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# Portfolio Management for Pension Funds

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**Abstract.** This paper introduces the use of dynamic stochastic optimisation pension fund management. The design of such products involves econometric modelling, economic scenario generation, generic methods of solving optimization problems and modelling of required risk tolerances. In nearly all the historical backtests using data over roughly the past decade the system described (with transactions costs taken into account) outperformed the benchmark S&P500.

## 1 Introduction

Defined benefit pension plans and most state schemes are becoming inadequate to cover the gap between the contributions of people while working and their pensions once retired. A long-term minimum guarantee return plan with a variable time-horizon and with the possibility of making variable contributions during the lifetime of the product in addition to the initial contribution is a new investment instrument aimed at attracting investors who are worried about the volatility of financial markets. Although potentially highly profitable for the provider, the design of such instruments is not a trivial task, as it encompasses the need to do long-term forecasting for investment classes and handling a number of stochastic factors together with providing a guarantee. This paper shows that dynamic stochastic optimisation methodology is an ideal technique to solve these kinds of problems.

This paper describes the approach and outcomes of a joint project with a leading firm operating in the European fund management industry to develop a state-of-the-art dynamic asset liability management (ALM) system for pension fund management. The liabilities considered in this paper take the form of guaranteed returns, which we refer to as *quasi-liabilities*.

## 2 Critical Issues for Pension Fund Management

Asset liability management concerns optimal strategic planning for management of financial resources in stochastic environments, with market, economic and actuarial risks all playing an important role. Some of the most important issues a pension fund manager has to face in the determination of the optimal asset allocations over the time to product maturity are given below.

*Stochastic nature of asset returns*
The future asset returns over the life of the product are unknown.
*Long investment horizons*
The typical investment horizon is very long (30 years). This means that the fund portfolio will have to be rebalanced many times and can make “buy&hold” Markowitz-style portfolio optimisation inefficient.
*Risk of under-funding*
There is a very important requirement to monitor and manage the probability of under-funding for both individual clients and the fund, that is the probability that the pension fund will not be able to meet its targets without resort to its parent guarantor.
*Management constraints*
The management of a pension fund is also dictated by a number of solvency requirements which are put in place by the appropriate regulating authorities. Moreover, since the fund’s portfolio must be actively managed, the markets’ bid-ask spreads, taxes and other frictions must also be modelled.

## 3 Pension Fund Management Through Stochastic Optimization

Most firms currently use *static* portfolio optimisation, such as the Markowitz mean-variance allocation [9], which is short-sighted and when rolled forward can lead to radical portfolio rebalancing unless constrained by the portfolio manager. In practice fund allocations are likely to be wealth dependent and face time-varying investment opportunities, path-dependent returns due to cash flows and transactions costs. Hence *all* conditions necessary for a sequence of myopic static model allocations to be dynamically optimal are likely to be violated [13, pp 9-11].

By contrast, the dynamic stochastic programming models incorporated in the system described below automatically hedge current portfolio allocations against future uncertainties over a longer horizon, leading to more robust decisions and previews of possible future problems and benefits. It is this feature and its ability to incorporate different attitudes to risk that make dynamic stochastic optimisation the most natural framework for the effective solution of pension fund ALM problems.

Strategic ALM requires the dynamic formulation of portfolio rebalancing decisions together with appropriate risk management in terms of a *dynamic stochastic optimisation problem*. In dynamic stochastic optimisation the unfolding uncertain future is represented by a large number of future scenarios (see e.g [8] and the references therein) and contingent decisions are made in stages according to tree representations of future data and decision processes. Each particular optimisation problem is formulated for a specific application combining the goals and the constraints reflecting risk/return relationships. The dynamic nature of stochastic optimisation: decisions – observed output – next decisions – etc … allows a choice of strategy which is the best suited for the stated objectives. For example, for pension funds the objective may be a guaranteed return with a low unexpected risk and decisions reviewed every year.

## Dynamic stochastic optimization model

We focus here on *strategic asset allocation* which is concerned with allocation across broad asset classes such as equity and bonds of a given country. The canonical pension fund problem is as follows:

> *Given a set of assets, a fixed planning horizon and a set of rebalance dates, find the trading strategy that maximizes utility subject to the constraints.*

Different pension plan instruments are given by alternative utility functions (fund risk tolerances) and the specification of risk management objectives through the constraints. A detailed specification of the model is given in [5].

We consider a discrete time and space setting. Assets take the form of *equity*, *bonds* and *cash*. Subject to the constraint structure, the fund acts by choosing the trading strategy which maximizes the (von Neumann-Morgenstern) *expected utility* of the wealth process which is assumed to be separable over time and states. *Utility functions* are used in our system to represent the general attitude to risk of the fund's participants over a specified fund horizon. In principle different attitudes to risk may be imposed at *each* decision point with the *additively separable* utility U through a sum of different *period utility functions*. We consider Exponential (CARA) and Downside-Quadratic period utility functions.

The basic constraints of the optimization model are those of the dynamic CALM model (*cf.* [2]):
- *Cash balance constraints.* These ensure that the net flow of cash at each time and state is zero.
- *Inventory balance constraints.* These give the amount invested in each asset at each time and state.
- *Wealth constraints.* These define the before and after rebalancing wealths at each time and state.

Besides these basic constraints, the fund may face the following portfolio restrictions:
- *Solvency constraints*. These constrain the fund wealth at each time to be non-negative.
- *Cash borrowing/short limits*. These limit the amount borrowed/shorted of an asset.
- *Position limits*. These limit the amount invested in an asset to be less than some proportion of the fund wealth.
- *Turnover (liquidity) constraints*. These limit the change in the amount invested in an asset from one period to the next.

For backtesting purposes we specify the following three types of constraint structures. T1 constraints have no position limits or turnover constraints, T2 constraints have position limits and no turnover constraints and T3 constraints contain both position limits and turnover constraints. Short selling and borrowing are not allowed in any of these constraint structures. Assuming that the simulated price processes are non-negative, this automatically enforces solvency constraints.

- *Guaranteed return constraints.* The return guarantee to an individual investor is absolute given the solvency of the guarantor. In the situation of a banking group such as the fund manager and its parent guarantor this necessitates strategies both to implement the absolute guarantee for individuals and to manage the investment strategy of the fund so as to ensure meeting the guarantee for all participants of the fund with a high probability.

Mathematically, this latter goal can be met by imposing a *probabilistic* constraint of the *value at risk* type on the wealth process at specific trading dates, computing expected shortfall across scenarios which fail to meet the fund guarantee and adding the corresponding penalty terms to period objective functions. For example, at the horizon T+1 or any intermediate date $t'$ this would take the form $P\left(\mathbf{w}_{t'} \geq w_{t'}^{*}\right) \geq 1-\alpha,$ where α: = 0.01 or 0.05, corresponding to respectively 99% or 95% confidence, and $w_{t'}^{*}$ is calculated from the initial wealth and the guaranteed period rate *r* as $w_0\left(1+r\right)^{t'}$. However, such scenario-based probabilistic constraints are extremely difficult to implement in that they convert the convex (deterministic equivalent) large scale optimisation problem to a *nonconvex* one. For practical purposes we have developed the *capital guaranteed products algorithm* implemented for a pension fund using parametric nested optimization techniques [7].

**Asset return statistical models**

Our main *asset return model* (BMSIM) used to generate scenarios for the ALM problem is based on a set of continuous time *stochastic differential equations* for the financial and economic dynamics of interest. We then discretise time to obtain the corresponding system of stochastic *difference* equations, *estimate* them econometrically (in the econometric estimation tradition initiated by Wilkie [14,15]) and *calibrate* the output of their simulation with history by various *ad hoc* or semi-formal methods of parameter adjustment. (See, for example, [4] and [12].)

The global structure of this model involves investments in the three major asset classes – cash, bonds and equities – in the four major currency areas – US, UK, EU and Japan (JP) – together with emerging markets (EM) equities and bonds. Each currency area is linked to the others *directly* via an exchange rate equation and *indirectly* through correlated innovations (disturbance or error terms). Detailed specifications of this model are given in [1] and [5]. All dependent variables in this specification are in terms of returns, while the explanatory variables are in original level or rate form. Although linear in the parameters, this model is second order autoregressive and nonlinear in the state variables, making its long run dynamics difficult to analyse and potentially unstable. Due to its linearity in the parameters this model may be estimated using the *seemingly unrelated regression* (SUR) technique, see e.g. [6, Chapter 11], recursively until a parsimonious estimate is obtained in which all non-zero parameters are statistically significant.

The formulation of the US economic model captures the interactions of the capital markets with the economy in each major currency area. For stability the specification is in terms of returns similar to the capital markets. This is again a second order autoregressive model in the state variables which is *linear* in parameters and *nonlinear* in variables. It may be estimated using the techniques mentioned above.

The emerging markets equity and bond index returns are modelled with AR(1) / GARCH(1,1). These models are estimated using maximum likelihood and the residuals are used to estimate the correlations of their innovations with those of the other state variables.

Various subsystems of this model have been estimated using the SURE model maximum likelihood estimation procedures of RATS 4.0. For each model the full set of model parameters was first estimated and insignificant (at the 5% level) variables sequentially removed to obtain a parsimonious final model with all statistically significant coefficients. The seemingly unrelated regression nature of the model is obvious as each currency area is directly related only through exchange rates and indirectly related through shocks. In light of Meese & Rogoff's [10, 11] classical view on the inefficacy of macroeconomic explanations of exchange rates even at monthly frequency, after considerable single equation and subsystem analysis we have found that interest rate parity expressed as inter-area short and long rate differences – *together* with other local capital market variables – has significant explanatory power, while purchasing power parity expressed various ways has less (*cf.* [7]).

We found model residuals correlated across equations and a similar level of correlation was also found in actual returns for the same variables and periods of time. Our main econometric finding is that the world's equity markets are linked simultaneously through shocks [1] and [5].

**Montel Carlo simulation**

In order to mirror reality the alternative unfolding future scenarios in the model must be organized in a *tree* form. Each path from the root to a leaf node in the tree *represents* a scenario and the nodes represent decision points – forward portfolio rebalances.

## 4 System Historical Backtests

A number of historical backtests have been run on variants of the global model, see [5] for complete details. The aims of these tests were several. First, we wished to evaluate how well the system would have performed had it been implemented in practice relative to a benchmark. Second, we wished to understand the impact of alternative utility functions on optimal portfolio decisions. Thirdly, we were interested in what effects imposing the practical diversification and liquidity (turnover) constraints would have on backtest returns. All portfolio rebalances are subject to a 1% value tax on transactions which of course does not apply to the benchmark index. Monthly data were available from July 1977 to August 2002.

Table 1 shows the results in terms of annualised returns of a typical backtest with a 2 year telescoping horizon and semi-annual rebalancing from February 1999 to February 2001 using a model with 8192 scenarios, a 128.16.2.2 branching structure and a terminal wealth criterion. During this period the S&P500 returned 0 percent. With no position limits the model tends to pick the best asset(s) and so in this case a high annual historical return to the chosen low diversification portfolios is an indication of the predictive merits of the tuned econometric model used to generate the scenarios. When more realistic constraints are imposed in this test portfolios become well diversified. However, performance is improved by the use of the emerging market asset returns even though they were actually not used in the optimal portfolios. Corresponding results for the addition of the US economic model to the system are mixed. When this backtest was extended one period to August 2001 – when the S&P500 annualised return over the 2.5 year period was –2.3% – similar results were obtained with the best position limited result being 6.8% per annum for the downside-quadratic utility and target wealth a 61% increase over the period.

**Table 1.** Asset allocation backtests: Annualised returns from February 1999 – 2001

| | **Capital Markets** | | **Capital Markets + Emerging Markets** | | **Capital Markets + Emerging Markets + US Economic Model** | |
|---|---|---|---|---|---|---|
| **Utility Function** | **No Limits** | **20% Limits** | **No Limits** | **20% Limits** | **No Limits** | **20% Limits** |
| **Linear** | 91% | 9% | 92% | 10% | 31% | 11% |
| **Downside-quadratic** | 54% | 9% | 70% | 11% | 29% | 9% |
| **Exponential** | 72% | 9% | 92% | 10% | 51% | 11% |

A summary of backtest results using the downside-quadratic utility function is given in Table 2.

**Table 2. Summary of historical backtests[1]**

| Period | Asset Return Model | Number of Scenarios k | Rebalance Frequency | Risk | Horizon | Annualized Return % | | | S&P 500 Benchmark Annualised Return % |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | T1 | T2 | T3 | |
| 1990-1995 | 3 areas (ex Japan) | 4 | annual | terminal | telescoping | 10.33 | 9.34 | - | 7.41 |
| 1996-2001 | 4 areas | 4 | annual | terminal | telescoping | 13.36 | 7.13 | - | 14.12 |
| 1999-2001 | 4 areas | 8.2 | semi-annual | terminal | telescoping | 27.89 | 6.48 | 2.69 | -2.3 |
| 1999-2001 | above + emerging markets | 8.2 | semi-annual | terminal | telescoping | 16.98 | 5.72 | 3.38 | -2.3 |

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[1] "-" entries not calculated.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1999-2001 | above + US economy | 8.2 | semi-annual | terminal | telescoping | 19.16 | 4.64 | -0.38 | -2.3 |
| 1996-2001 | 4 areas | 8.2 | annual | all periods | telescoping | 8.54 | - | 8.37 | 14.12 |

## 5 Conclusions

This paper has described the use of dynamic stochastic optimisation methodology for structured products for pension fund management. Practical solutions to the design of guaranteed return investment products for pension funds have been outlined. In nearly all the historical backtests using data over roughly the past decade the global asset allocation system outperformed the S&P500 when transactions costs are taken into account. Nearly all system returns for the nonlinear statistical model were positive.

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