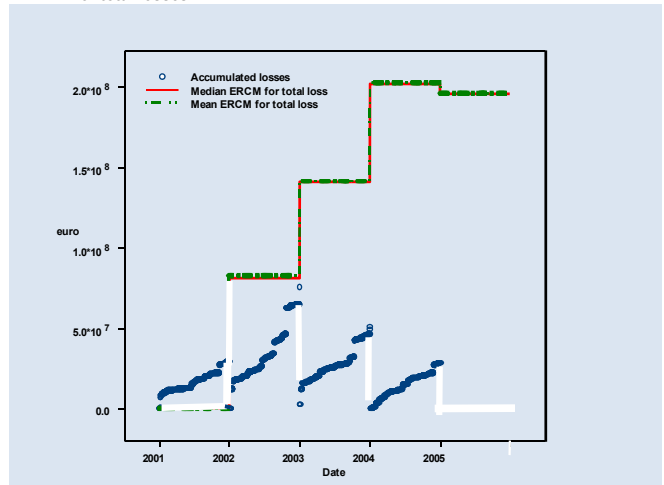
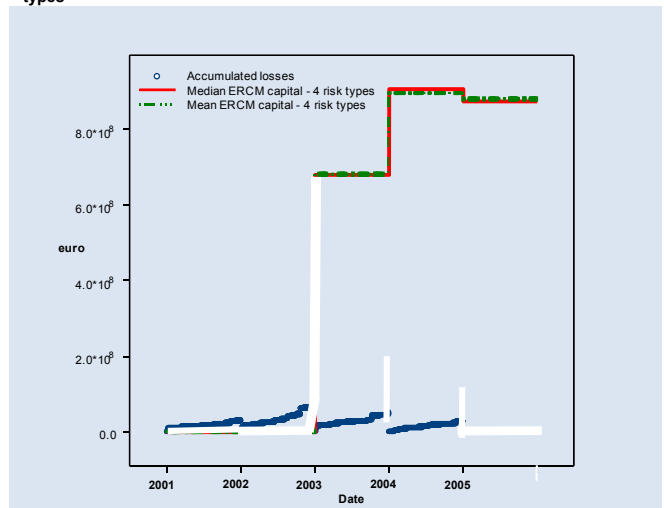


Figure 13 – Backtesting annual OR capital levels using the ERCM using four risk types



the GPD gives a reasonable fit to the underlying distribution's tail. Clearly the lognormal distribution of the LDA does not (Figure 6). The GPD using hierarchical Bayesian estimates also appears to give reasonable fits for the four risk individual categories and the very highest observed losses are properly captured. The lognormal distribution on the other hand tends to underestimate the probabilities of large losses for both the total and all four risk type losses.

Results

A summary of median OR capital values derived from the ERCM with various datasets is presented in Table 4. The hierarchical structure of the Bayesian model provides a more transparent risk assessment of the four risk types. For instance, the OR capital for system losses based on the four-year data series is €50 million if considered on a standalone basis, whereas considered together with the other risk types this figure increases to €181 million. Likewise, when dependencies are considered between risks types the capital for external, process and people losses increase respectively from €70 million to €195 million, €205 million to €316 million and from €149 million to €183 million. Clearly taking account of the statistical interdependencies of individual risk types' losses is important. The higher risk found by the ERCM's Bayesian hierarchical structure shows that dependencies

Figure 14 - Comparison of regulatory, BIA, LDA and EVCM capital levels with actual accumulated losses

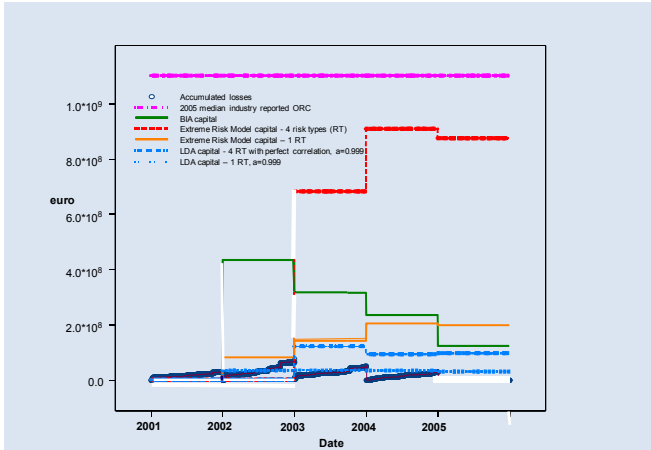
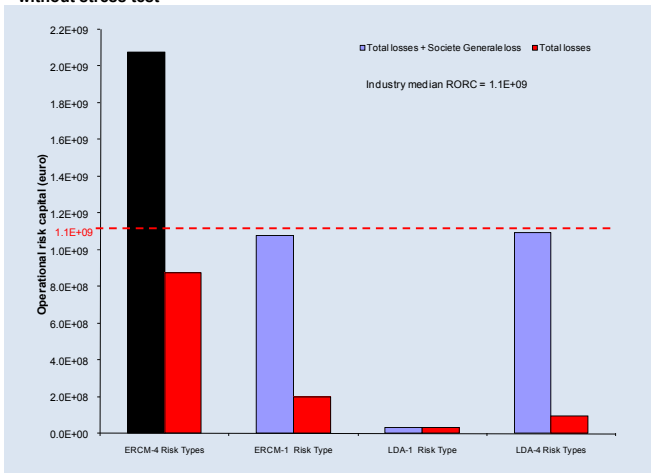


Figure 15 - Comparison of regulatory OR, LDA and ERCM capital levels with and without stress test



between the extreme (tail) risks of the four types have a large positive impact on the total OR capital required. Considering risk types separately and simply summing the results (i.e. assuming perfect correlation between risk types) gives a lower OR capital level than is appropriate in this situation. If we examine the total losses year by year in Table 4, we see the highest OR capital was required in 2002, €105 million, while by 2004 they only required €66 million.

Backtests

We first assess the EVCM OR capital estimates for total losses by historical backtesting to project the capital estimates for years 2001, 2003 and 2004 a year ahead and compare them with actual accumulated daily losses over each year from 2002 to 2005. Figure 12 shows clearly that the projected annual capital estimates for total losses using the ERCM cover the actual accumulated daily total losses. Figure 13 shows a comparison of accumulated daily losses with OR capital levels calculated with ERCM based on the four risk types for years 2003 and 2004 (due to data limitations). We observe a clear increase in the capital level projected for 2004 over the 2003 level due to the increase of extreme losses in 2003. The ERCM OR capital estimate based on the four risk types data is about 4.5 times higher than that for total losses due to risk type losses tail interdependencies⁹.

9 Due to the limited sample size we cannot perform an evaluation on the level of total capital of the impact of the length of the loss data series divided into its four risk types.

Stress tests

Analogous to the stress tests for the loss distribution approach above, we perform worst case scenario analysis on total ERCM OR capital estimates by adding external loss data to our dataset. We add individually extreme losses experienced at Deutsche Bank (U.S.\$325 million), AXA (U.S.\$600 million), Citigroup (U.S.\$2.58 billion) and Sumitomo (U.S.\$2.6 billion) and re-calculate the OR capital estimate. The results show significant increases in median OR capital: 52% (Deutsche Bank), 95% (AXA), 417% (Citigroup) and 438% (Sumitomo).

In summary, we have demonstrated that the multivariate ERCM with hierarchical Bayesian parameter estimation using MCMC techniques applied to a large bank's operational losses is a prudent and robust model which produces OR capital estimates which are plausible and consistent with actual experience.

Comparison of alternative OR capital models

Next we compare the LDA and the ERCM OR capital estimates for the Bank with its BIA capital, its reported regulatory OR capital, and the median industry OR capital¹⁰.

Capital levels

Table 5 summarizes the alternative model OR capital estimates. Comparing the 2005 capital levels for total losses calculated using the LDA (with a 99.9 percentile confidence level) and the ERCM we find that the ERCM capital estimate is about 6.5 times larger than the LDA capital estimate. For 2005 the ERCM capital level of €195.76 for total losses is twice as large as that of the LDA at €95.62 (based on a 99.9 percentile confidence level and perfect correlation between risk types), whereas the ERCM capital level for the four risk types of €875.16 is 9.2 times larger than the 2005 LDA capital estimate. These calculations also show that the BIA based OR capital estimate exhibits a decreasing trend and has reduced by about 72% between 2002 and 2005, and 46% from 2004 to 2005. Both the LDA and the ERCM capital estimates for total losses first increase and then decrease like the assessment of total losses in Figure 13. The overall pattern of changes in capital level over time, and with an increasing pool of losses, suggest that both the ERCM- and LDA-based capital calculations are more risk sensitive and robust than the BIA. However, the 95 percent confidence interval for the LDA-based OR capital estimates show higher dispersion than the ERCM for total loss estimates.

All the capital estimates calculated with BIA, LDA, and ERCM are significantly below the Bank's (industry median) OR regulatory capital. The ERCM four risk types OR capital estimate in 2005 is 80% of the Bank's regulatory OR capital, whereas the capital estimates for ERCM for total losses and the BIA are only 18% and 11%, respectively. The LDA capital estimate for total losses with 99.9 percentile confidence levels is a mere 2.7% of the regulatory OR capital but using the perfectly correlated four risk types this estimate increases to 8.7%.

These results are summarized in Figure 14, which shows that the capital estimates calculated with the BIA, LDA, and ERCM (except for the capital estimate calculated using the LDA based on total losses) all cover the accumulated daily total losses for each year. The ERCM's capital level using four risk types is significantly higher than the other models' capital levels due to its ability to account for interdependencies between risk type extremes¹¹.

10 The median is €1,100 million calculated from data in the 50 bank survey above.
11 In fact, its year 2005 capital estimate is about 4.47 times larger than the 2005 ERCM capital estimate for total losses.

Table 5 - OR capital estimates calculated with the BIA, LDA, ERCM for total losses and four risk type losses

Year	Capital _{BIA} € Mil	Percentile	Capital _{LDA} Total	Capital _{LDA} Four risk types	Capital _{ERCM} Total	Capital _{ERCM} Four risk types
2002	433	99.90	33.4 (32.9,34.0)	NA	80.8 (80.7,80.9)	NA
		99.97	37.5 (36.3,39.1)			
2003	316	99.90	34.3 (33.9,34.7)	120.6 (113.5,128.7)	140.8 (140.6,140.9)	680.9
		99.97	37.2 (36.4,38.4)	179.4 (159.3,203.5)		
2004	234	99.90	33.0 (32.7,33.4)	91.8 (87.2,97.4)	202.0 (201.8,202.2)	907.2
		99.97	35.6 (34.8,36.6)	128.8 (117.4,143.5)		
2005	121	99.90	30.1 (30.2,30.9)	95.6 (90.6,101.5)	195.8 (195.6,196.0)	875.2
		99.97	32.8 (32.1,33.6)	136.6 (122.8,152.7)		

Given the short loss data series and the scarcity of extreme losses we next pose the question: what is the impact of a single extreme loss on the capital levels of the alternative models in relation to the Bank's year 2005 regulatory OR capital? To investigate the answer we append to our loss dataset in 2004 one of the largest known extreme losses in financial history, Société Générale's €4,900 million rogue trading loss¹², and recalculate the capital estimates for the models. Figure 15 summarizes our results and shows the reported median OR capital level across banks for years 2001-05 at €1,100 million euros (in red). The ERCM OR capital level using the four risk types is €2,971 million, or €1,871 million above the year 2005 reported figure. The LDA capital level accounting for the four risk type losses separately is only €101 million, or €999 million below the year 2005 regulatory level.

How much capital should the Bank be allocating for operational risk? Unfortunately, there is no simple answer. The results of our analysis suggest that if the Bank's self-assessment reveals that its risk profile is higher than expected, relative to comparable banks in the industry, then it should consider setting the capital level to cover extreme operational risks at the higher end of the capital range from €875 million to €2,971million. If the Bank's self-assessment reveals that its risk profile is low in comparison with the rest of the industry, then it should consider setting the OR capital level at the lower end of the capital range from €196 million to €875 million. Our finding in Table 5 that all models gave lower capital estimates than its reported OR regulatory capital level of about €1,100 million suggests that when it determines the OR capital level the Bank is placing the majority of weight on the use of relevant external data and/or scenario analysis and the inclusion of factors reflecting the business environment and internal control systems. Thus, the capital estimate derived from the Bank's AMA might actually be reflecting the industry's risk exposures rather than its own. A possible reason for this is the common shortage of internal loss data at banks. Large internationally active banks are allocating anywhere between U.S.\$2 billion to U.S.\$7 billion for operational risk depending on their nature, size, risk profile, and organizational complexity [de Fontnouvelle et al. (2004)]. Until sufficient bank level data is available, industry level data must be at least considered.

Capital adequacy

The Bank's regulatory OR capital level of about €1,100 million in 2005 corresponds to a medium risk profile. To further assess the Bank's risk profile in relation to other banks in the industry we examine its OR capital adequacy in terms of the relationship

between its capital provisions and assets. If the bank has more equity capital than its estimated risk capital, this could result in inefficiencies as the excess capital might be better used elsewhere. Conversely, if the bank has less equity capital than the estimated risk capital, then it is probably taking on too much risk and should consider reducing its balance sheet or raising new capital. Thus the actual values of risk capital must be periodically compared against the bank's capital for risk coverage. The total disposable capital for risk coverage primarily comprises balance sheet equity less goodwill. We calculated the amount of the Bank's total risk capital used for operational risk coverage and found that it has steadily declined, while the OR proportion allocated has increased over the same time period (2002 12.7%, 2003 12.0%, 2004 13.9%, and 2005 14.5%). A risk buffer of at least 20% between the Bank's overall risk capital and the disposable capital available for its OR coverage is considered prudent [Commerzbank (2007)]. Operational risk taking capacity is defined as disposable capital available for operational risk coverage minus reported OR capital. The operational risk taking capacity of the bank was very low (2.7%) in 2002, but has since increased (16.9% in 2003 and 37.2% in 2004) and was significantly higher than the 20 percent risk buffer in 2005 (44.8%).

Finally, we relate the Bank's risks to its assets and assess its standing relative to other banks in the industry. We consider the relationship between the frequency of losses exceeding various thresholds and three measures of bank size: total assets, Tier 1 capital, and gross income. This is analogous to a study performed by the U.S. Federal Reserve and the thrift regulatory agencies, which examined 24 banks with a presence in the U.S based on the 2004 operational risk loss data collection exercise [FRB et al. (2005)]. We also consider the relationship between the severities of losses (i.e., the average annual losses exceeding a U.S.\$20,000 threshold) and the three measures of bank size. The corresponding ratios calculated for the Bank are compared with the results reported by the U.S regulatory agencies. In particular, we study cross-bank median values in order to assess the Bank's operational risk in relation to a typical bank in the global market. An examination of the inter quartile ranges (IQR) in the Fed study enables us to assess the consistency of the reported ratios across banks. The Bank's relation to other banks is determined by its ratio lying below, within, or above the typical bank's IQR.

Table 6 illustrates that the Bank experiences a considerably lower number of losses per year than is typical in terms of total assets, but in terms of Tier 1 capital or gross income the loss frequency for very extreme losses appears to be higher than the industry average. The Bank's average annual losses as a percentage of its size appears to be in line with the industry average.

¹² The Bank's regulatory OR capital is about 22 percent of this external €4,900 million loss.

Our limited examination of the Bank's risk ratio suggests that the Bank has a medium risk profile in relation to other banks in the industry. We propose that the bank uses a multiple risk type extreme risk capital model to determine the baseline OR capital, and, thereafter, adjust the capital estimate in accordance with the results of its rigorous self-assessment process and relevant stress testing. As a result, given that the Bank is a typical bank with a medium risk profile in 2005, it should have set its OR capital level at about €875 million.

Conclusion

In this paper we propose that the extreme risk capital model improves the measurement and understanding of interdependencies in operational risks in banking and guides the determination of OR capital and the corresponding regulatory capital. We compared the operational risk models and capital estimates determined by the BIA, LDA, and multiple risk type ERCM (with hierarchical Bayesian parameter estimation using MCMC techniques). In the modeling we used four years of internal operational loss data from a large internationally active bank. As part of the process, we found the appropriate threshold levels for extreme value theory parameter estimation. We also calculated operational risk capital for extreme loss data corresponding to distributions with tail index greater than one, i.e., with very heavy tails.

The loss distribution approach has emerged as the most common form of measuring operational risk in large banks. However, a key concern with the LDA is its limited ability to accurately capture extreme operational risk events. Moreover, inclusion of other data sources than limited internal ones leads to additional problems, such as difficulties with scaling external loss data and combining data from different sources.

It has been demonstrated in this paper that the ERCM is a prudent and robust model, which produces plausible estimates consistent with actual experience. The comparative analysis showed that the ERCM model outperformed the BIA and LDA regarding accurate stable operational risk capital estimates and that it has some predictive power in extrapolating historical data. The ERCM does not require particular assumptions on the nature of the distributions underlying the observations and the general lack of operational loss data favors OR capital estimates based on Bayesian hierarchical MCMC simulation, which provides robust parameter estimates of extreme distributions even with small sample sizes. Another major advantage of the ERCM over the LDA is its ability to take into account expert judgment and external data through well specified prior parameter distributions. The ERCM enables aggregation of risks from different organizational units and risk types into an overall measure of operational risk, and in doing so, takes account of the ways in which different extreme events interact with each other. We found that dependencies between extreme risks of all types lead to a significant increase in the estimated OR capital required. Considering risk types separately and then aggregating them using a simple sum (with perfect correlation between risk types, as is often recommended by regulators following Basel II) results in inappropriate lower OR capital levels.

Model risk with all methods can be considerable when they are calibrated on unreliable or sparse data. We are aware of the usual claims that LDA- and EVT-based models are of limited use for capital calculation due to lack of data and inconsistency of the data with the modeling assumptions. Nevertheless in this paper we have shown some promising results from the simulations with our ERCM on the group wide level. Although our findings only apply to one specific bank (because the ERCM was applied to its internal losses) this framework can easily be tailored to other banks, and even across banks to study intra-industry interdependencies [Acharya and Richardson (2009)]. Given a

larger loss dataset, further research could include a more granular analysis using the ERCM hierarchical model on business line by event type loss data such as is proposed in Basel II. Further research could also include extensions to the ERCM model by allowing for non-stationarity and clustering effects. Additionally, research involving losses from different banks is needed to lead to correct scaling of external data and risk attitude assessment across the industry.

Central banks, financial supervisory authorities, international organizations such as the Bank for International Settlements, and perhaps certain financial industry loss data consortia, are expected to take the lead in researching these issues. A benchmark capital level, i.e., a prudent and reasonable baseline OR capital level calculated using the ERCM, would aid regulators in their review and evaluation of a bank's capital adequacy. Regulators and marketplace participants would thus gain sufficient understanding of a bank's operational risk to reward the bank for managing its risks prudently or to penalize it. Our examination of the top 50 international banks' economic and regulatory capital supports greater transparency about banks' risk exposures and better disclosure of their institutional arrangements for risk management and risk quantification.

In summary, we propose that banks use the ERCM to determine a baseline OR capital allocation and thereafter adjust it in accordance with the results from its self-assessment process and relevant stress testing. We also advocate the ERCM as a tool especially suitable for decision makers at the top levels of a bank who need an integrated and holistic view of the bank's extreme operational risk exposure and an estimate of its required future capital coverage.

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