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Managing Guarantees

Providing protection but maintaining access to potentially higher returns.

MICHAEL A.H. DEMPSTER, MATTEO GERMANO, ELENA A. MEDOVA, MURIEL I. RIETBERGEN, FRANCESCO SANDRINI, and MARK SCROWSTON

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When the equity bubble of the second half of the 1990s deflated, there was a change in the attitude of many investors toward risk. Investors began to search for protection that would transfer risk from them to a guarantor in exchange for a fee.

Both investment banks and asset managers are developing long-term guaranteed products allowing the sharing of risk between the fund manager and the investor. The purpose of these products is to take advantage of new sources of performance while simultaneously limiting the liability to the client. This liability could be capital protection or a guaranteed return, either fixed or linked to inflation.

Investment banks might issue structured notes with, for example, constant-proportion portfolio insurance (CPPI) features and automatic trading rules, adjusting the exposure of a portfolio to risky markets depending on the gap between the net asset value (NAV) of the portfolio and a bond floor. One problem with this approach is that CPPI and similar products tend to follow the market rather than anticipating it; they provide us a riskless approach of reaching the guarantee, but forgo any upside potential. This is where the asset manager has a potential advantage. He or she can provide the protection while still exposing the client to higher-risk markets through active asset allocation to potentially higher returns.

The other role of the asset manager regarding capital-protected segregated accounts is to provide risk control, usually in conjunction with investment banks or insurance companies that act as guarantor of the fund. The usual approaches, such as Markowitz mean-variance optimization, are inappropriate for this task because of their buy-and-hold static view of managing investment risk only at the horizon and not continually over the life of the fund.

For asset managers to do their job effectively, there is clearly a need for dynamic asset allocation on the investment side and for valuing liabilities at market rates constantly over time. Since guarantees constitute a liability for the fund, the market value of the minimum guarantee represents a benchmark for the fund manager that must be exceeded and in the worst case matched.

Our approach to portfolio construction for protected products is *dynamic stochastic programming (optimization)*, known in financial applications as *asset liability management* (ALM). Examples appear in Kouwenberg [2001] and Mulvey, Pauling, and Madey [2003].

Dynamic stochastic programming models will automatically hedge current portfolio allocations against projected future uncertainties in asset returns and costs of liabilities over the time horizon (see Dempster et al. [2003]). They also are flexible enough to take into account multiple time periods, portfolio constraints such as the prohibition of short-selling, and varying degrees of risk aversion in the portfolio allocation.

Dynamic stochastic optimization requires a variety of skills and techniques, particularly in the modeling of asset returns, liabilities, and related economic factors in order to generate scenarios for all underlying processes. The formulations of asset liability management optimization problems vary under different regulatory regimes and for different fund sponsors. They involve econometric modeling, economic scenario generation, and solutions of stochastic optimization problems within required risk tolerances. Fabozzi, Focardi, and Jonas [2004, p. 14] note that "ALM technology can be a challenge even to sophisticated users," but proprietary modular software systems can help with a variety of problems under uncertainty.

We illustrate an ALM implementation and solution results using a simple example of a closed-end guaranteed return fund—a starting point for the construction of more complex guaranteed products with different risk management strategies.

There are several different guarantees available in the market. The most common is the *nominal guarantee*, which assures a fixed percentage of the initial wealth at a specified date in the future. There are also funds with a "real" or *flexible guarantee* linked to an inflation index or some other capital market index. Sometimes the guarantee of a minimum rate of return may be set in relation to the performance of other funds. A minimum rate of return guarantee may be termed a (lower) *barrier* since the fund's portfolio return at any time must be greater than or at least equal to that level.

We analyze the optimal strategic asset allocation for a *closed-end guaranteed return fund* over the fund's life. In such a fund no contributions are allowed after the initial cash outlay. The time horizon of the fund is T years, and the guarantee is G% return on the initial wealth. Eight different assets a are chosen for the fund to hold: coupon-bearing Treasury securities with maturities equal to 1, 2, 3, 4, 5, 10, and 30 years and an equity index to boost fund performance. We use a five-year horizon ($T = 5$) for illustration, although the techniques described can be applied to much longer horizons at the cost of increased computation time.

We simulate the future returns of the eight assets and construct a scenario tree where each path of asset returns through the tree corresponds to a scenario ω in Ω, and each node in the tree corresponds to a decision time along one or more scenarios. An example scenario tree is given in Exhibit 1.

The probability $p(\omega)$ of scenario ω in Ω is the reciprocal of the total number of scenarios as the data paths are generated with equal probability by Monte Carlo simulation. The times are given in months and years $\left\{0, \frac{1}{12}, \frac{2}{12}, ..., 1, ..., 2, ..., T\right\}$. Certain times s, e.g., $s := t = 0, 1, 2, \ldots$, correspond to the (annual) decision times at which the fund will trade to rebalance its portfolio. T corresponds to the time the guarantee will be paid out.

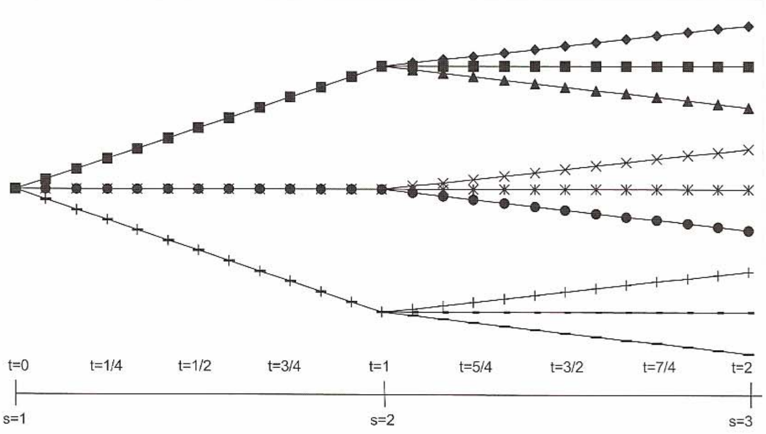
To represent the scenario tree structure we use a *treestring*, which is a string of integers specifying for each stage s the number of branches for each node in that stage. This specification gives rise to *balanced scenario trees* where each subtree in the same period has the same number of branches. The balanced scenario tree of Exhibit 1 can be described by the treestring 3.3, giving a total of $3 \cdot 3 = 9$ scenarios.

MODELING THE GUARANTEE LIABILITY

Starting with an initial wealth W_0 and a nominal guarantee of G% annually, the guarantee liability at the planning horizon T is given by:

$$W_0 (1 + G)^T \qquad (1)$$

EXHIBIT 1
Representation of Scenarios



To price the liability at time t, consider the yield for scenario ω of a zero-coupon bond that pays 1 at time T, i.e., $Z_T(\omega) = 1$. The zero-coupon bond price at time t in scenario ω (assuming continuous compounding) is given by:

$$Z_t(\omega) = e^{-y_{t,T}(\omega)(T-t)} \tag{2}$$

where $y_{t,T}(\omega)$ is the zero-coupon Treasury yield with maturity T at time t in scenario ω.

This gives us a formula for the nominal or fixed guarantee barrier at time t in scenario ω:

$$\begin{aligned} L_t^F(\omega) &= W_0(1+G)^T Z_t(\omega) \\ &= W_0(1+G)^T e^{-y_{t,T}(\omega)(T-t)} \end{aligned} \tag{3}$$

This formulation of the barrier shows that short-horizon funds are likely to attract more risk-averse participants than long-horizon funds, whose participants can afford to tolerate more risk in the short run. This natural distinction between short- and long-horizon funds is incorporated in the problem setup, as the barrier will initially be lower for long-term funds than for short-term funds.

This feature of the model is demonstrated in Exhibit 2, where the top line can be interpreted as the barrier for a fund that guarantees a 2% return throughout the life of the product, and the bottom two lines represent the barriers if the 2% is guaranteed only at the horizon. At time $t = 0$, the gap between the top line (the initial portfolio wealth) and the bottom line (the barrier) is wider for the five-year guarantee than for the one-year guarantee, indicating that the five-year portfolio can afford to take a more risky asset allocation.

This formulation agrees with what is required by the Financial Accounting Standards Board (and has been proposed by leading professionals): "a high-quality zero-coupon bond whose par value matches the liability payment amount, and whose maturity matches the liability payment date" (see Ryan and Fabozzi [2002, p. 9).

In the case of an inflation-indexed guarantee, the final guarantee at time T is given by:

$$W_0 \prod_{s=\frac{1}{12}}^{T} \left(1 + i_s^{(m)}(\omega)\right) \tag{4}$$

where $i_s^{(m)}(\omega)$ represents the monthly inflation at time s in scenario ω.

Contrary to the nominal guarantee, at time $t < T$ the final inflation-linked guarantee is still unknown. We propose to approximate the final guarantee by using the inflation rates that are known at time t, combined with the expected inflation at time t for the period $\left[t+\frac{1}{12}, T\right]$.

The inflation-indexed barrier at time t is now given by:

$$\begin{aligned} L_t^I(\omega) &= W_0\left(\prod_{s=\frac{1}{12}}^{t}\left(1+i_s^{(m)}\right)\right)\mathbb{E}\left(\prod_{s=t+\frac{1}{12}}^{T}\left(1+i_s^{(m)}(\omega)\right)\right)Z_t(\omega) \\ &= W_0\left(\prod_{s=\frac{1}{12}}^{t}\left(1+i_s^{(m)}\right)\right)\mathbb{E}\left(\prod_{s=t+\frac{1}{12}}^{T}\left(1+i_s^{(m)}(\omega)\right)\right)e^{-y_{t,T}(\omega)(T-t)} \end{aligned} \tag{5}$$

EXHIBIT 2
Barrier for One-Year and Five-Year 2% Guaranteed Fund

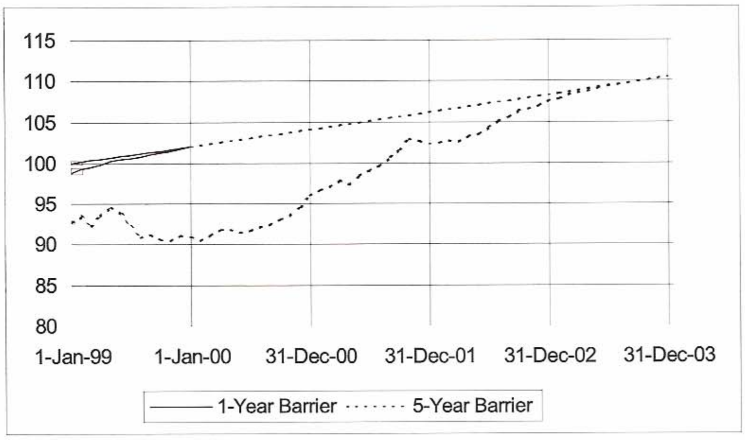
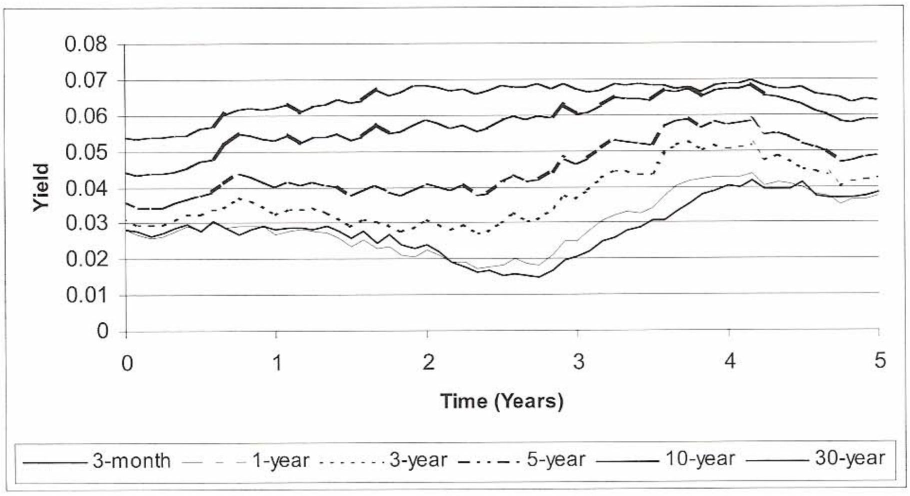


EXHIBIT 3
Five-Year Simulation Using Gaussian Three-Factor Model



For the fixed guaranteed return fund, the end value is known from the start. Because of different yield curve scenarios, though, this guarantee will be discounted at different values, thereby creating different barriers for each scenario. In the case of an inflation-linked guarantee, both the expected inflation and the expected yields will vary across scenarios, giving different barriers and different terminal values for the guarantee for each scenario.

The challenge is to derive and simulate the scenarios for the yield curve according to these requirements. We use a Gaussian three-factor economic model with a closed-form solution for the yields with parameters estimated using the Kalman filter. Yields of 16 different maturities ranging from 1 month to 30 years are used in the estimation process to capture the dynamics of the entire term structure. The yields of the bonds in which the fund invests must be a subset of these data.

A typical yield curve simulation is shown in Exhibit 3.

MODELING ASSET RETURNS

As the products we discuss are aimed at the European market, the assets used in the model are denominated in euros. The equity is the Dow Jones Eurostoxx 50 index, which is assumed to follow a geometric Brownian motion. The returns of individual Treasury securities are obtained from the yield curve, which is used to generate the barrier, i.e., for pricing the liability. Thus both asset returns and liabilities are priced in a consistent way across all future scenarios.

As sufficient historical data on coupon-bearing euro Treasury securities are difficult to obtain, the returns of relevant securities are obtained from the zero-coupon yield curve. Coupons on newly issued securities are closely related to the corresponding spot rate at the time, so we will use the current zero yield with maturity T as a proxy for the coupon rate of a coupon-bearing security with maturity T, i.e., the coupon rate $\delta_2^{B^{10}}(\omega)$ on a newly issued ten-year Treasury security at time $t = 2$ will be set equal to the projected ten-year spot rate $y_{2,10}(\omega)$ at time $t = 2$, thereby creating par securities.

Treasury securities are assumed to pay coupons semiannually and to be rolled over on an annual basis, so that a coupon will be received after six months and again after a year just before selling the security. This forces us to distinguish between the price at which we will sell the security and the price for which we buy the new security at decision times.

For inflation-linked guarantees, we propose to approach the problem in the same general way as the nominal guaranteed return funds, still using a three-factor economic factor model estimated using the Kalman filter to simulate the nominal yields and using a mean-reverting Ornstein-Uhlenbeck (OU) process to model inflation. By using this approach, we obtain flexibility and allow the model to be applied to virtually any index-tracking guarantee. With little data available on inflation-linked Treasury securities in the European market, we perceive this to be a better approach than having to simulate break-even rates and from those derive expected inflation rates.

MULTISTAGE ALM

Exhibit 4 defines the variables and parameters of the stochastic programming problem.

Constraints

The basic constraints of the dynamic model considered for the minimum guaranteed return problem are:

- *Cash balance constraints.* These constraints ensure that the net cash flow at each decision time and at each scenario is equal to zero:

$$\sum_{a\in A} fP_{0,a}^{\text{buy}}(\omega)x_{0,a}^{+}(\omega) = W_0(\omega) \qquad \omega\in\Omega \qquad (6)$$

$$\sum_{a\in A\setminus\{S\}} \frac{1}{2}\delta_{t-1}^{a}(\omega)F^{a}x_{t,a}^{-}(\omega) + \sum_{a\in A} gP_{t,a}^{\text{sell}}(\omega)x_{t,a}^{-}(\omega) = \sum_{a\in A} fP_{t,a}^{\text{buy}}(\omega)x_{t,a}^{+}(\omega)$$
$$\omega\in\Omega \quad t\in T^{d}\setminus\{0\} \qquad (7)$$

- *Short sale constraints.* These eliminate the possibility of short-selling the equity and bond assets:

$$x_{t,a}(\omega)\geq 0 \qquad a\in A \quad \omega\in\Omega \quad t\in T^{\text{total}}\setminus\{T\} \qquad (8)$$

$$x_{t,a}^{+}(\omega)\geq 0 \qquad a\in A \quad \omega\in\Omega \quad t\in T^{\text{total}}\setminus\{T\} \qquad (9)$$

$$x_{t,a}^{-}(\omega)\geq 0 \qquad a\in A \quad \omega\in\Omega \quad t\in T^{\text{total}}\setminus\{0\} \qquad (10)$$

- *Wealth constraint.* This constraint determines the portfolio wealth at each time:

$$W_t(\omega) = \sum_{a\in A} fP_{t,a}^{\text{buy}}(\omega)x_{t,a}(\omega)$$
$$\omega\in\Omega \quad t\in T^{\text{total}}\setminus\{T\} \qquad (11)$$

$$W_T(\omega) = \sum_{a\in A} gP_{T,a}^{\text{sell}}(\omega)x_{T-\frac{1}{12},a}(\omega) + \sum_{a\in A\setminus\{S\}} \frac{1}{2}\delta_{T-1}^{a}(\omega)F^{a}x_{T-\frac{1}{12},a}(\omega) \quad \omega\in\Omega \qquad (12)$$

- *Accounting balance constraints.* These constraints give the quantity invested in each asset at each time and for each scenario:

$$x_{0,a}(\omega) = x_{0,a}^{+}(\omega) \qquad a\in A \quad \omega\in\Omega \qquad (13)$$

$$x_{t,a}(\omega) = x_{t-\frac{1}{12},a}(\omega) + x_{t,a}^{+}(\omega) - x_{t,a}^{-}(\omega)$$
$$a\in A \quad \omega\in\Omega \quad t\in T^{\text{total}}\setminus\{0\} \qquad (14)$$

- *Information constraints.* These constraints ensure that the portfolio allocation cannot be changed between decision times:

$$x_{t,a}^{+}(\omega) = x_{t,a}^{-}(\omega) = 0$$
$$a\in A \quad \omega\in\Omega \quad t\in T^{i}\setminus T^{c} \qquad (15)$$

- *Coupon reinvestment constraints.* We assume that the coupon paid each six months will be reinvested in the same coupon-bearing Treasury bond:

$$x^{+}_{t,a}(\omega) = \frac{\frac{1}{2}\delta^{a}_{t}(\omega)F^{a}x_{t-\frac{1}{12},a}(\omega)}{fP^{\text{buy}}_{t,a}(\omega)} \qquad x^{-}_{t,a}(\omega) = 0$$

$$x^{+}_{t,S}(\omega) = x^{-}_{t,S}(\omega) = 0$$

$$a \in A \setminus \{S\} \quad \omega \in \Omega \quad t \in T^{c} \tag{16}$$

- *Annual rollover constraint.* This constraint ensures that at each decision time all the coupon-bearing Treasury security holdings are sold:

$$x^{-}_{t,a}(\omega) = x_{t-\frac{1}{12},a}(\omega)$$

$$a \in A \setminus \{S\} \quad \omega \in \Omega \quad t \in T^{d} \setminus \{0\} \tag{17}$$

- *Barrier constraints.* These constraints determine the shortfall of the portfolio at each time and in each scenario as defined in Exhibit 4:

$$h_t(\omega) + W_t(\omega) \geq L_t(\omega) \qquad \omega \in \Omega \quad t \in T^{\text{total}} \tag{18}$$

$$h_t(\omega) \geq 0 \qquad \omega \in \Omega \quad t \in T^{\text{total}} \tag{19}$$

- *Non-anticipativity constraints.* These constraints prevent foresight of uncertain future events and in the nodal problem representation used here are taken into account implicitly.

As the objective of the stochastic program will put a penalty on any shortfall, optimizing will ensure that $h_t(\omega)$ will be zero if possible and as small as possible otherwise.

To obtain the maximum shortfall for each scenario, we need to add one of two constraints:

$$H(\omega) \geq h_t(\omega) \qquad \omega \in \Omega \quad t \in T^{d} \cup \{T\} \tag{20}$$

$$H(\omega) \geq h_t(\omega) \qquad \omega \in \Omega \quad t \in T^{\text{total}} \tag{21}$$

Constraint (20) must be added if the maximum shortfall is taken into account on a yearly basis, while (21) considers the maximum shortfall on a monthly basis.

Objective Functions

The risk-return trade-off is incorporated into the objective function. Defining *shortfall* as the amount by which the portfolio's wealth falls below the barrier, the risk of the policy is quantified in two ways. First, we consider the average shortfall over time for each scenario and then take the expectation over all scenarios (the *expected average shortfall* approach). Second, we look at the maximum shortfall over time for each scenario, and then as before take the expectation over all scenarios (the *expected maximum shortfall* approach). A scaling factor, which can be interpreted as a measure of risk aversion, links the portfolio wealth and the shortfall/risk factor for the guarantee in the objective function.

The objective function of the *expected average shortfall* (EAS) model is given by:

$$\max_{\substack{x_{t,a}(\omega),x^{+}_{t,a}(\omega), \\ x^{-}_{t,a}(\omega): a\in A, \\ \omega\in\Omega, t\in T^{d}\cup\{T\}}} \left\{ \sum_{\omega\in\Omega} \sum_{t\in T^{d}\cup\{T\}} p(\omega)\left((1-\beta)W_t(\omega) - \beta \frac{h_t(\omega)}{|T^{d}\cup\{T\}|} \right) \right\} =$$

$$\max_{\substack{x_{t,a}(\omega),x^{+}_{t,a}(\omega), \\ x^{-}_{t,a}(\omega): a\in A, \\ \omega\in\Omega, t\in T^{d}\cup\{T\}}} \left\{ \begin{array}{l} (1-\beta)\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{d}\cup\{T\}} W_t(\omega) \right) - \\ \beta\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{d}\cup\{T\}} \frac{h_t(\omega)}{|T^{d}\cup\{T\}|} \right) \end{array} \right\} \tag{22}$$

That is, we maximize the expected sum of wealth over time while exacting a penalty every time wealth falls below the barrier. In this case, only the shortfalls at decision times are taken into account, and any serious loss in portfolio wealth in between decision times is ignored.

From the fund manager's perspective, however, the position of the portfolio's wealth relative to the fund's barrier is always significant, and serious or repeated drops below this barrier might force the purchase of expensive insurance. To capture this feature specific to minimum guaranteed return funds, we also use an objective function that considers the shortfall of the portfolio on a monthly basis.

For the expected average shortfall with monthly checking (EAS MC) model, the objective function is given by:

$$\max_{\substack{x_{t,a}(\omega),x^{+}_{t,a}(\omega),x^{-}_{t,a}(\omega): \\ a\in A, \omega\in\Omega, t\in T^{d}\cup\{T\}}} \left\{ \begin{array}{l} (1-\beta)\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{d}\cup\{T\}} W_t(\omega) \right) - \\ \beta\left(\sum_{\omega\in\Omega} p(\omega) \sum_{t\in T^{\text{total}}} \frac{h_t(\omega)}{|T^{\text{total}}|} \right) \end{array} \right\} \tag{23}$$

Note that although we still rebalance only once a year, shortfall is now measured in the objective on a monthly basis so that the annual decisions must also take

Exhibit 4
Variables and Parameters of the Model

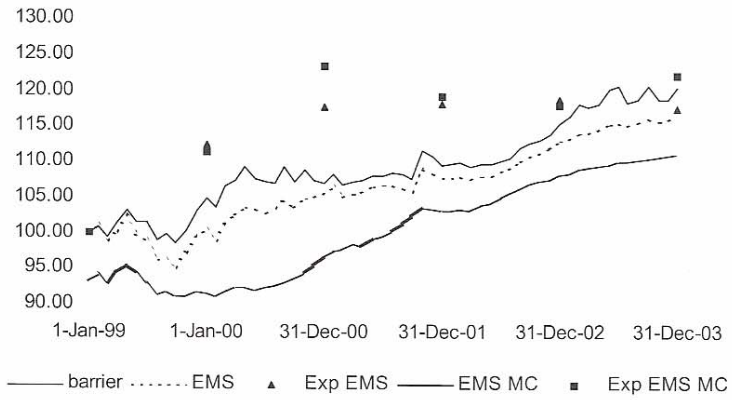
Time Sets	
$T^{\text{total}} = \left\{0, \frac{1}{12}, ..., T\right\}$	set of all times considered in the stochastic program
$T^{d} = \{0, 1, ..., T-1\}$	set of decision times
$T^{i} = T^{\text{total}} \setminus T^{d}$	set of intermediate times
$T^{c} = \left\{\frac{1}{2}, \frac{3}{2}, ..., T-\frac{1}{2}\right\}$	times a coupon is paid out in between decision times
Instruments	
$S_t(\omega)$	Dow Jones Eurostoxx 50 index level at time t in scenario ω
$B_t^T(\omega)$	EU Treasury security with maturity T at time t in scenario ω
$\delta_t^{B^T}(\omega)$	coupon rate of EU Treasury security with maturity T at time t in scenario ω
F^{B^T}	face value of EU Treasury security with maturity T
$Z_t(\omega)$	EU zero-coupon Treasury security price at time t in scenario ω
Risk Management Barrier	
$y_{t,T}(\omega)$	EU zero-coupon Treasury yield with maturity T at time t in scenario ω
G	annual guaranteed return
$L_t(\omega)$	barrier at time t in scenario ω
Portfolio Evolution	
A	set of all assets
$P_{t,a}^{buy}(\omega) / P_{t,a}^{sell}(\omega)$	buy/sell price of asset $a \in A$ at time t in scenario ω
f/g	transaction costs of buying/selling
$x_{t,a}(\omega)$	quantity held of asset $a \in A$ between time t and $t + 1/12$ in scenario ω
$x_{t,a}^{+}(\omega) / x_{t,a}^{-}(\omega)$	quantity bought/sold of assets $a \in A$ at time t in scenario ω
$W_t(\omega)$	portfolio wealth at time $t \in T^{\text{total}}$ in scenario ω
$h_t(\omega) := \max(0, L_t(\omega) - W_t(\omega))$	shortfall at time t in scenario ω

into account the possible forward effects they will have on monthly shortfall.

The value of $0 \leq \beta \leq 1$ can be chosen freely and sets the level of risk aversion. The higher the value of β, the more importance given to shortfall and the less to the expected sum of fund values over time, and hence the more risk-averse the optimal portfolio allocation will be. The two extreme cases are represented by $\beta = 0$, the unconstrained situation, which is indifferent to the probability of falling below the barrier, and $\beta = 1$, when only the shortfall is penalized and the expected sum of fund values is ignored.

EXHIBIT 5

Backtest 1999–2004: 2% Annual Guarantee Using Expected Maximum Shortfall Objective



512.2.2.2.2 tree.

The second model we consider uses the *expected maximum shortfall* (EMS) objective given by:

$$\max_{\left\{\substack{x_{t,a}(\omega),x^+_{t,a}(\omega),x^-_{t,a}(\omega):\\ a\in A,\omega\in\Omega,t\in T^d\cup\{T\}}\right\}} \left\{ \begin{array}{l} (1-\beta)\left(\sum_{\omega\in\Omega} p(\omega)\sum_{t\in T^d\cup\{T\}} W_t(\omega)\right) - \\ \beta\left(\sum_{\omega\in\Omega} p(\omega)H(\omega)\right) \end{array} \right\} \tag{24}$$

using constraint (20) to define $H(\omega)$.

For the expected maximum shortfall with monthly checking (EMS MC) model, the objective function remains the same, but $H(\omega)$ is now defined by (21). In this model we penalize the expected maximum shortfall over time assuming that for each scenario $\omega \in \Omega$, $H(\omega)$ is as low as possible. Combining this with the constraints (20) or (21) ensures that $H(\omega)$ is equal to the maximum shortfall for scenario ω.

The EMS model focuses on limiting the maximum shortfall and therefore does not penalize portfolio wealth falling just slightly below the barrier several times. The EAS model, on the other hand, incurs a penalty every time the portfolio's wealth falls below the barrier, but does not differentiate between a substantial shortfall at one time and a series of small shortfalls over time. So one model limits fund wealth from falling below the barrier *substantially*, and the other limits the *number* of times it does so.

RESULTS

To illustrate the methodology we propose, we describe some backtests over January 1999 to January 2004. In this period we saw a major correction in the equity market and a period of declining interest rates, resulting in increased present values of the liability. While this would represent a difficult time to earn high portfolio returns, it is an excellent time to see how the model performs under adverse market conditions with its risk management assumptions in place.

Included in the exhibits are the one-year ahead in-sample expectations of the portfolio's wealth. Implementing the first-stage decisions, we calculate the portfolio's wealth for each scenario in the simulated tree one year later, and then take an expectation over the scenarios.

The initial in-sample overestimation of the model is likely due mainly to the short time series for parameter estimation, which leads to hugely inflated stock return expectations. As time progresses and we have more data points to recalibrate the model, the model expectation and real-life realization very closely approximate one another.

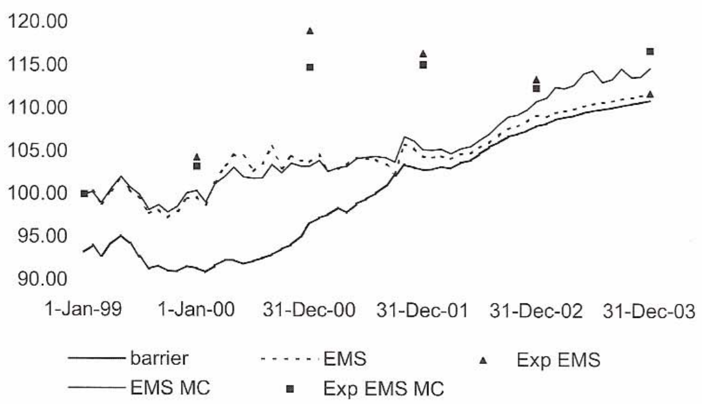
Three different sets of experiments are considered:

- 2% nominal guarantee, no transaction costs;
- 2% nominal guarantee, 50 basis point bid-ask spread;
- Inflation-linked guarantee, no transaction costs.

The 2% nominal guarantee experiments in Exhibit 5 show a clear difference between the expected maximum

EXHIBIT 6

Backtest 1999–2004: 2% Annual Guarantee with Transaction Costs Using Expected Maximum Shortfall Objective



32.4.4.4.4 tree.

EXHIBIT 7

Portfolio Allocation for 2% Annual Guarantee with Transaction Costs Using Expected Maximum Shortfall Objective

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.79**	**0.14**	0	**0.07**
Jan 00	0	0	0	0	0	**0.17**	**0.52**	**0.31**
Jan 01	0	0	0	0	**0.60**	**0.17**	0	**0.23**
Jan 02	0	**0.53**	0	0	**0.42**	0	0	**0.05**
Jan 03	**0.96**	**0.04**	0	0	0	0	0	0

32.4.4.4.4 tree.

shortfall approach using just yearly checks and monitoring the shortfall on a monthly basis.

As the system with annual rollover involves the selling and buying of a new Treasury security portfolio each year, including transaction costs puts significant downward pressure on the portfolio's wealth. The terminal wealth is now more in the region of 115, rather than the 120 of earlier experiments.

Exhibit 6 shows that, even under this increased downward pressure, the model performs well, staying above the barrier at all times. Exhibits 7 and 8 reflect this need for a more conservative portfolio allocation in the diminished reliance on equity.

For the inflation-linked guarantees, Exhibits 9 and 10 give the portfolio allocations for the 512.2.2.2.2 tree using the maximum shortfall objective functions. In both cases, we can identify a tendency for the portfolio to move to the safer, shorter-term assets as time progresses. This is naturally built into the model (as explained by Exhibit 2).

Comparing these inflation-linked guarantee portfolio allocations to the portfolio allocations of the nominal guarantee, we see that the portfolios are not as well diversified. As time progresses, we also do not see as clear a move toward shorter-maturity securities. A possible explanation is that, as time progresses, inflation remains uncertain, so even one year away from maturity a one-year security might not be sufficient to hedge the inflation risk. The optimizer therefore chooses slightly longer-term securities with a better guarantee of higher returns.

EXHIBIT 8
Portfolio Allocation for 2% Annual Guarantee with Transaction Costs Using Expected Maximum Shortfall with Monthly Checking Objective

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.90**	**0.02**	0	**0.08**
Jan 00	0	0	0	0	**0.75**	**0.02**	0	**0.23**
Jan 01	0	0	0	0	**0.77**	**0.06**	0	**0.17**
Jan 02	**0.23**	**0.01**	0	0	**0.71**	0	0	**0.05**
Jan 03	**0.39**	**0.01**	0	0	**0.59**	0	0	**0.01**

32.4.4.4.4 tree.

EXHIBIT 9
Portfolio Allocation for Inflation-Linked Guarantee Using Expected Maximum Shortfall Objective

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	0	**0.62**	0	**0.38**
Jan 00	0	0	0	0	0	**0.33**	0	**0.67**
Jan 01	0	0	0	0	0	**0.60**	0	**0.40**
Jan 02	0	0	0	0	**0.83**	**0.08**	0	**0.09**
Jan 03	**0.77**	0	**0.23**	0	0	0	0	0

512.2.2.2.2 tree.

Furthermore, for decisions to be made on January 2002–2003, portfolio wealth is significantly closer to the barrier for the EMS model than for the EMS MC model. The model takes this increased risk for the fund into account, resulting in an investment in safer short-term securities. While the EMS MC model invests mainly in securities with a maturity in the range of four to five years, the EMS model stays in the one- to three-year range, and hence for both models the portfolio's wealth manages to stay above the barrier.

For reference, Exhibit 11 describes the performance of the Eurostoxx 50 to give some indication of how the stock market performed over the backtesting period.

CONCLUSION

We have demonstrated the construction of investment products that give a nominal or real minimum guaranteed return. Our main focus has been on designing the liability side of the product.

Overall, the expected maximum shortfall with monthly checking objective outperforms the other objectives, in terms of both portfolio performance over the life of the product and terminal portfolio wealth. Higher initial branching factors tend to improve the performance, although the expected maximum shortfall with monthly checking already performs well even with low initial branching factors (the objective functions with yearly checking perform quite poorly with low initial branching factors). Transaction costs unsurprisingly reduce the portfolio's terminal wealth, but the model continues to perform well, choosing more conservative portfolio asset allocations, and therefore remaining above the barrier at all times.

These backtests show that the stochastic optimization framework we describe carefully considers the risks created by the guarantee. The expected maximum shortfall with monthly checking model in particular produces well-diversified portfolios that do not change drastically from one year to the next and results in a dynamic portfolio allocation that even in a period of economic downturn and uncertainty caused by increasing inflation rates remains above the barrier. The dynamic risk

EXHIBIT 10

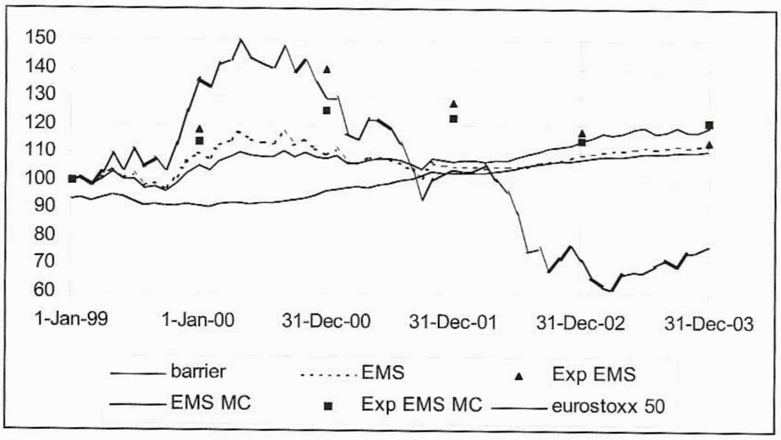
Portfolio Allocation for Inflation-Linked Guarantee Using Expected Maximum Shortfall with Monthly Checking Objective

	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.36**	**0.38**	0	**0.26**
Jan 00	0	0	0	0	**0.58**	0	0	**0.42**
Jan 01	0	0	0	0	**0.40**	**0.29**	0	**0.31**
Jan 02	0	0	0	**0.07**	**0.85**	0	0	**0.08**
Jan 03	**0.15**	0	0	**0.07**	**0.77**	0	0	**0.01**

512.2.2.2.2 tree.

EXHIBIT 11

Backtest 1999–2004: Inflation-Linked Guarantee Using EMS—Comparison to Eurostoxx 50



control we propose using monthly time steps may be generalized to any time step appropriate to the style and asset choices of the fund.

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