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A FRAMEWORK TO MEASURE
INTEGRATED RISK

E A Medova & R G Smith

WP 09/2003

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Dr E A Medova
Centre for Financial Research
Judge Institute of Management
University of Cambridge
Tel: +44 (0) 1223 339593
Fax: +44 (0) 1223 339652
Email: eam28@cam.ac.uk

R G Smith
Centre for Financial Research
Judge Institute of Management
University of Cambridge
Tel: +44 (0) 1223 339651
Fax: +44 (0) 1223 339652
Email: rgs22@cam.ac.uk

Please address enquiries about the series to:

Research Support Manager
Judge Institute of Management
Trumpington Street
Cambridge CB2 1AG, UK
Tel: 01223 339611 Fax: 01223 339701
E-mail: research-support@jims.cam.ac.uk

A Framework to Measure Integrated Risk

E A Medova

e-mail: eam28@cam.ac.uk

R G Smith

e-mail: rgs22@cam.ac.uk

Centre for Financial Research

Judge Institute of Management

University of Cambridge

Cambridge CB2 1AG

web: www-cfr.jims.cam.ac.uk

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Abstract: Risk management has often consisted of managing different types of risk separately as a consequence of the traditional internal bank organisation by asset class groups. However, the limitations of this segregated approach have increasingly become recognised, and many firms are now looking to use enterprise-wide risk management systems that integrate different types of risk into one framework. A framework underlying several models that measure the credit risk of a portfolio is extended in this paper to allow the integration of market risk and credit risk. Using a Monte Carlo methodology, the proposed integrated framework calculates a distribution for the value of a portfolio at a series of future time horizons. To reduce some of the data deficiencies that continue to be prevalent, particularly with the credit quality of firms, the proposed implementation of the framework uses market data where possible. Default probabilities are calculated using a structural model where some of the key parameters are estimated from equity data. As an illustration, the model developed in this paper is applied to a foreign exchange forward where there is a significant probability of default by the counterparty.

Keywords: risk measurement, market risk, credit risk, pre-settlement risk, integrated risk, structural models.

1. Introduction

Banks have traditionally organised themselves by asset class groups for money markets, bonds, loans, foreign exchange, equity, commodities, and so on. Accordingly, risk management in firms has often consisted of managing different types of risk separately. The academic literature has reflected this, with models concentrating on measuring only one type of risk. However, the limitations of this approach have increasingly become recognised, and firms are now looking to use enterprise-wide risk management systems that integrate many types of risk into one framework.

The almost universal methodology for the measurement of market risk has become the Value at Risk (VaR) technique (see Jorion 2000 for a review of VaR concept and methods). VaR summarises the maximum unexpected loss from adverse market movements over a given holding period¹ within a given confidence level under normal market conditions. This gives a single statistic able to measure the effects of many types of market risk factors, including the four major types: interest rate, equity, foreign exchange and commodity risk.

From 1997, a number of credit portfolio models were developed (see e.g. Uhrig-Homburg, 2002, for a survey of single-name credit models). Credit portfolio models include CreditMetrics, a multi-state, microeconomic model developed by JP Morgan and documented by Gupton, Finger & Bhatia (1997), CreditRisk+, a two-state model developed by Credit Suisse Financial Products and described in Wilde (1997), and CreditPortfolioView, a multi-state econometric model developed by Wilson (1997a,b) and then implemented by McKinsey & Company. All these models estimate the probability distribution for the value of a static portfolio at a future horizon date, which is often taken to be a year in the future. Various statistics of this distribution are then used for risk management purposes. Although the above credit portfolio models have different sets of fundamental assumptions, Koyluoglu & Hickman (1998) observed that all the models fit a generalised underlying framework.

One of the major limitations of credit portfolio models is that they only allow deterministic exposures, i.e. the exposures are not allowed to vary with market or macroeconomic variables. This is not a major problem when investigating the risk of credit instruments, for

¹ Market risk is often measured over short holding periods; the Basel Accord sets a holding period of ten days.

which this set of models was predominately designed, as exposures are reasonably constant. However, it is difficult to investigate the credit risk of market instruments, which have volatile exposures, using these models; allowing the exposure of instruments to be stochastic will be one of the major benefits of an integrated model.

The rest of this paper is structured as follows. In Section 2, a framework that allows the measurement of integrated risk is proposed. The measurement of the risk of a foreign exchange forward is investigated in Section 3. First, the market risk and the pre-settlement credit risk of the forward are calculated separately. When investigating credit risk, a structural model that estimates default probabilities from market data is proposed, and this model plays a crucial rôle when the risk of the forward is calculated using the integrated framework in the next part of Section 3. The section finishes with an example of how an integrated risk calculation can model wrong-way credit exposure. Section 4 concludes.

2. Framework for an Integrated Model

An integrated model should calculate the distribution for the future value of a portfolio, as this is the emphasis in both market and credit risk modelling. However, one major difference between the measurement of market and credit risk is the time horizon that is used: market risk is usually measured over a time horizon of a small number of days, while a longer time horizon, such as a year, is used to measure credit risk. Therefore, any model that incorporates both market and credit risk cannot have a single time horizon. Instead, the model would need to calculate the value of a portfolio at a series of time horizons, both short-term where market risk is the dominant risk component, and longer-term, where credit risk would become more important.

Rather than simply extend a widely-used credit portfolio model to allow stochastic exposures, the proposed integrated framework uses the three basic components of credit portfolio models (Koyluoglu & Hickman, 1998). Therefore, it consists of the following three steps:

1. *Generate a 'state of the world'*: market variables are projected one period forward. Also, macroeconomic variables may be simulated if this is required. The simulations can be done by one of the simulation methods that are commonly used in market risk modelling.
2. *Calculate the conditional distribution of the value of the portfolio*: the mark-to-market value of each instrument in the portfolio is calculated and default probabilities are estimated using the projected market and macroeconomic variables. The exposure to each obligor is calculated using the mark-to-market value of the instruments, although a netting agreement may be applicable if a firm has more than one contract with a counterparty. Conditional on the projected state of the world, obligors are assumed to be independent. Hence, correlations between credit events are assumed to be entirely driven by correlations between market movements or changes in the macroeconomy. As a result, structural models that use market or macroeconomic variables to determine the credit risk of single obligors can be used to estimate the default probabilities of the obligors.

It should be noted that a reduced-form model could also be used in this integrated framework to model the default event. As an example, the yield spread on a firm's debt could be modelled in step 1, and then used to calibrate a reduced-form model, which in turn is used to model the default event. However, as is pointed out in the original paper on reduced-form models, Jarrow & Turnbull (1995), this set of models was developed to price derivative securities involving credit risk, while Chen (2003) points out that structural models tend to be used for default prediction. For this reason, the use of single-name structural models rather than reduced-form models to estimate default probabilities and model defaults will be explored further here.

The market variables are then projected a further period ahead, and step 2 is repeated to calculate the value of the portfolio at the second time horizon. This process is repeated until the market variables have been projected to the furthest time horizon of interest to a risk manager. This describes one simulation.

3. *Calculate the unconditional distribution of the future value of the portfolio*: A large number of simulations takes place and aggregation takes place over these simulations.

Once this has been done, a distribution for the value of the portfolio at every period has been produced. The risk manager can then take statistics of these distributions that are of particular interest.

The integrated framework is dependent on two key components: an accurate projection of market and macroeconomic variables, and a model that determines default probabilities for single obligors using market and macroeconomic data. The first component is crucial to market risk modelling and much research has taken place into this already. However, there has been less research into the use of structural models to estimate default probabilities², although one recent paper in this area is Leland (2002). A structural model for single firms is outlined in Section 3.2 that extends a method developed by Finger *et al.* (2002) to estimate default probabilities using market data.

The following diagram illustrates the relationship between risk factors involved in the proposed framework for an integrated model.

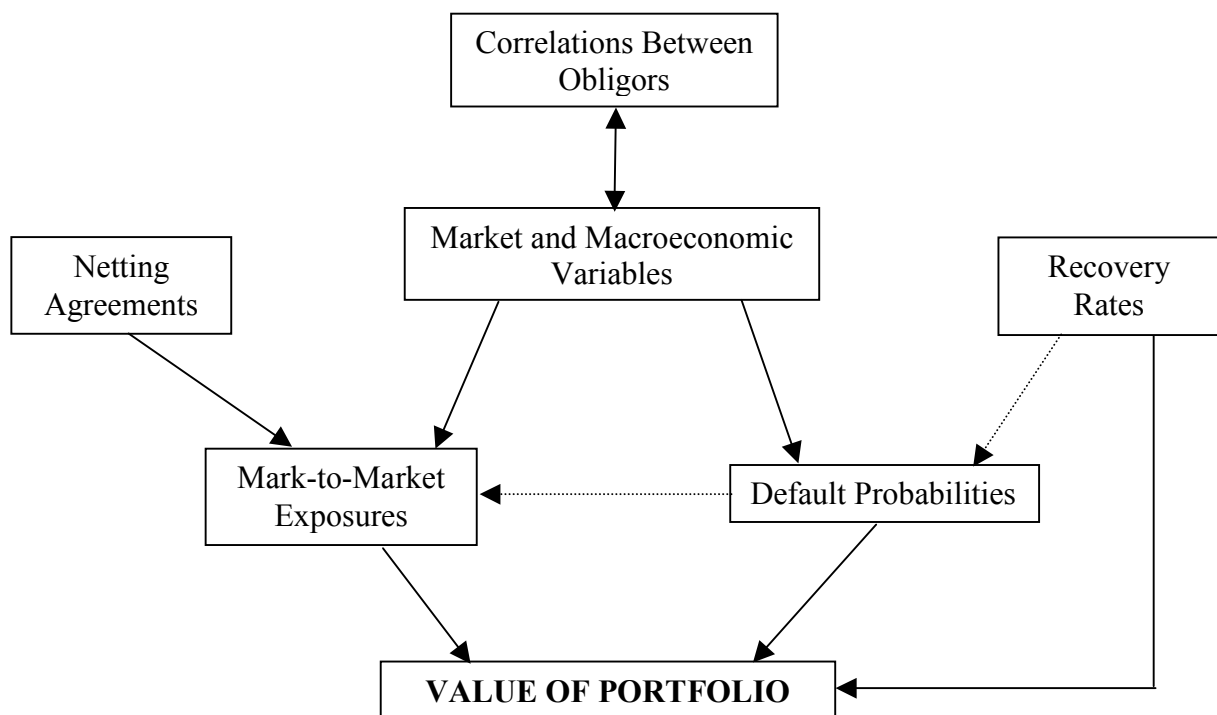


Figure 2.1: Relationship Between Risk Factors

² The majority of research into structural models has focused on estimating yield spreads.

In the above framework, the measurement of market risk is linked to mark-to-market exposures, whereas the estimation of default probabilities is principal to the evaluation of credit risk.

Due to the short time horizons involved in market risk modelling, a problem that rarely has to be considered is the expiry of an instrument. However, in an integrated framework, expiry cannot be ignored since the overall risk of the instruments is measured over longer time horizons. To do this, a cash balance is introduced. All cash flows, including debt payments and from the expiry of derivatives, will be paid into the cash balance. By investing the cash balance in liquid, virtually default-free securities, such as US Treasurys, it can be assumed that the cash balance earns interest at the risk-free short rate.

Market risk and credit portfolio models usually measure the risk of static portfolios. However, an integrated framework measures the risk of all instruments over a range of time horizons, including the risk of liquid assets over the long time horizons that are normally used to measure the risk of less volatile credit instruments. This is problematic for heavily traded instruments which may only be held for a short period of time. One approach to solving this problem is that of a dynamic trading strategy, which makes use of the multi-step feature of the integrated framework. Under this approach, an instrument is assumed to be held until a condition is met, e.g. the instrument is sold at a certain time, for instance in the case of a hedge to a derivative, if its value changes by a certain amount, or a more complicated condition.

There is evidence that the exposure of some instruments depends upon the credit quality of the obligor. A loan commitment is composed of a drawn portion, which is the amount currently borrowed, and an undrawn portion. Therefore, loan commitments give an obligor the option of changing the size of the loan. Asarnow & Marker (1995) showed that the size of the drawn portion is closely related to the credit quality of the obligor. This is intuitive: if the credit quality of an obligor deteriorates, it is likely to draw down additional funds, while if its prospects improve, it is unlikely to need extra borrowings, and may repay some of its existing loan. The multi-step nature of the proposed framework allows exposure to depend on the credit quality of an obligor: the exposure could be increased or decreased as the calculated default probability of an obligor changes.

3. Measuring the Risk of a Foreign Exchange Forward

This section illustrates the integrated risk framework described in the previous section by measuring the risk of a British pound / US dollar FX forward. The delivery time – three years forward – is chosen to be far enough into the future for default by the counterparty to become a significant factor. As a result, there are both significant market risk and credit risk components. We start by calculating the market risk and the credit risk of the instrument separately.

3.1 Market Risk

At the delivery time T , a US bank will deliver $\$K$ to a counterparty and receive $\pounds 1\text{m}$. Thus, the value of the FX forward in US dollars at time t is given by

$$Value_t = \left(X_t \times \pounds 1,000,000 \times \exp[-R_{[t,T]}^{GBP}(T-t)] \right) - \left(\$K \times \exp[-R_{[t,T]}^{USD}(T-t)] \right) \quad (1)$$

where $R_{[t,T]}^{GBP}$ and $R_{[t,T]}^{USD}$ are the time- t British pound and US dollar continuously-compounded risk-free rates respectively for the period $[t, T]$, and X_t is the time- t exchange rate in terms of dollars per pounds. The strike rate K is chosen so that the value of the contract at time 0 is zero. The exchange rate will be modelled using a driftless geometric Brownian motion,

$$\frac{dX_t}{X_t} = \sigma_X dW_t^X, \quad (2)$$

and the interest rates will be modelled using the single factor model proposed by Cox, Ingersoll & Ross (1985). In this model, the term structure of interest rates in both countries can each be represented by a single sufficient statistic, the short interest rate in each country r^{GBP} and r^{USD} , which are assumed to follow

$$dr_t^i = \kappa^i (\beta^i - r_t^i) dt + \sigma^i \sqrt{r_t^i} dW_t^i, \quad (3)$$

where $i \in \{USD, GBP\}$. The time- t price $P^i(r^i, t, T)$ of a zero-coupon bond in currency i that matures at time T satisfies

$$\frac{1}{2} (\sigma^i)^2 r^i \frac{\partial^2 P^i}{\partial (r^i)^2} + \kappa^i (\beta^i - r^i) \frac{\partial P^i}{\partial r^i} + \frac{\partial P^i}{\partial t} - \lambda^i r^i \frac{\partial P^i}{\partial r^i} - r^i P^i = 0 \quad (4)$$

with the boundary condition

$$P^i(r^i, T, T) = 1. \quad (5)$$

The bond price $P^i(r^i, t, T)$ was shown to have a closed-form expression in Cox, Ingersoll & Ross. The required interest rate is calculated using

$$R_{[t,T]}^i = -\frac{\ln[P^i(r^i, t, T)]}{T-t} \quad (6)$$

The convention that a year consists of 360 days will be used. The exchange rate and interest rates are projected ahead according to the above processes using time steps of one day, and the mark-to-market value of the forward at a future time t is calculated using the projected market variables in (1).

The parameters of the market variables are as follows:

Description of Parameter		Symbol	Value
Delivery time of contract		T	3 years
Strike rate		K	\$1,622,404
Initial exchange rate		X_0	£1 = \$1.65
Annual exchange rate volatility		σ_X	8%
British interest rate	Initial short rate	r_0^{GBP}	5%
	Volatility	σ^{GBP}	$0.015/\sqrt{0.06}$
	Long-term mean	β^{GBP}	6%
	Speed of mean reversion	κ^{GBP}	0.25
	“Market” risk parameter	λ^{GBP}	0
US interest rate	Initial short rate	r_0^{USD}	4%
	Volatility	σ^{USD}	$0.02/\sqrt{0.065}$
	Long-term mean	β^{USD}	6.5%
	Speed of mean reversion	κ^{USD}	0.25
	“Market” risk parameter	λ^{USD}	0

Table 3.1.1: Parameters of FX Forward

The Brownian motions driving the three processes will be allowed to be correlated:

	Exchange Rate	British Interest Rate	US Interest Rate
Exchange Rate	1	-0.6	-0.75
British Interest Rate	-0.6	1	0.9
US Interest Rate	-0.75	0.9	1

Table 3.1.2: Correlation Matrix

Although a distribution can be calculated for the value of the FX forward on every day up to a time horizon of three years, only three time horizons will be illustrated here: 14 days, one year and three years. Since it is being assumed that a year consists of 360 days, a 14-day time horizon is approximately equivalent to a time horizon of 10 trading days, the holding period used in the regulatory capital calculation of market risk. The distributions are calculated using 500,000 trials.

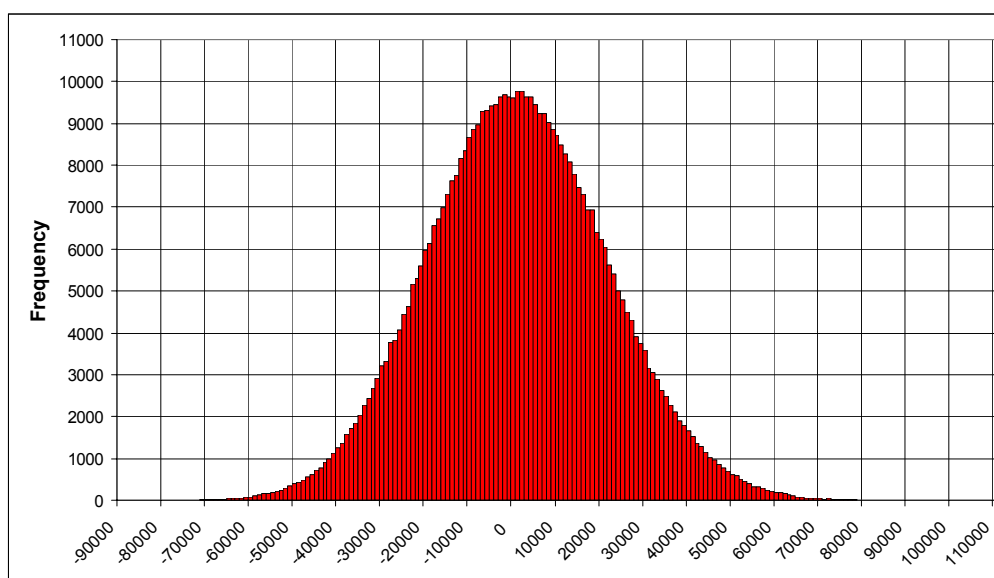


Figure 3.1.1: Market Risk Over a 14 Day Time Horizon

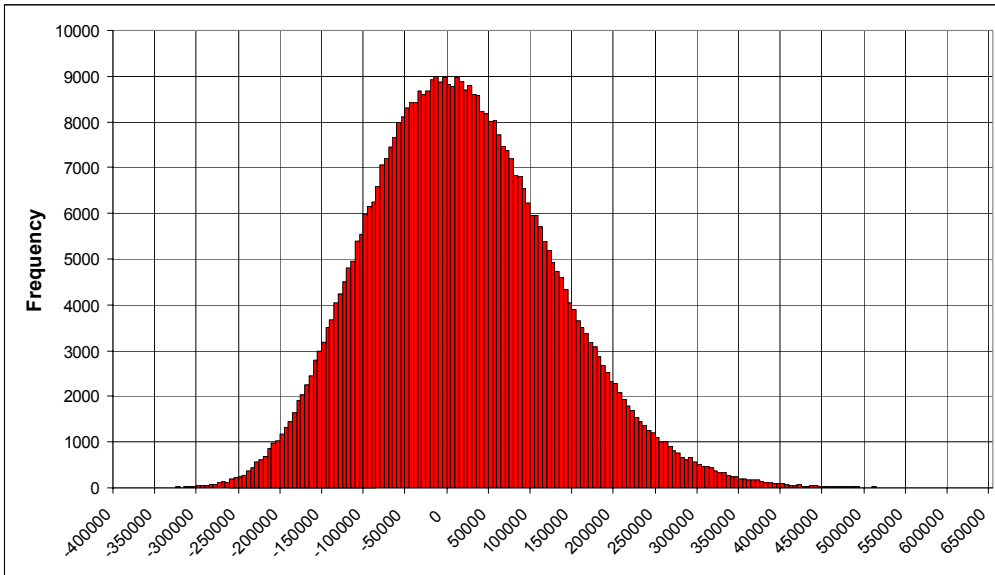


Figure 3.1.2: Market Risk Over a One Year Time Horizon

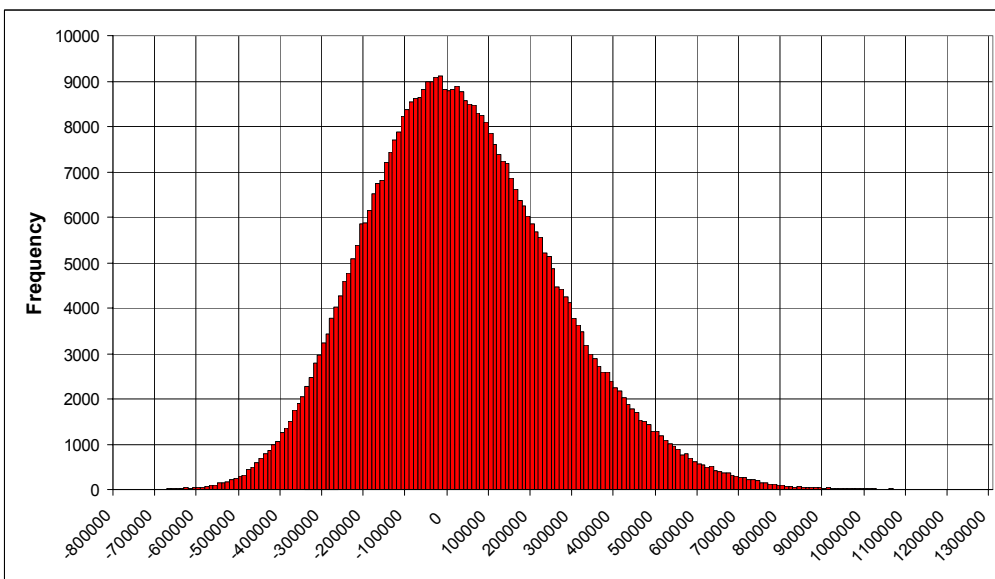


Figure 3.1.3: Market Risk Over a Three Year Time Horizon

Various summary statistics of the three distributions are given in the following table:

Time Horizon	14 Days	One Year	Three Years
Mean	\$607.82	\$13,674.80	\$27,664.51
Standard Deviation	\$20,510.61	\$112,267.22	\$230,155.08
0.1 th Percentile	-\$60,625.27	-\$276,000.77	-\$574,391.88
0.5 th Percentile	-\$50,708.91	-\$235,449.48	-\$483,313.32
1 st Percentile	-\$45,782.55	-\$216,320.06	-\$442,516.04
5 th Percentile	-\$32,577.30	-\$158,751.81	-\$321,444.39

Table 3.1.3: Summary Statistics

Note that the mean future value of the contract increases as the time horizon increases. This is a result of the values of the market variables used in this example rather than a feature inherent in FX forwards. As the forward approaches the delivery time, its value to the US bank is approximately equal to $(\text{£}1\text{m} \times X) - \K ; thus, if the exchange rate has the same value at the delivery time as at time 0, the contract would now have a positive value.

3.2 Credit Risk

As is described in Picoult (2002), counterparty risk, i.e. the risk that the counterparty to a market instrument could default in its obligations, occurs in two forms: settlement risk and pre-settlement risk. Settlement risk is the risk that a firm delivers on its obligations at settlement, while the counterparty fails in its obligations, i.e. a ‘one-sided’ trade. Pre-settlement risk is the risk that the counterparty defaults before the settlement of the contract. In this case, a firm makes a credit loss if the contract has a positive value to the firm. This subsection investigates the measurement of pre-settlement risk.

Counterparty credit models were developed to estimate the risk that future changes in market prices could increase credit exposure. These models make statistical estimates of the maximum probable value that the derivative can reach over a specified time horizon to a given confidence level. The Federal Reserve System (2002) in its *Trading and Capital-Markets Activities Manual* suggests a multi-step simulation method. Using this technique, market variables are projected one day ahead, and the exposure of a transaction is calculated using these projected values. The market variables are then projected a further day ahead,

and again, the exposure of the contract is calculated. This is continued until the contract has matured. This represents one scenario. The process is repeated a number of times so that a distribution for the value of the contract on each day in the future can be obtained. From each distribution, the 95% value (or an alternative confidence level) is taken, and this represents the maximum value of the contract on each day. The maximum exposure is then simply calculated from

$$\text{maximum exposure} = \max(\text{maximum value of contract}, 0) \quad (7)$$

since a bank only makes a credit loss if the instrument has a positive value.

The method described above was implemented on the FX forward described in Section 3.1 using 50,000 simulations. The parameters of the underlying risk factors are the same as those given in Table 3.1.1. The graph below shows the maximum exposure (to a 95% confidence level) on each day over the remaining life of the contract:

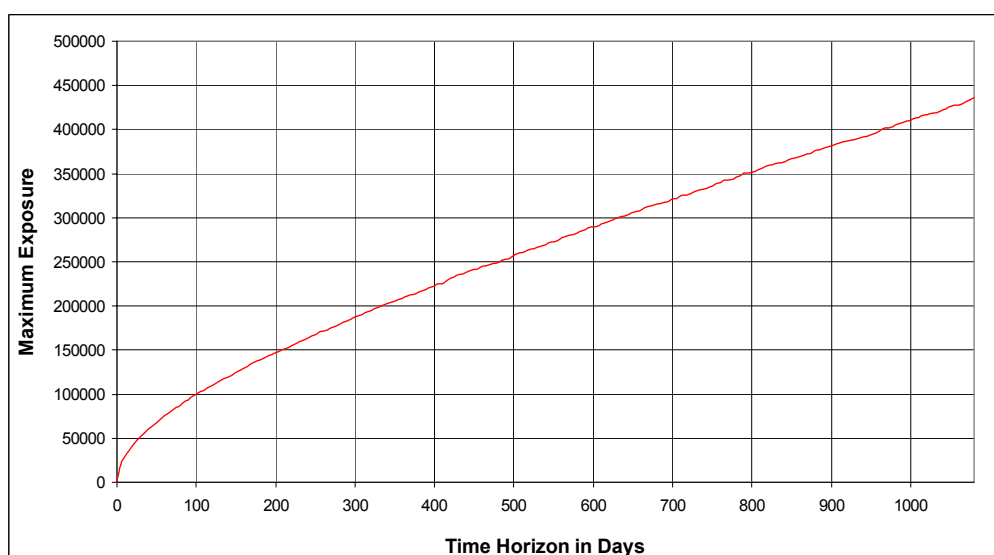


Figure 3.2.1: Maximum Exposure (to a 95% Confidence Level) On Each Day

The *Trading and Capital-Market Activities Manual* suggests that the pre-settlement exposure should be taken to be the peak exposure within a time interval. However, Picoult (2002) suggests that the mean of the maximum exposure over time could be used, i.e. the pre-settlement exposure over an interval $[0, T]$ is calculated using

$$PSE_{[0,T]} = \frac{1}{T} \sum_{t=1}^T (\text{Maximum Exposure on day } t). \quad (8)$$

The following table shows the pre-settlement exposure of the FX forward over the same three time horizons used in Section 3.1; both the ‘peak’ and ‘average’ exposure measures for the pre-settlement exposure are given.

Time Horizon	‘Peak’ Exposure	‘Average’ Exposure
14 Days	\$34,763.37	\$22,966.49
1 Year	\$208,759.25	\$131,539.29
3 Years	\$435,769.78	\$260,648.00

Table 3.2.1: Pre-Settlement Exposures Over Different Time Horizons

However, counterparty credit models only calculate potential exposures to counterparties, and do not model probabilities of default. As was mentioned in the introduction, the emphasis in credit modelling has moved towards calculating a distribution for the value of the portfolio (included losses due to defaults) at a future time horizon. One way this can be done is to fix the exposure to a counterparty at the pre-settlement exposure of a transaction, and then use either a credit portfolio model or a single-name credit risk model, such as the structural model described below, to calculate the credit risk of the contract; indeed, this is the method suggested by Gupton, Finger & Bhatia (1997) to handle market instruments in CreditMetrics. However, the fundamental problem with this technique is that the two risk components of a default event, the time and the magnitude of default, are assumed to be independent: regardless of the time of default, the magnitude of default is the same.

Following many papers, such as Leland & Toft (1996), it will be assumed in the model proposed here that the asset value of a firm V evolves as the following lognormal process

$$\frac{dV_t}{V_t} = (\mu_t - \delta)dt + \sigma dW_t, \quad (9)$$

where μ is the expected total rate of return on assets V , δ is the fractional payout rate on assets to both shareholders and liabilities-holders, and W is a Brownian motion under the real-world measure. Note that this is a special case of the process suggested by Merton (1974), with the payout rate being proportional to the value of the firm. As is pointed out in Leland (2002), the expected rate of return on assets can be split into two components,

$$\mu_t = r_t + \gamma, \quad (10)$$

where r is the short risk-free interest rate in the country where the firm is based and γ is the asset risk premium. The asset risk premium, the fractional payout rate and the asset volatility

are assumed to be constant over time. However, we will drop the assumption often made that the risk-free interest rate is constant over time, and allow r to be stochastic³ and follow the process described by (3).

The principal value of the firm's debt D will be assumed to be constant over time. This assumption is made in many papers. Some papers, such as Merton (1974) and Black & Cox (1976), assume that a firm has outstanding only equity and a single bond issue with a promised final payment of D upon maturity. Other papers, such as Leland & Toft (1996), assume that a firm continuously sells a constant principal amount of new debt so that the total principal value of all outstanding debt is D . Also, debt will be assumed to be non-callable⁴.

Both the asset value and the level of debt can be taken to be on a per share basis. As will be seen later, a major advantage of making calculations on a per share basis is that the share price of the firm can be used as an input. Henceforth, V will be taken to be the asset value per share of the firm, where the asset value per share is defined simply as the asset value of the firm divided by the total number of shares, and D will be taken to be the principal value of the debt on a per share basis.

Default is triggered when the firm-value process first reaches a default barrier, which is defined exogenously. Thus, default can occur before the maturity of a debt and occurs on all the debt contracts of a firm simultaneously. As a result, equity can be viewed as a down-and-out barrier option on the firm's assets.

Let $L \in [0, 1]$ be the firm-wide recovery rate averaged over different debts and different seniorities. Then, if default occurs, holders of the firm's debt receive a total of LD per share. It will be assumed that the firm loses a proportion of its asset value when default occurs, and that the default barrier is given by

$$V_B = [L + \alpha(1 - L)]D, \quad (11)$$

where $\alpha \in [0, 1]$ is a default cost factor, so that the default barrier lies between the amount recovered by bond-holders (LD) and the principal value of the firm's debt (D). This

³ Other papers that allow stochastic interest rates include Kim, Ramaswamy & Sundaresan (1993) and Longstaff & Schwartz (1995).

⁴ Papers that have allowed debt to be callable or convertible, and for it to be dynamically restructured, include Leland (1998), Goldstein, Ju & Leland (2001) and Collin-Dufresne & Goldstein (2001).

formulation allows all values for the recovery rate between 0% and 100%, while ensuring that default cannot occur if the value of a firm's assets is greater than the firm's debt⁵. A consequence of this expression is that a firm with a low recovery rate loses a greater proportion of its assets upon default than a firm with a high recovery rate, *ceteris paribus*. Also note that the assumption of a constant principal value for the firm's debt implies that the default barrier is constant over time, as in Longstaff & Schwartz (1995).

Suppose that the share price \mathbf{S} satisfies the following stochastic differential equation

$$d\mathbf{S}_t = \mu_s \mathbf{S}_t dt + \sigma_s \mathbf{S}_t d\mathbf{W}_t \quad (12)$$

where μ_s and σ_s may be dependent on time. By assuming that the value of a firm's assets is a function of the value of a firm's equity and time

$$V_t = V(\mathbf{S}_t, t), \quad (13)$$

Itô's Lemma can be applied to (12) to give

$$dV = \left(\frac{\partial V}{\partial t} + \mu_s \mathbf{S} \frac{\partial V}{\partial \mathbf{S}} + \frac{1}{2} \sigma_s^2 \mathbf{S}^2 \frac{\partial^2 V}{\partial \mathbf{S}^2} \right) dt + \sigma_s \mathbf{S} \frac{\partial V}{\partial \mathbf{S}} d\mathbf{W}. \quad (14)$$

Comparing (9) and (14) and re-arranging terms gives the following general formula linking the asset volatility and the equity volatility:

$$\sigma = \sigma_s \frac{\mathbf{S}}{V} \frac{\partial V}{\partial \mathbf{S}}. \quad (15)$$

Crosbie & Bohn (2002) demonstrated how to estimate the value of a firm's assets from the equity value of the firm under the assumptions made by Merton (1974). Nickell, Perraudin & Varotto (2001) considered the estimation of the value of a firm's assets in the case that default is assumed to have occurred when the value of the assets hits some low level relative to the level of debt, similar to the model described above. However, to solve the resulting expressions requires a computationally demanding maximum likelihood estimation procedure. An alternative approach was taken by Finger *et al.* (2002), who considered the same problem as Nickell, Perraudin & Varotto, but produced a simple, approximate expression for the value of a firm's assets. Their method is extended here to allow positive default costs.

⁵ Some papers assume that firms lose a fixed proportion π of its asset value upon default. However, this method does not allow recovery rates above $1-\pi$ if it is assumed that default can only occur when the value of a firm's assets is below the firm's debt.

If V is close to the default barrier V_B it is assumed that the share price S is close to zero,

$$V|_{S=0} = V_B = [L + \alpha(1-L)]D, \quad (16)$$

where L is a realisation of the firm-wide recovery rate. Far from the default barrier, it is assumed that

$$\frac{S}{V} \uparrow 1 \quad \text{as} \quad S \rightarrow \infty. \quad (17)$$

The simplest expression for V that satisfies the boundary conditions (16) and (17) is

$$V = S + [L + \alpha(1-L)]D. \quad (18)$$

Thus, at time $t = 0$, the initial asset value of the firm, V_0 , can be estimated from the firm's share price, S_0 , using

$$V_0 = S_0 + [L + \alpha(1-L)]D. \quad (19)$$

Using (15) and (19) gives the following expression that estimates the asset volatility:

$$\sigma = \sigma_S^{(0)} \frac{S_0}{S_0 + [L + \alpha(1-L)]D}, \quad (20)$$

where $\sigma_S^{(0)}$ is the equity volatility at time 0. Since it is being assumed that asset volatility is constant over time, the asset volatility is fixed at the value estimated at time 0. This implies that equity volatility varies over time according to

$$\sigma_S^{(t)} = \sigma \frac{S_t + [L + \alpha(1-L)]D}{S_t}, \quad (21)$$

so that equity volatility is negatively correlated to a firm's share price.

It should be noted that in the case of constant interest rates, the conditional default probability has a closed-form expression. This is derived in Appendix A. However, no such expression exists if interest rates follow the Cox, Ingersoll & Ross model. Instead, the default probability is found by simulating V directly according to (9) and (10).

Using this model, a default probability curve is generated for a counterparty that is assumed to be based in the US. Thus, in this case, the interest rate in (10) is the US risk-free short rate r^{USD} , which follows the process given by (3) and whose parameters are given in Table 3.1.1.

The firm-wide average recovery rate, L , will be distributed as a beta random variable⁶ with a mean of 56.7% and a standard deviation of 29.3%; these are the values for the firm-wide average recovery rates that were found by Hamilton & Carty (1999). Other parameters of the counterparty are given in the table below:

Description of Parameter	Symbol	Value
Initial stock price	S_0	\$30
Debt per share	D	\$15
Initial equity volatility	$\sigma_S^{(0)}$	50%
Asset risk premium	γ	4%
Fractional payout rate	δ	6%
Default cost factor	α	25%

Table 3.2.2: Parameters of Counterparty

The value of the asset risk premium and the payout rate are those used by Leland (2002). A default cost factor of 25% implies that if the firm-wide average recovery rate is 56.7%, i.e. the mean found by Hamilton & Carty, the firm loses 16% of the value of its assets upon default. This is within the range of values that have been found from empirical studies, such as Andrade & Kaplan (1998), which have shown that default costs are approximately 10% to 20% of the firm's value. Also, the Brownian motions driving the firm-value process and the US interest rate will be assumed to have an instantaneous correlation of -0.1 , since most stocks exhibit a long-term correlation between their price and the short rate of between -0.05 and -0.15 .

The default probability curve of the counterparty is calculated using 500,000 simulations, and illustrated on the graph below:

⁶ The use of a beta distribution ensures that recovery rates are between 0% and 100%. There are some examples of recovery rates that are higher than 100%, e.g. TIE/Communications, Inc. had a firm-wide average recovery rate of 131.9%, but such examples are rare and will be excluded here.

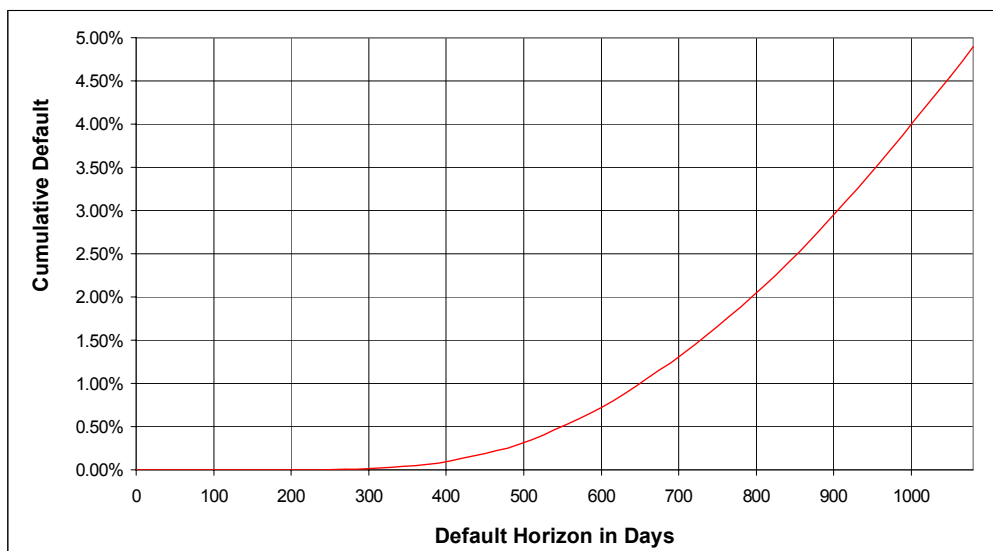


Figure 3.2.2: Default Probability Curve of the Counterparty

The default probabilities for the three time horizons used in Section 3.1 are:

Time Horizon	Defaults	Default Probability
14 Days	0	0.0000%
1 Year	244	0.0488%
3 Years	24,471	4.8942%

Table 3.2.3: Number of Defaults

The distribution of losses due to default by the counterparty is not particularly enlightening as the exposure was assumed to be fixed throughout the lifetime of the contract. Instead, the expected credit loss will be calculated for each of the three time horizons used previously. Since the exposure of the forward is assumed to be fixed at the pre-settlement exposure, the expected loss of each contract is given by

$$\text{pre-settlement exposure} \times \text{probability of default} \times \text{loss given default.}$$

The loss given default is usually taken to be 100% in the case of market instruments, and this assumption is made here⁷.

⁷ Since the firm-wide recovery rate L is an average over different debts and seniorities, it is possible for some debts to have a recovery rate of 0% (or equivalently, a loss given default of 100%) and L to be strictly positive.

For each of the time horizons, both the ‘peak’ and ‘average’ measures of the pre-settlement exposure calculated earlier are used:

Time Horizon	Probability of Default	Exposure	Expected Credit Loss
14 Days	0.0000%	\$34,763.37	\$0.00
		\$22,966.49	\$0.00
1 Year	0.0488%	\$208,759.25	\$101.87
		\$131,539.29	\$64.19
3 Years	4.8942%	\$435,769.78	\$21,327.44
		\$260,648.00	\$12,756.63

Table 3.2.4: Expected Credit Loss Over Three Time Horizons

3.3 Integrated Risk

Here, the integrated risk of the FX forward is measured when the counterparty is a firm that may default prior to the contract maturity. As in the previous section, the counterparty will be assumed to be based in the US so that the value of the counterparty’s assets V satisfies the stochastic differential equation

$$\frac{dV_t}{V_t} = (r_t^{USD} + \gamma - \delta)dt + \sigma dW_t, \quad (22)$$

where r_t^{USD} is the time- t US short interest rate being simulated as part of the calculation of the future mark-to-market value of the FX forward. Therefore, there are three variables (r^{USD} , r^{GBP} , X) that are used to determine the future mark-to-market value of the forward, while r^{USD} is also used in the simulation of the firm-value process for the counterparty (V). Unlike the previous two subsections, the exchange rate, interest rates and V are all projected forwards together in the integrated risk calculation.

If the value of the counterparty’s assets V is above the default barrier V_B , it is assumed that the counterparty has not defaulted, and the value of the forward contract is equal to the mark-to-market value given in (1). However, if V reaches V_B , it is assumed that the counterparty has defaulted. In this case, if the mark-to-market value of the contract is negative, the instrument has a positive value to the counterparty, and thus, a bankruptcy court would want the contract to perform to help the counterparty pay its debt. Therefore, the value of the

contract to the bank is equal to its mark-to-market value. On the other hand, if the mark-to-market value is positive, the bank would not receive this payment from the counterparty (as such a payment would harm the counterparty's ability to pay its debt) and thus the value of the contract to the bank is zero.

The initial values and parameters of the market variables and the counterparty are taken to be the same as those in Section 3.1 and Section 3.2 respectively, and the distributions are again calculated using 500,000 trials. During the simulations, the number of trials that result in a default before each time horizon are counted. Also, a count is kept of the number of trials where default occurs when the forward has a positive mark-to-market value to the bank, as it is these defaults that result in a credit loss.

The following two graphs illustrate the distribution of the future value of the FX forward at two time horizons: one year and three years into the future. The 14-day time horizon is not illustrated here as no defaults had occurred in the simulations by that time. Therefore, the results for this horizon are exactly the same as in Section 3.1, within sampling error.

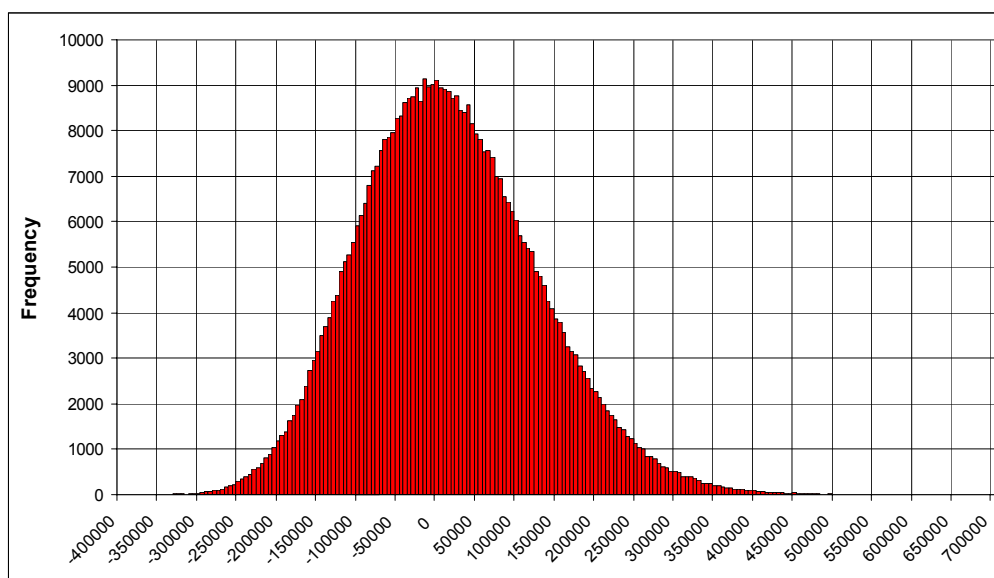


Figure 3.3.1: Integrated Risk Over a One Year Time Horizon

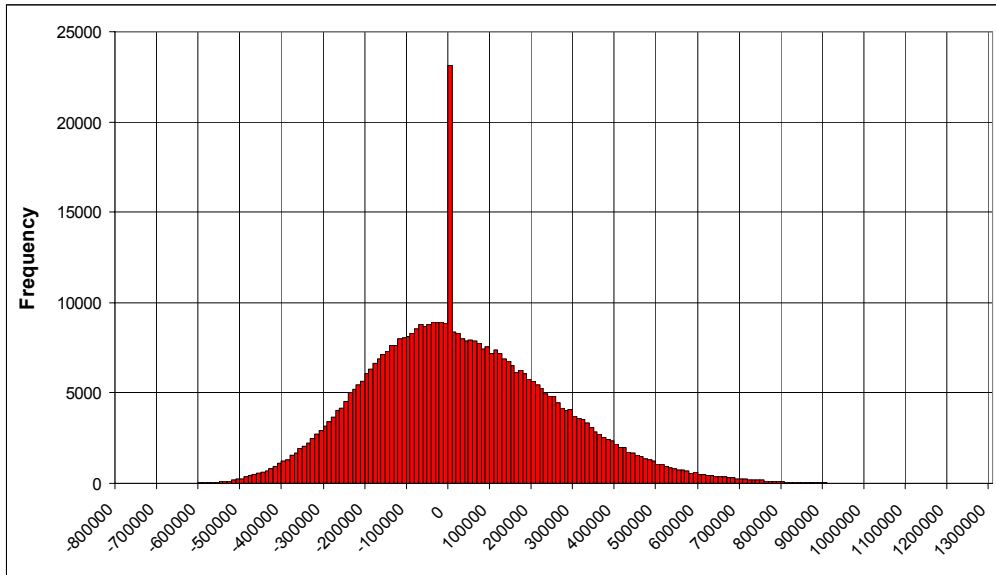


Figure 3.3.2: Integrated Risk Over a Three Year Time Horizon

The following table gives various summary statistics of the above two graphs, including the number of simulations that ended in a default before each time horizon:

Time Horizon	One Year	Three Years
Number of Defaults	284	24,883
Number of Defaults when MtM Value of Forward is Positive	176 (62% of all defaults)	14,346 (58% of all defaults)
Mean	\$13,791.67	\$22,476.98
Standard Deviation	\$112,197.57	\$224,705.55
0.1 th Percentile	-\$279,288.42	-\$558,151.17
0.5 th Percentile	-\$237,734.20	-\$476,322.53
1 st Percentile	-\$217,414.39	-\$435,770.37
5 th Percentile	-\$157,957.20	-\$318,928.40

Table 3.3.1: Summary Statistics

The number of simulated defaults in this integrated risk calculation are within sampling error of the number of defaults, as given in Table 3.2.3, when the default probability of the counterparty was calculated and exposure was not considered.

From Table 3.1.3, the expected future value of the contract in three years' time is \$27,664.51 if credit risk is ignored, while the expected future value of the forward is \$22,476.98 under an integrated calculation, a reduction of \$5,187.53. Note that this reduction is significantly less

than the expected credit loss calculated in Table 3.2.4, using either the peak or average measures of the pre-settlement exposure, demonstrating that the pre-settlement exposure is an extremely conservative measure of credit risk.

As described above, if a default occurs when the mark-to-market value of the contract is negative, the value of the transaction is equal to its mark-to-market value. As a result, the percentiles in the lower tail of the distribution as given in Tables 3.1.3 and 3.3.1 are very similar whether a market risk calculation or an integrated risk calculation is made; the differences in the percentiles produced by the two calculations are within sampling error. However, if a default occurs when the mark-to-market value of the forward is positive, the value of the contract falls to zero at default. This has two effects that can be seen in Figure 3.3.2. First, there is a significant spike at 0, with the size of the spike being fully explained by the 14,346 simulated defaults when the mark-to-market value of the forward was positive. Secondly, the frequencies when the future value of the forward is positive are noticeably lower under an integrated risk calculation than under a market risk calculation.

3.4 Wrong-Way Credit Exposure

As well as allowing a more accurate measurement of the risk of the contract, an integrated model makes it easier for variables determining the probability of default by the counterparty to be correlated with variables determining the mark-to-market value of the contract. In this section, we will allow the Brownian motion that drives the exchange rate and the Brownian motion driving the value of the counterparty's assets to be negatively correlated. Therefore, as the exchange rate goes up, and thus the value of the forward increases, the value of the counterparty's assets fall, increasing the default probability: this is known as *wrong-way credit exposure*. There have been examples of such transactions in recent years, including a famous case, reported in many sources such as Baumohl (1998), which involved Thai baht-related derivatives between JP Morgan and various Korean counterparties in 1998. Unfortunately for JP Morgan, the value of the counterparties' assets were also linked to the baht, so that when the baht collapsed, JP Morgan had a \$500m exposure to counterparties whose assets had significantly fallen in value.

The parameters of the processes describing the market variables and the firm-value process are exactly the same as in the previous section, except that the Brownian motions that drive the exchange rate and the value of the counterparty's assets are now assumed to have a correlation of -0.5 .

The following graph illustrates the distribution of the value of the FX forward three years into the future:

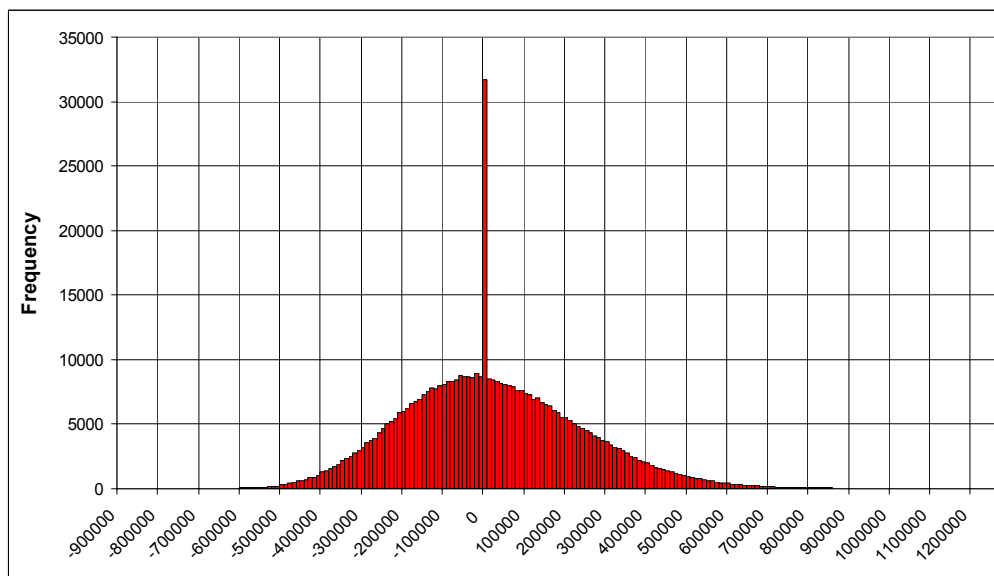


Figure 3.4.1: Integrated Risk Over a Three Year Time Horizon When the Exchange Rate and the Value of the Counterparty's Assets are Negatively Correlated

The following table gives some summary statistics of this distribution:

Time Horizon	Three Years
Number of Defaults	24,473
Number of Defaults when MtM Value of Forward is Positive	23,117 (94% of all defaults)
Mean	\$14,249.06
Standard Deviation	\$216,897.24
0.1 th Percentile	-\$562,909.65
0.5 th Percentile	-\$477,303.78
1 st Percentile	-\$437,021.52
5 th Percentile	-\$321,056.72

Table 3.4.1: Summary Statistics

The above graph and table illustrate two particularly noticeable points. First, the spike in the histogram at 0 is larger when the value of the FX forward and the probability of default are positively correlated than in the previous section. Previously, as shown in Table 3.3.1, there were 14,346 simulated defaults when the mark-to-market value of the forward was positive. However, when wrong-way credit exposure is introduced into the calculation, the number of defaults that occurs when the mark-to-market value of the forward is positive increases substantially to 23,117 defaults, even though there were slightly fewer defaults in this simulation than in Section 3.3.

This leads to the second point, which is the expected value of the FX forward at the delivery time of the contract. If a higher proportion of defaults occur when the exposure to the counterparty is positive, the credit risk of the contract is higher and thus the expected future value of the contract decreases. This is illustrated in Tables 3.3.1 and 3.4.1: in the previous section, the expected future value of the forward was \$22,476.98, but fell dramatically to \$14,249.06 after the introduction of wrong-way credit exposure.

4. Conclusions

This paper gives a framework for a multi-step integrated model that measures both market and credit risk. This was done by extending the Koyluoglu & Hickman framework that is underlying the most widely implemented credit portfolio models. The integrated framework consisted of two key components: the modelling of changes in market variables to generate a state of the world, and a credit model, such as the structural model described in Section 3.2, that uses market data to determine the default probability of a single obligor, conditional on the state of the world. These two components form the first two parts of the integrated framework. Aggregation takes place by making many simulations of the market.

This framework was used to calculate the risk of a foreign exchange forward. It was seen that the expected future value of the contract fell when moving from a market risk calculation to an integrated risk calculation, but by a smaller amount than the expected credit loss as calculated by the pre-settlement exposure method. Further, it was seen that in this particular case, the lower tail of the distribution of future values for the contract, as used in the calculation of capital adequacy, was very similar under both market risk and integrated risk

calculations. Therefore, moving from a market risk calculation to an integrated risk calculation did not affect one of the tails, but did affect the expected future value. This illustrates the difficulties in estimating the risk of a contract by measuring market and credit risk separately and then attempting to combine them in some manner, and shows why an integrated risk calculation is crucial. The last section demonstrated that the framework proposed in this paper can model wrong-way credit exposure, as well as illustrating its dangers, including a dramatic fall in the expected future value of the contract.

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Appendix A

Assume that the short rate, r , is constant over time, so that the value of the firm's assets (on a per share basis) satisfies

$$\frac{dV_t}{V_t} = (r + \gamma - \delta)dt + \sigma dW_t. \quad (23)$$

As a result, for an initial value V_0 , the asset value per share at time t is given by

$$V_t = V_0 \exp \left[\sigma W_t + \left(r + \gamma - \delta - \frac{1}{2} \sigma^2 \right) t \right]. \quad (24)$$

Let L be a realisation of the firm-wide average recovery rate, so that the default barrier is given by

$$V_B = [L + \alpha(1-L)]D. \quad (25)$$

Then the probability that the firm survives until time t is

$$\bar{P}(t | V_0, D, \sigma, L, \gamma, \delta, \alpha) = \mathbb{P}(V_s > [L + \alpha(1-L)]D \quad \forall s < t) \quad (26)$$

Combining (24) and (26),

$$\bar{P} \left(t \mid \frac{D}{V_0}, \sigma, L, \gamma, \delta, \alpha \right) = \mathbb{P} \left(\sigma W_s + \left(r + \gamma - \delta - \frac{1}{2} \sigma^2 \right) s > \log \left(\frac{[L + \alpha(1-L)]D}{V_0} \right) \quad \forall s < t \right) \quad (27)$$

Using (19), the expression for the survival probability becomes

$$\bar{P} \left(t \mid \frac{D}{S_0}, \sigma, L, \gamma, \delta, \alpha \right) = \mathbb{P} \left(\sigma W_s + \left(r + \gamma - \delta - \frac{1}{2} \sigma^2 \right) s > -\log \left(1 + \frac{S_0}{[L + \alpha(1-L)]D} \right) \quad \forall s < t \right) \quad (28)$$

A result from probability (proved in Musiela & Rutkowski, 1998, for instance) is

$$\mathbb{P}(as + bW_s > y \quad \forall s < t) = \Phi \left(\frac{at - y}{b\sqrt{t}} \right) - e^{2ay/b^2} \Phi \left(\frac{at + y}{b\sqrt{t}} \right) \quad (29)$$

Using (29), an expression for the conditional survival probability is obtained:

$$\bar{P} \left(t \mid \frac{D}{S_0}, \sigma, L, \gamma, \delta, \alpha \right) = \Phi \left(\frac{\left(r + \gamma - \delta - \frac{1}{2} \sigma^2 \right) t + \log(d)}{\sigma\sqrt{t}} \right) - d^{1 - \frac{2(r + \gamma - \delta)}{\sigma^2}} \Phi \left(\frac{\left(r + \gamma - \delta - \frac{1}{2} \sigma^2 \right) t - \log(d)}{\sigma\sqrt{t}} \right) \quad (30)$$

where

$$d = 1 + \frac{S_0}{[L + \alpha(1-L)]D}. \quad (31)$$

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