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DC pension fund benchmarking with fixed-mix portfolio optimization

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1. Introduction

Corporate sponsored *defined benefit* (DB) pension schemes have recently found themselves in hot water. Accounting practices that led to over-exposure to equity markets, increases in longevity of the scheme participants and low interest rates have all contributed to the majority of schemes in the EU and the UK finding themselves underfunded. In essence, a DB scheme promises to pay its participants an annuity at retirement that gives them a pension equal to a proportion of their final salary (the proportion depending on the number of years of service). Therefore the responsibility to meet these promises (liabilities) rests firmly with the scheme's trustees and ultimately with the corporate sponsor.

The management of these corporate schemes was greatly affected in the past by quarterly earnings reports which directly impacted stock prices in the quest for 'shareholder value'. Consequently DB scheme sponsors resorted to a management style that was able to keep the liabilities, if not off the balance sheet, then at least to a minimum. One sanctioned tactic that achieved these aims was the ability to discount liabilities by the expected return of the constituent asset classes of the fund. In other words, by holding a large part of the fund in equities, the liabilities could be discounted away at over 10% p.a. The recent performance of the equity markets and the perception of equity as a long-horizon asset class assisted in justifying this asset-mix in the eyes of the scheme's trustees. However, with the collapse of the equity-market bubble in 2001, many funds found their schemes grossly underfunded and were forced to crystallize their losses by panicked trustees. Consequent tightening of the regulations has made the situation even worse (e.g. all discounting must be done by the much lower AA credit quality bond yield rates in the UK FRS17 standard).

As a result many DB schemes have closed and are now being replaced with *defined contribution* (DC) schemes.† In this world of corporate sponsored DC pension schemes the liability is separated from the sponsor and the market risk is placed on the shoulders of the participants. The scheme is likely to be overseen by an investment consultant and if the scheme invests in funds that perform badly over time a decision may be made by the consultant to move the capital to another fund. However, any losses to the fund will be borne by the participants in the scheme and not by the corporate sponsor.

Since at retirement date scheme participants will wish to either purchase an annuity or invest their fund payout in a self-managed portfolio, an obvious need arises in the market place for real return guaranteed schemes which

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†DC pension scheme participants typically make a lump sum initial payment and regular contributions to the pension fund which are employer matched.

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are similar to those often found in life insurance policies. These guarantees will typically involve inflation protection plus some element of capital growth, for example, inflation rate plus 1% per annum. From the DC fund manager's viewpoint provision of the relevant guarantee requires very tight risk, control, as the recent difficulties at Equitable Life so graphically illustrate.

The question addressed in this article is how consultants or DC fund-managers can come to a sensible definition of an easily understandable liability-related benchmark against which the overall fund performance for a DC scheme can be measured. Performance of both fund and benchmark must be expressed to fund participants in easy-to-understand concepts such as probability of achieving some target wealth level above the scheme guarantee – a measure easily derived from the solutions of the models discussed in this article.

2. Current market practice

Currently, DC pension funds are market-benchmarked against either a fixed-mix of defined asset classes (total return bond and equity indices) or against some average performance of their peers. The benchmark is not defined in terms of the pension liability and investment is not liability-driven. The standard definitions of investment risk—standard deviation, semi-variance and downside risk—do not convey information regarding the probability of missing the scheme participants' investment goals and obligations.

For example, the macro-asset benchmark may be defined as 20% of certain equity indices and 80% of particular bond indices, but may not reflect the risk that the scheme participants are willing to take in order to attain a specific *substitution rate* between their final salary and pension income (pension earnings/final salary).

3. Optimal benchmark definition for DC funds

In line with current market practice, we wish to find a definition of a fixed asset mix benchmark similar to that given in the example above but with an asset mix that optimizes returns against user-defined risk preferences. Specifically, for participants that are willing to take a certain amount of risk in order to aim for a given substitution rate between final salary and pension, we should be able to 'tune' the asset mix in an optimal way to reflect the participants desire to reach this substitution rate. The risk could then be defined as the probability of not reaching that substitution rate.

In contrast to dynamic multi-stage portfolio optimization, where the asset-mix is changed dynamically over time to reflect changing attitudes to risk as well as market performance (dynamic utility), a *fixed-mix rebalance strategy* benchmark in some sense reflects an average of this dynamic utility over the fund horizon. For such a strategy the realized portfolio at each decision stage is rebalanced back to a fixed set of portfolio weights. In practice for DC pension schemes we want the returns to be in line with salary inflation in the sense that the required substitution rate is reached with a given probability. The solution to this problem will entail solving a fixed-mix dynamic stochastic programme that reflects the long run utility of the scheme participants.

In general multi-period *dynamic stochastic optimization* will be more appropriate for long-term investors. Single-period models construct optimal portfolios that remain unchanged over the planning horizon while fixed mix rebalance strategies fail to consider possible investment opportunities that might arise due to market conditions over the course of the investment horizon. Dynamic stochastic programmes on the other hand capture optimally an investment policy in the face of the uncertainty about the future given by a set of scenarios.

Cariñno and Turner (1998) compare a multi-period stochastic programming approach to a fixed-mix strategy employing traditional mean-variance efficient portfolios. Taking a portfolio from the mean-variance efficient frontier, it is assumed that the allocations are rebalanced back to that mix at each decision stage. They also highlight the inability of the mean-variance optimization to deal with derivatives such as options due to the skewness of the resulting return distributions not being taken into account. The objective function of the stochastic programme is given by maximizing expected wealth less a measure of risk given by a convex cost function. The stochastic programming approach was found to dominate fixed-mix in the sense that for any given fixed-mix rebalance strategy, there is a strategy that has either the same expected wealth and lower shortfall cost, or the same shortfall cost and higher expected wealth. Similar results were found by Hicks-Pedrón (1998) who also showed the superiority in terms of final Sharpe ratio of both methods to the constant proportion portfolio insurance (CPPI) strategy over long horizons.

Fleten *et al.* (2002) compare the performance of four-stage stochastic models to fixed-mix strategies of in- and out-of-sample, using a set of 200 flat scenarios to obtain the out-of-sample results. They show that the dynamic stochastic programming solutions dominate the fixed-mix solutions both in- and out-of-sample, although to a lesser extent out-of-sample. This is due to the ability of the stochastic programming model to adapt to the information in the scenario tree in-sample, although they do allow the fixed-mix solution to change every year once new information has become available, making this sub-optimal strategy inherently more dynamic.

Mulvey *et al.* (2003) compare buy-and-hold portfolios to fixed-mix portfolios over a ten-year period, showing that in terms of expected return versus return standard deviation, the fixed-mix strategy generates a superior efficient frontier, where the excess returns are due to portfolio rebalancing. Dempster, Evstigneev and Schenk-Hoppé (2007a) discuss the theoretical cause of this effect (and the historical development of its understanding) under the very general assumption of stationary ergodic returns. A similar result was found in Mulvey *et al.* (2004) with respect to including alternative investments into the portfolio. In particular they looked at the use of the Mt. Lucas Management index in multi-period fixed-mix strategies. A multi-period optimization will not only identify these gains but also take advantage of volatility by suggesting solutions that are optimal in alternative market scenarios. In Mulvey *et al*. (2007) the positive long term performance effects of new asset classes, leverage and various overlay strategies are demonstrated for both fixed-mix and dynamically optimized strategies.

4. Fund model

The dynamic optimal portfolio construction problem for a DC fund with a performance guarantee is modelled here at the strategic level with annual rebalancing. The objective is to maximize the expected sum of accumulated wealth while keeping the expected maximum shortfall of the portfolio relative to the guarantee over the 5 year planning horizon as small as possible. A complete description of the *dynamic stochastic programming model* can be found in Dempster *et al*. (2006). In the *fixed-mix model* the portfolio is rebalanced to fixed proportions at all future decision nodes, but not at the intermediate time stages used for shortfall checking.

This results in annual rebalancing while keeping the risk management function monthly, leading to the objective function for both problems as

$$\max_{\left\{\substack{x_{t,a}(\omega),\, x^+_{t,a}(\omega),\, x^-_{t,a}(\omega):\\ a\in A,\, \omega\in\Omega,\, t\in T^d\cup\{T\}}\right\}} \left\{(1-\beta)\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^d\cup\{T\}} W_t(\omega)\right) - \beta\left(\sum_{\omega\in\Omega} p(\omega) \max_{t\in T^{\text{total}}} h_t(\omega)\right)\right\}, \quad (1)$$

where

- $p(\omega)$ denotes the *probability* of scenario ω in Ω—here $p(\omega) := 1/N$ with N scenarios,
- $W_t(\omega)$ denotes the *portfolio wealth* at time $t \in T^{\text{total}}$ in scenario ω,
- $h_t(\omega)$ denotes the *shortfall* relative to the barrier at time t in scenario ω.

For the *nominal* or *fixed guarantee*, the *barrier* at time t in scenario ω, below which the fund will be unable to meet the guarantee, is given by

$$L_t^F(\omega) = W_0(1+G)^T Z_t(\omega) = W_0(1+G)^T \mathrm{e}^{-y_{t,T}(\omega)(T-t)}, \quad (2)$$

where

- G denotes the *annual nominal guarantee*
- $Z_t(\omega)$ denotes the *zero-coupon Treasury bond* price at time t in scenario ω.

For simplicity we model closed-end funds here, but see Dempster *et al*. (2006, 2007) and Rietbergen (2005) for the treatment of contributions. We employ a five-period (stage) model with a total of 8192 scenarios to obtain the solutions for the dynamic optimization and fixed mix approaches.†

For this article five different experiments were run on a five-year closed-end fund with a minimum nominal guarantee of 2% and an initial wealth of 100 using a 512.2.2.2.2 tree.‡ (See Dempster *et al*. (2006, 2007) for more details on this problem.) The parameter of risk aversion β is set to 0.99 and the parameter values used were estimated over the period June 1997–December 2002. The Pioneer CASM simulator was used to generate the problem data at monthly intervals.

The five experiments run were as follows.

- Experiment 1: *No* fixed-mix constraints. Objective function: fund wealth less expected maximum shortfall with monthly checking.
- Experiment 2: Arbitrary fixed-mix: 30% equity and 10% in each of the bonds.
- Experiment 3: The fixed-mix is set equal to the root node decision of Experiment 1.
- Experiment 4: The fixed-mix is set equal to the root node decision of Experiment 1 but only applied after the first stage. The root node decision is optimized.
- Experiment5: The fixed-mix is determined optimally.

Experiments 2–4 with fixed-mixed constraints are '*fixed* fixed-mix' problems in which the fixed-mix is specified in advance in order to keep the optimization problem convex. Finally Experiment 5 uses fixed-mix constraints without fixing them in advance. This renders the optimization problem non-convex so that a global optimization technique needs to be used. In preliminary experiments we found that although the resulting unconstrained problems are multi-extremal they are "near-convex" and can be globally optimized by a search routine followed by a local convex optimizer. For this purpose we used Powell's (1964) algorithm followed by the SNOPT solver. Function evaluations involving all fixed-mix rebalances were evaluated by linear programming using CPLEX. This method is described in detail in Scott (2002).

As the fixed-mix policy remains the same at all rebalances, theoretically there is no reason to have a scenario tree which branches more than once at the beginning of the first year. A simple fan tree structure would be perfectly adequate as the fixed-mix approach is unable to exploit the perfect foresight implied after the first stage in this tree. However for comparison reasons we use the same tree for both the dynamic stochastic programme (Experiment 1) and the fixed-mix approach (Experiments 2–5).

5. Nominal guarantee results

Table 1 shows the expected terminal wealth and expected maximum shortfall for the five experiments.

†In practice 10 and 15 year horizons have also been employed.
‡Assets employed are Eurobonds of 1, 2, 3, 4, 5, 10 and 30 year maturities and equity represented by the Eurostock 50 index.

As expected, Experiment 1 with no fixed-mix constraints results in the highest expected terminal wealth and lowest expected maximum shortfall. Whereas Experiments 2 and 3 underperform, Experiment 3 in which the initial root node solution of Experiment 1 is used as the fixed-mix is a significant improvement on Experiment 2 (arbitrary fixed-mix) and might serve as an appropriate benchmark. Experiment 4 resulted in a comparable expected terminal wealth to Experiment 3, but the expected maximum shortfall is now an order of magnitude smaller. Finally in Experiment 5 global optimization was used which correctly resulted in an improvement relative to Experiment 3 in both the expected terminal wealth and the expected maximum shortfall.

Table 1. Expected terminal wealth and maximum shortfall for the nominal guarantee.

	Expected terminal wealth	Expected maximum shortfall
Experiment 1	126.86	8.47 E-08
Experiment 2	105.58	14.43
Experiment 3	120.69	0.133
Experiment 4	119.11	0.014
Experiment 5	122.38	0.122

Table 2. Root node solutions for the nominal guarantee.

	1y	2y	3y	4y	5y	10y	30y	Stock
Exp 1	0	0	**0.97**	0	0	0	**0.02**	**0.01**
Exp 2	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.30**
Exp 3	0	0	**0.97**	0	0	0	**0.02**	**0.01**
Exp 4	0	0	**0.06**	**0.94**	0	0	0	0
Exp 5	0	0	**0.96**	**0.04**	0	0	0	0

Table 2 shows the optimal root node decisions for all five experiments. With the equity market performing badly and declining interest rates over the 1997–2002 period, we see a heavy reliance on bonds in all portfolios.

Figures 1 and 2 show the efficient frontiers for the dynamic stochastic programme and the fixed-mix solution, where the risk measure is given by expected maximum shortfall. Figure 1 shows that the dynamic stochastic programme generates a much bigger range of possible risk return trade-offs and even if we limit the range of risk parameters to that given for the fixed-mix experiments as in figure 2, we see that the DSP problems clearly outperform the fixed-mix problems.

We also considered the distribution of the terminal wealth as shown in figures 3 and 4. From figure 3 we observe a highly skewed terminal wealth distribution for Experiment 1 with most of the weight just above the

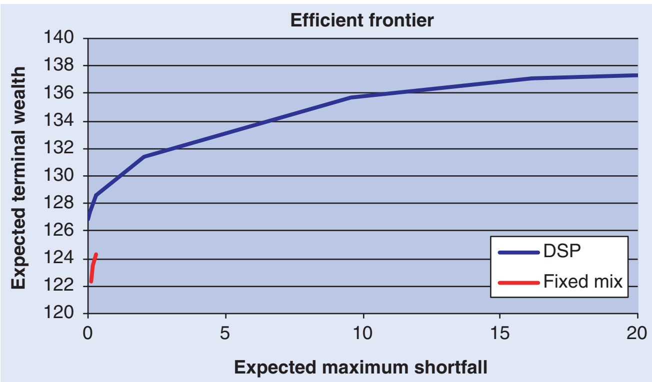

Figure 1. Efficient frontier for the nominal guarantee.

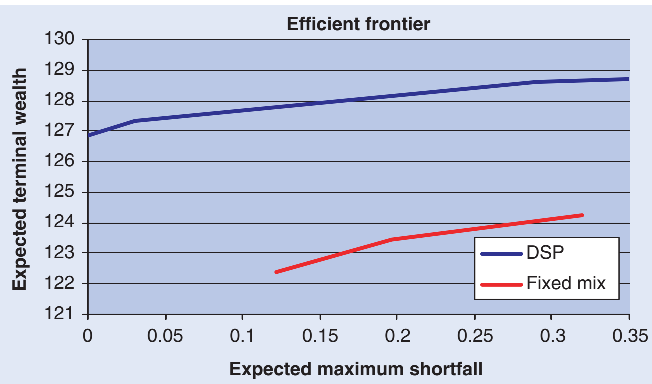

Figure 2. Efficient frontier for the nominal guarantee.

Figure 3. Terminal wealth distribution for optimal dynamic stochastic policy of Experiment 1.

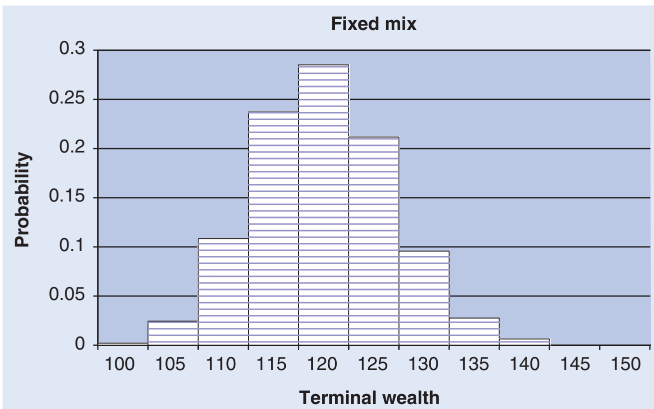

Figure 4. Terminal wealth distribution for the optimal fixed-mix policy of Experiment 5.

guaranteed wealth of 110.408. The effect of dynamic allocation is to alter the overall probability distribution of the final wealth. Cariño and Turner (1998) also note this result in their experiments. For Experiments 2 and 3 in which there is no direct penalty in the optimization problem for shortfall, we see the more traditional bell-shaped distribution. Using the initial root node solution of Experiment 1 as the fixed-mix portfolio results in a distribution with a higher mean and lower standard deviation (the standard deviation drops from 20.51 to 6.43). In Experiment 4 we see an increase again in the probability of the terminal wealth ending up just above the minimum guarantee of 110 as the optimization problem has flexibility at the initial stage. The standard deviation is further reduced in this experiment to 4.04. The mean and standard deviation of Experiment 5 is comparable to that of Experiment 3, which is as expected since the portfolio allocations of the two experiments are closely related.

6. Inflation-linked guarantee results

In the case of *an inflation-indexed guarantee* the final guarantee at time T is given by

$$W_0 \prod_{s=1/12}^{T} \left(1 + i_s^{(m)}(\omega)\right), \tag{3}$$

where $i_s^{(m)}(\omega)$ represents the *monthly inflation* rate at time s in scenario ω.

However, unlike the nominal guarantee, at time $t < T$ the final inflation-linked guarantee is still *unknown*. We propose to *approximate* the final guarantee by using the inflation rates which are known at time t, combined with the expected inflation at time t for the period $[t + (1/12), T]$.

The *inflation-indexed barrier* at time t is then given by

$$\begin{aligned} L_t^{\mathrm{I}}(\omega) &= W_0 \left(\prod_{s=\frac{1}{12}}^{t} \left(1 + i_s^{(m)}(\omega)\right) \right) \left(\prod_{s=t+\frac{1}{12}}^{T} \left(1 + i_s^{(m)}(\omega)\right) \right) Z_t(\omega) \\ &= W_0 \left(\prod_{s=\frac{1}{12}}^{t} \left(1 + i_s^{(m)}(\omega)\right) \right) \left(\prod_{s=t+\frac{1}{12}}^{T} \left(1 + i_s^{(m)}(\omega)\right) \right) \mathrm{e}^{-y_{t,T}(\omega)(T-t)}. \end{aligned} \tag{4}$$

In general the expected terminal wealth is higher for the inflation-linked barrier, but we also see an increase in the expected maximum shortfall (see table 3). This reflects the increased uncertainty related to the inflation-linked guarantee which also forces us to increase the exposure to more risky assets. With an inflation-linked guarantee the final guarantee is only known for certain at the end of the investment horizon. Relative to the nominal guarantee results of table 2, table 4 shows that the initial portfolio allocations for the inflation-linked guarantee are more focused on long-term bonds.

As in figure 3, figure 5 shows that there is a noticeable pattern of asymmetry in the final wealth outcomes for

Table 3. Expected terminal wealth and maximum shortfall for the inflation-linked guarantee.

	Expected terminal wealth	Expected maximum shortfall
Experiment 1	129.88	0.780
Experiment 2	122.81	13.60
Experiment 3	129.34	1.580
Experiment 4	129.54	1.563
Experiment 5	128.23	1.456

Table 4. Root node solution for the inflation-linked guarantee.

	1y	2y	3y	4y	5y	10y	30y	Stock
Exp 1	0	0	0	0	**0.77**	**0.23**	0	0
Exp 2	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.30**
Exp 3	0	0	0	0	**0.77**	**0.23**	0	0
Exp 4	0	0	0	0	**0.88**	**0.12**	0	0
Exp 5	0	0	0	0	**0.94**	**0.06**	0	0

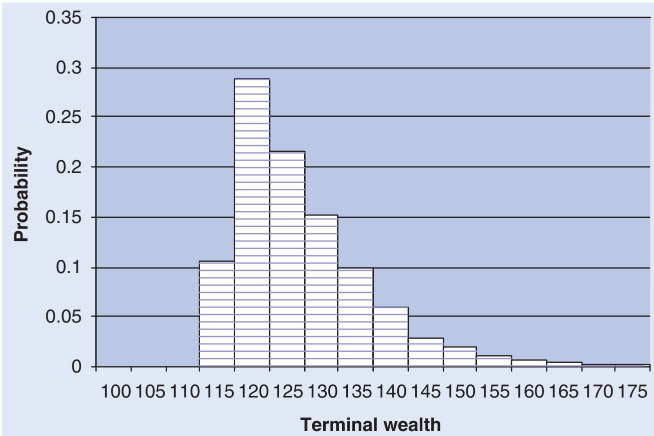

Figure 5. Terminal wealth distribution for Experiment 1 for the inflation-linked guarantee.

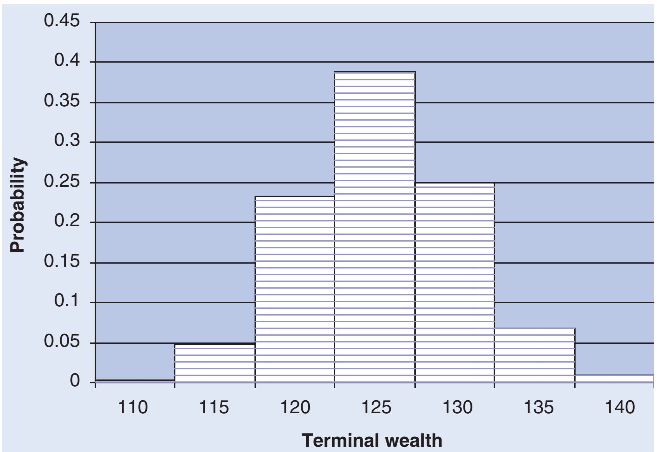

Figure 6. Terminal wealth distribution for Experiment 5 for the inflation-linked guarantee.

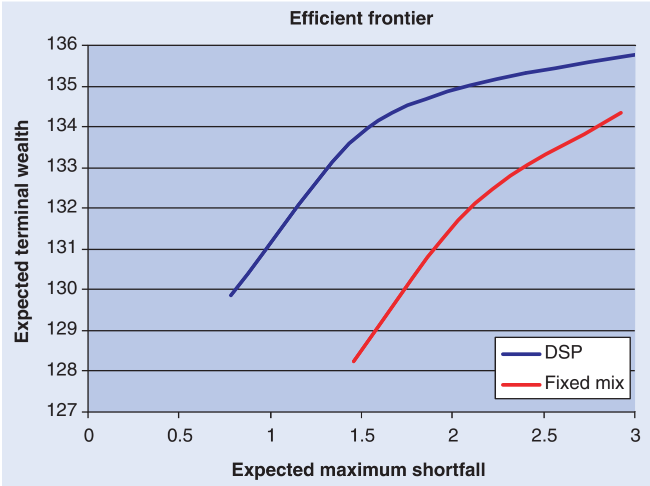

Figure 7. Efficient frontier for the inflation-linked guarantee.

the dynamic stochastic programme of Experiment 1. However the skewness is not so marked. This is due to the fact that for the inflation-linked guarantee problems inflation rates differ on each scenario and the final guarantee is scenario dependent which results in a different value of the barrier being pursued along each scenario. This symmetrizing effect is even more marked for the inflation-linked guarantee as shown in figure 6 (*cf*. figure 4).

Figure 7 shows for the inflation-linked guarantee problem similar out-performance of DSP relative to the optimal fixed-mix policy as in figure 2.

7. Conclusion

In this article we have compared the performance of two alternative versions of a dynamic portfolio management model for a DC pension scheme which accounts for the liabilities arising from a guaranteed fund return. The results show that a fixed-mixed rebalance policy can be used as a benchmark for the dynamic stochastic programming optimal solution with less complexity and lower computational cost. Whereas the risk-return trade-off for a fixed-mix portfolio rebalancing strategy is constant over the planning horizon, for the dynamic stochastic programming solutions portfolio allocations shift to less volatile assets as the excess over the liability barrier is reduced. The resulting guarantee shortfall risk for the easy-to-explain fixed-mix portfolio rebalancing strategy is therefore higher and its portfolio returns are lower than those of the dynamic optimal policy. On a percentage basis however these differences are sufficiently small to be able to use the easier-to-compute fixed-mix results as a conservative performance benchmark for both in-sample (model) and actual out-of-sample fund performance. For out-of-sample historical backtests of *optimal* dynamic stochastic programming solutions for these and related problems the reader is referred to Dempster *et al*. (2006, 2007b). Perhaps the easiest way to explain both benchmark and actual fund performances to DC pension scheme participants is to give probabilities of achieving (expected) guaranteed payouts and more. These are easily estimated *a priori* by scenario counts in both models considered in this paper for fund design and risk management. It is of course also possible to link final fund payouts to annuity costs and substitution rates with the corresponding probability estimates.

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