University Finance Seminar
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Operational Risk Measures and Bayesian Simulation Methods for Capital Allocation

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Outline

1. Definitions, discussion and directions
2. Our interpretation of operational risk
3. Calculating the capital allocation for operational risk
4. Some results from extreme value theory
5. Bayesian simulation methods for multiple risk types
6. Examples

What is operational risk?… Other risks?

- British Bankers' Association sample definitions
  - The risk associated with human error, inadequate procedures and control, fraudulent and criminal activities,...;
  - the risks caused by technological shortcomings, system breakdowns;
  - all risks which are not 'banking' and arising from business decisions as competitive action, pricing, …;
  - legal risk and risk to business relationships, failure to meet regulatory requirements or an adverse impact on the bank's reputation,... ;
  - 'external factors' includes: Acts of God, natural disasters, terrorist attacks and fraudulent activity,… .
- all risks that are not market or credit risk are operational risks
- '…a semantic Wild West -- by defining in terms of what it is not, one fails to say what it is!' [R Jameson, Risk, 1998]
  - → precise definitions of market and credit risk need to be established

Regulatory Capital Charge for Operational Risk

- Banks' objectives are to develop an accurate allocation of economic capital and to avoid hard regulatory over-provision that will make their business less competitive
- Banking supervisors' objectives are national and global financial stability. The main concern is to capture all types of risk and to ensure that there is sufficient capital in a bank to provide protection up to a certain confidence level
- If in the future capital is to be allocated there must be some definitional common ground!

Directions (FSA informal working party discussion paper) Selected methodologies

- Using the balance sheet and the profit and loss account
  - Capital charges are based on weighted items in the financial statements -- annual overall cost, staff cost, total assets, total revenue, non interest income, …
- Causal modelling for operational risk
  - Nodes represent variables / key performance drivers and the links imply causality / conditional probabilities updated using a Bayesian approach given a particular value for any of variables -- attempt to model the cause-effect relationship (Algorithmics)
- Box approach
  - Risk score assignment to business units and into number of categories: settlement, legal, retail,...

- Statistical modelling including modelling extreme events
- Capital asset pricing model (CAPM)
- Business risk earnings volatility
- Brand: if possible to find a brand value for a bank, might be possible to attach a regulatory capital requirement -- proxy for reputational risk
- The use of self-assessment in allocation of capital. Regulators could adjust any charge

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Problems

- Most definitions include the solution for risk -- namely systems and controls as a part of the definition
- Right regulatory capital avoids real risks and prevents banks 'managing' charges rather than risk
- No industry consensus on any sort of quantitative measure for operational risk

- Need to clarify the roles of credit, market and other risk capital

  Two categories of losses:

  low value but frequently occurring -- control procedures

  significant in value but rare -- capital provision, reinsurance

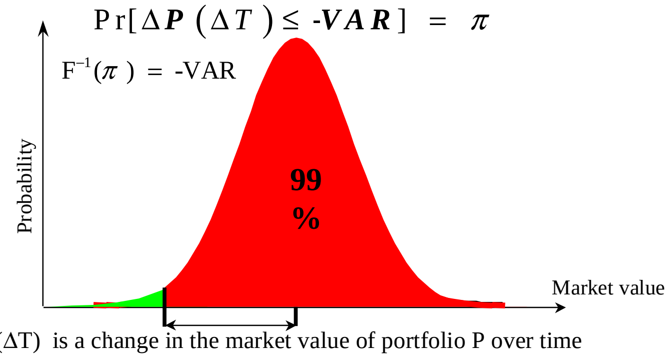
  Time horizon
- Integration with market and credit models

Market, Credit & Operational Risks

RiskMetrics *Value at Risk* (*mean*) $= W_0 \alpha \sigma \sqrt{t}$

- Value at Risk is precisely defined mathematically
- Under normal market conditions VaR provides a measure of market risk

$$\Pr[\Delta \boldsymbol{P}(\Delta T) \leq -\boldsymbol{VAR}] = \pi$$



Δ **P**(ΔT) is a change in the market value of portfolio P over time horizon ΔT with probability π.

Market, Credit & Operational Risks

- Under normal conditions (credit ratings higher than BBB) credit models provide a measure for credit risk. Credit risk requires a larger capital allocation than market [P. Jorion]

Choosing Equity Coverage from the Credit Rating

(Multiples of Annual Standard Deviation)

Desired Rating (Moody's)	1-Year Probability of Default %	Equity Coverage Normal	t(6)	t(4)
Aaa	0.009	3.75	9.26	15.96
Aa1	0.015	3.65	8.45	14.03
Aa2	0.022	3.51	7.89	12.72
Ba1	1.25	2.24	3.52	4.31
B1	6.14	1.54	2.30	2.58

Economic capital provision for operational risk

- Three types of required data for operational risk capital allocation:
  - key risk indicators
  - size of losses (severity)
  - frequency of losses
- Risk measures for a random loss are derived from statistics of profit and loss (P&L) distribution at different levels of a financial institution
- Preference for risk types

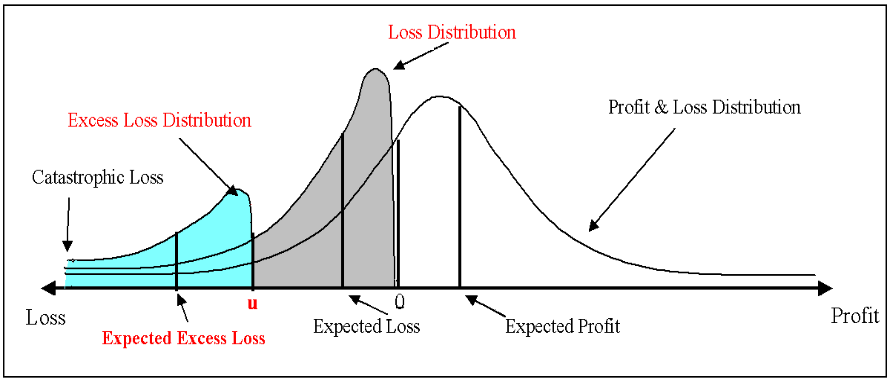
  The distribution of losses due to different risk types should be considered according to individual business preferences

→ Value at Risk due to factors other than market or credit exposures may be defined as a probabilistic statement

Operational risk measure

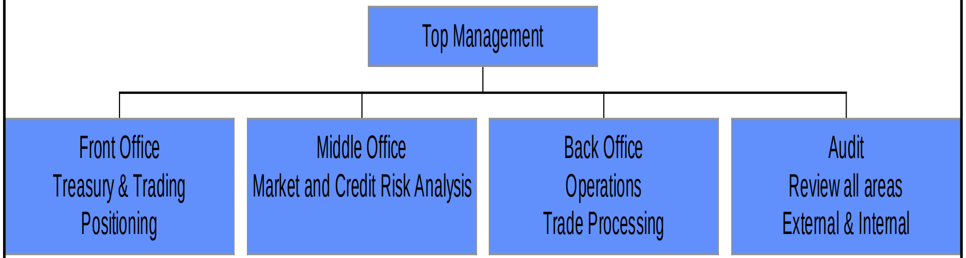
- For integrated P&L data for business unit consider the following thresholds for losses under normal market conditions:
  - the level of loss due to market risk with probability π -- VaR
  - the level of loss due to both credit and market risks with probability ρ -- TailVaR
- Further losses by definition belong to operational risk categories
- Operational risk must be measured as an excess over levels for market and credit risk

Business Unit



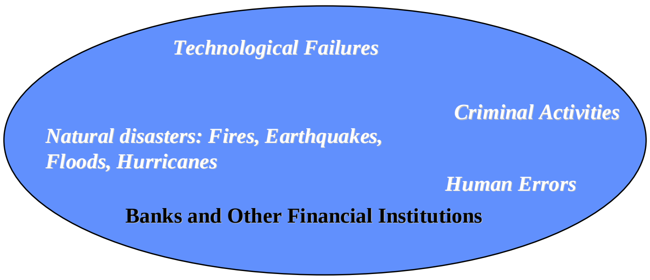
What do we need for aggregation of risk?

- Structural model of the business



- Models of processes contributing to operational risk
- Information flow control structure / model

Database Modeling: Public domain general losses over $1m



Empirical loss distributions for each risk type

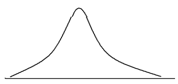
Pasting, scaling and adjusting to financial business units

Summing business unit distributions back to total loss distribution over period for bank

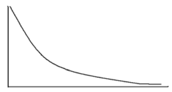
Data issues: quality of data, categorization, cross-reference, database design

Pasting, Scaling, Adjusting, Summing

- *VaR* and *Elementary Reliability Theory* use *Gaussian* (normal) severity and *Poisson* frequency (negative exponential inter-event) distributions

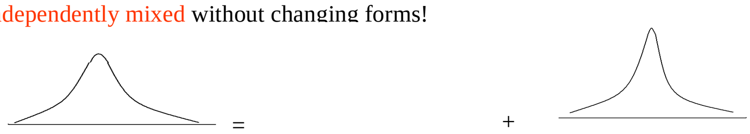


Loss



Time to next loss

- Why? Essentially only *Gaussian* and *Poisson* distributions can be sliced, diced and independently mixed without changing forms!



Mean μ
Variance σ^2

Mean μ_1
Variance σ_1^2

Mean μ_2
Variance σ_2^2

- Problem! Only high frequency / low severity losses are Gaussian.

  Many significant business unit operational losses will be highly correlated

Solution to Pasting, Scaling, Adjusting and Summing Hierarchical Units

- Use *Markov* process description and Monte Carlo simulation methods
- *State dependent loss event rate* captures correlation between individual business unit losses
- *Loss severity* distribution of any appropriate type may be utilized

Solution to forecasting of unexpected losses

Extreme Value Theory

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Applying Extreme Value Theory (EVT)

- By estimating accurately the extreme losses and their corresponding probabilities, one can manage extreme operational risks -and other types of- risk effectively
- Extreme quantiles and tail-probabilities can be estimated by fitting an extreme value statistical model to a set of extreme-event data

➢ Data availability, model applicability, time horizon for capital allocation,…?

LTCM [P. Jorion]:

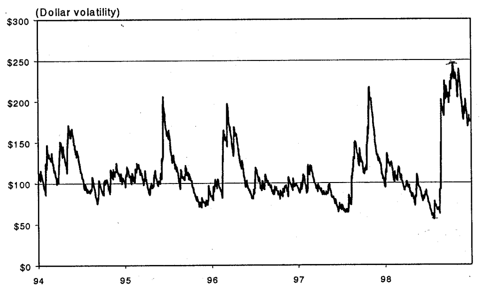
- LTCM claimed to be no more risky than an unleveraged investment of US equities with \$45 million as the target daily volatility.
- For the 1997th LTCM capital base of \$4.7 billion and 15% annual average volatility of the S&P 500 over 1978 to 1997 , a daily volatility is \$44 million:

$\$4700 \cdot 0.15 / \sqrt{252} = \$44$

- Assuming a normal distribution the daily 99% (the associated multiplier of 2.33) VaR is \$105 millions. Applying the Basle rules with 10 days period, this translates into a minimum capital level of $3 \cdot \$105 \cdot \sqrt{10} = \$993$m .

- With actual daily volatility around \$100 million, the Basle minimum capital would be \$2.2 billion. This is now closer to the actual loss of \$1.7 billion in August.

Simulated Daily Volatility of the Fund



The fund's volatility had wide variations from a low of $55 million in July 1998 to a high of $245 million in October 1998.

Source:P. Jorion- Risk Management Lessons from LTCM *UCLA Working Paper (1999)*

Market data for analysis of operational risk

- 'Black Monday'

The worst day for the S&P showed a 23% loss:
for normal distribution the probability of such event $p=6.7 \times 10^{-131}$,
or 24 standard deviation away from the mean

- LTCM

By August 31, the portfolio had lost $1.7billion in one month only.

For normal distribution and $45 million daily standard deviation (or $206m monthly) this translates into a 8.3 standard deviation event.

Such event would occur once every 800 trillion years!

Extreme Value Theory

- Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. random variables with common distribution function F.
- Define sample maxima:

  $M_n = \max(X_1, X_2, \ldots, X_n)$ for $n \geq 2$
- Define the distribution function of the maximum M_n:

  $P(M_n \leq x) = P(X_1 \leq x, \ldots, X_n \leq x) = F^n(x),\ x \in R,\ n \in N$

  $F^n(x) \rightarrow 0$ for any x such that $F(x) < 1$ as $n \rightarrow \infty$

- Under some assumptions as $n \rightarrow \infty$ the tail of the maximum determines the tail of the sum -- subexponential distributions

The GEV $(H_\xi)_{\xi \in R}$ describes the limit distributions of normalized maxima $\frac{M_n - d_n}{c_n} \xrightarrow{d} H$

$$F^n(c_n x + d_n) \approx H_\xi(x)$$

where $c_n > 0$ and $d_n \in R$ are the normalizing and centering constants

The Generalised Extreme Value Distribution (GEV)

- One-parameter representation of the three standard cases in one family of distribution functions $(H_\xi)_{\xi \in R}$

$$H_\xi = \begin{cases} \exp(-(1+\xi x)^{-1/\xi}) & \text{if } \xi \neq 0 \\ \exp(-\exp(-x)) & \text{if } \xi = 0 \end{cases} \quad \text{where } (1+\xi x) > 0$$

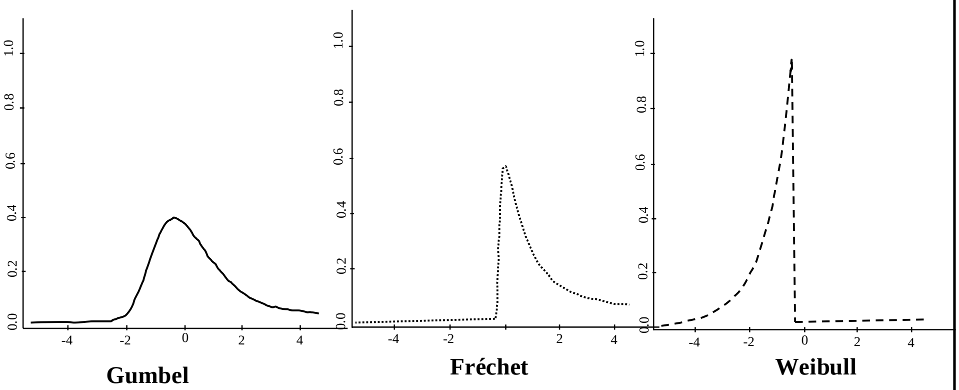
- x may be replaced by $(x-\mu)/\sigma$ to obtain a standard GEV
- Introducing a parameter ξ these can be represented as:

  $\xi = 0$: Gumbel distribution Λ

  $\xi = \alpha^{-1} > 0$: Fréchet distribution Φ_α

  $\xi = -\alpha^{-1} < 0$: Weibull distribution Ψ_α

Densities of the Standard Extreme Value Distributions



The Generalised Pareto distribution (GPD) -- the limit distribution of excesses *y =X-u* over high thresholds u

$$F_u(y) = P\ (X - u \le y \mid X > u) = \frac{F(y+u) - F(u)}{1 - F(u)} \quad \textit{for} \quad 0 \le y < x_F - u$$

For a large class of underlying distributions we can find a function β (u) such that

$$F_u(y) \approx G_{\xi,\beta}(y)$$

$$(GPD) \quad G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x / \beta)^{-1/\xi} & \xi \neq 0, \\ 1 - \exp(-x/\beta) & \xi = 0 \end{cases}$$

For given realizations of X, the GPD is fitted to the N excesses to obtain estimates $\hat{\xi}$ and $\hat{\beta}$ by choosing a sensible u

Peaks over Threshold Model (POT)

- The Peaks over Threshold (POT) model is suitable for the excesses $X \sim GPD$ with parameters $\xi<1$ and β
- The mean excess function for $x_F>u$

$$e(u) = E(X-u \mid X>u) = \frac{\beta+\xi u}{1-\xi}$$

- Using POT we model the number of exceedances over a threshold u and the exceedance times by a Poisson point process with intensity

$$\lambda_u = \left(1+\xi\,\frac{u-\mu}{\sigma}\right)^{-1/\xi}$$

- Excesses and exceedance times are independent of each other

Data requirement for operational risk analysis

- Accounting profit and loss over long fixed period
- Loss distribution of interest are the exceedances over threshold
- Defining the single value as a suitable threshold for unexpected losses

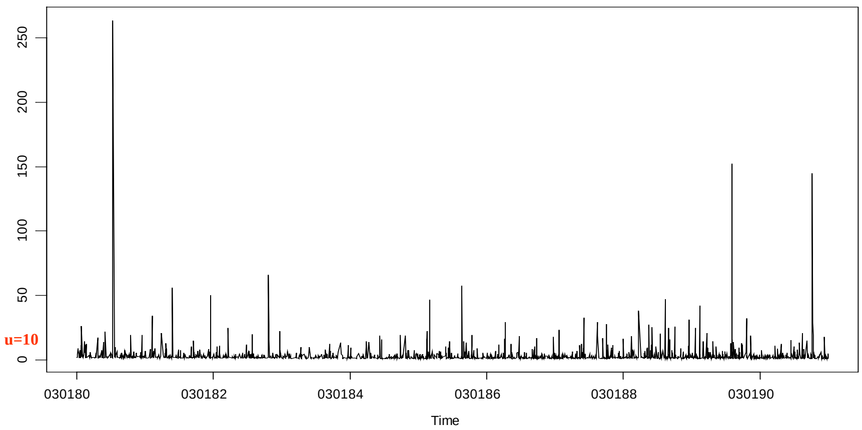
Actuarial practice: expected excess loss

Example
Danish Fire Insurance Claims in Danish Kroner (m)
(S. Resnick, P. Embrechts *et all*; Alexander McNeil; Richard Smith)

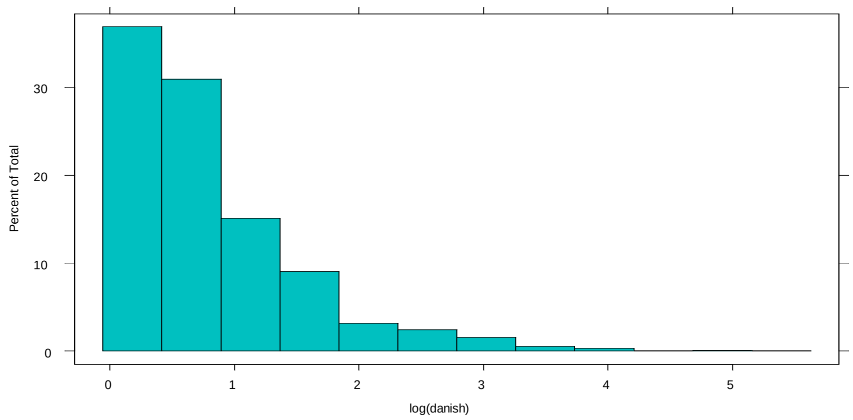
Summary Statistics:

- 2167 data points from 03/01/1980 to 31/12/1990 11 years
- min 1
- max 263.25
- median 1.778
- mean 3.385
- standard deviation 8.507

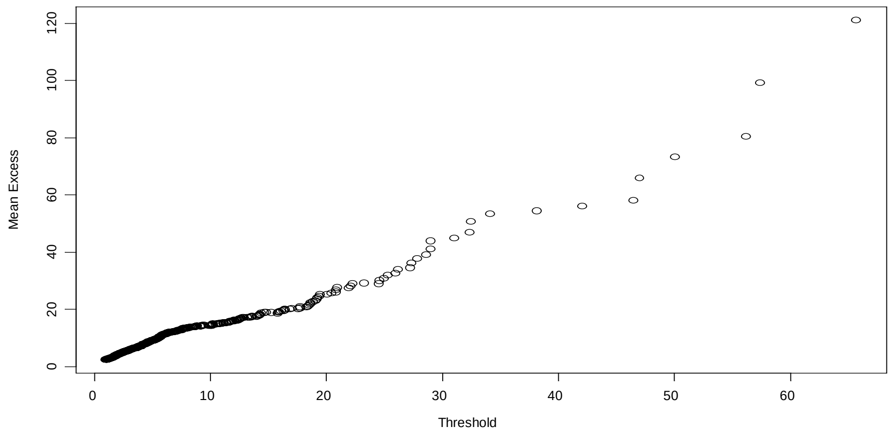
Time-series plot of Danish Fire Insurance Data



Histogram of log (data)



Expected Excess



Peaks Over Threshold (POT)

- Using POT model the expected number of exceedances over a threshold u is modelled as an intensity of a Poisson point process
- In the example:

  The average number of fire insurance claims higher than 10 is 0.027 per day

  (Poisson parameter λ=0.027)

  A claim happens on average every 37 days

  $(\tau = 1/\lambda = 37)$

- Using the threshold u=10 and the corresponding *Generalized Pareto Distribution* estimated parameters calculate the (conditional) expected excess

  $$E(X\text{-}u|\ X>u) = 24$$

- Total capital required to support fire insurance risk per event is 10 + 24 = 34

  and excess capital per annum is

  24 x 0.027 x 365 = 56.76

Capital Allocation for Operational Risk

- Set the capital allocated for market and credit risks as a threshold
- Estimate the expected excess i.e the extra capital required to support operational risk
- Calculate the total capital to support business unit and the firm-wide risk

Problems

- very limited data
- relations between risk types, relations between different business units

Bank Trading Losses Analysis in the Period of the Russian Crisis

- Statistical analysis on the aggregated loss data time-series
- Analyze losses from the four trading desks separately

Objectives:

- Break down the risks involved in four financial instruments and compare them with the overall assessed risk of the aggregated loss data
- Split time-series data prior to the extreme event(s) and predict the forthcoming losses using the EV distribution fitted in the first part

Aggregated Loss Data - Statistics

- **Min:** 0.002 **Max:** 2942.028
- **Mean:** 247.203 **Median:** 120.002 **Std Dev.:**380.069

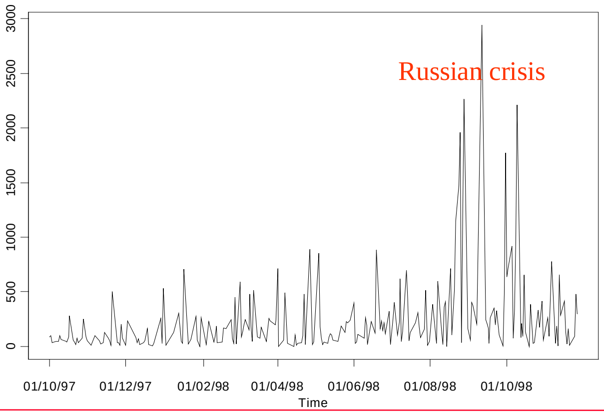
- **1st Qu.:** 42.799
- **3rd Qu.:** 265.575
- **Total N:** 237.000 **Total Loss Size:** 58600

timespan: 421 days or 1.15 yrs

start end

01/10/97 ... 26/11/98

Time Series plot of Aggregated Loss data



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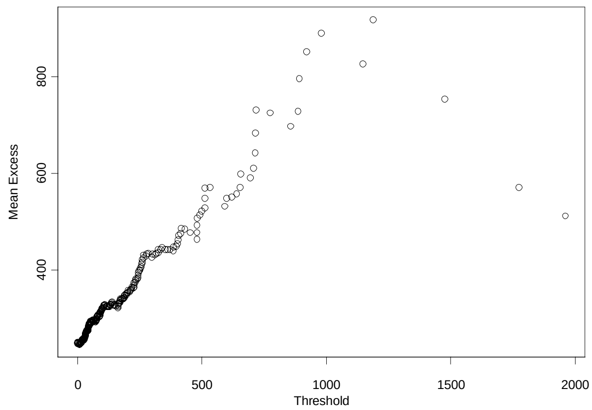
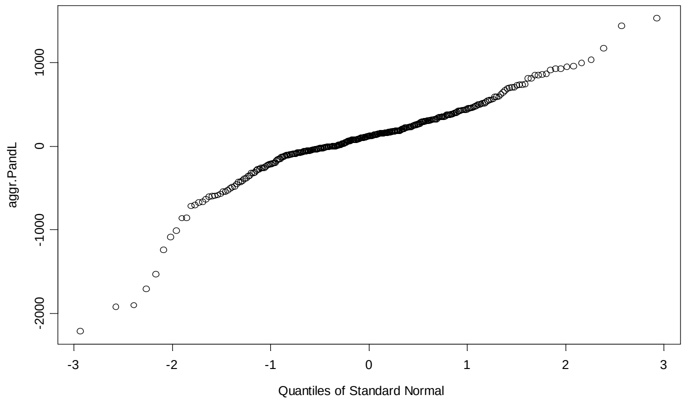
Mean excess plot of aggregated loss data



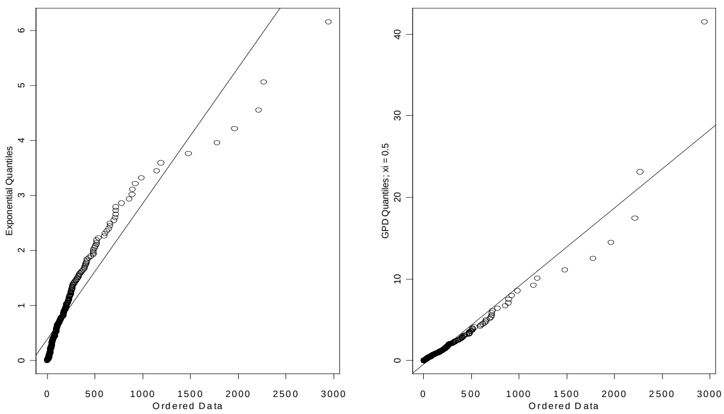
Q-Q(Normal) plot of aggregated Profit & Loss



- Bent-down left part indicates that losses have heavier tail than the Normal

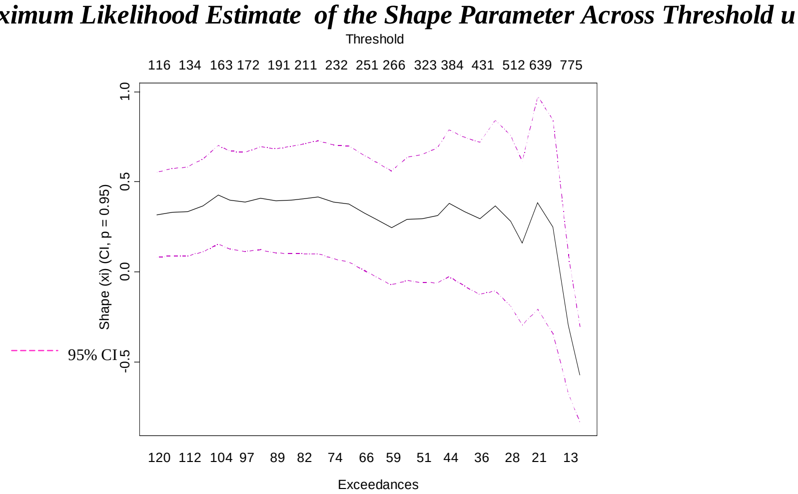
Q-Q (GPD) plot of aggregated data
shape parameter range:

$\xi=0$ (exp) $\xi=0.5$



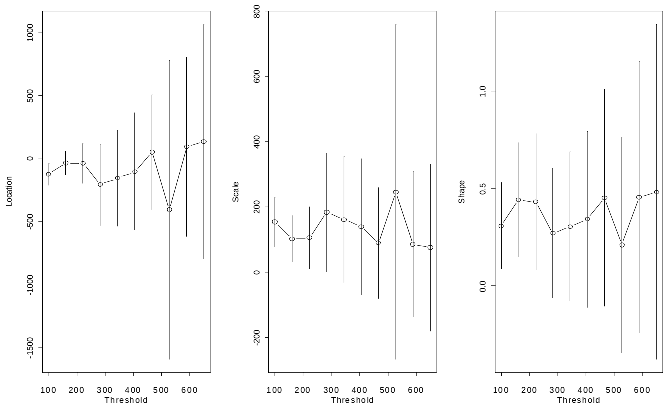
Optimal choice of threshold u

Maximum Likelihood Estimate of the Shape Parameter Across Threshold u



- Choose threshold u over a stable MLE range

POT Model: location (μ), scale (σ) and shape (ξ) MLE's across threshold u



Statistical Toolkit

- Maximum Likelihood numerical procedures for parameters estimation of POT model

  -- only appropriate when number of exceedances sufficiently large (~100)
- Monte Carlo Markov Chain Simulation
- Bayesian hierarchical modeling for parameter estimation of GPD distributions

MCMC Bayesian Hierarchical Model

- Generate a Markov chain of EV parameters whose stationary distribution is the posterior distribution of interest (R. Smith, 1999)
  - Gibbs Sampler and Metropolis-Hastings Algorithm
  - Gibbs sampler is used in Bayesian setting
- Take the Markov chain output to represent a sample drawn from that posterior distribution
- Use Monte Carlo integration to approximate the population mean by the sample mean
- Inference or prediction for individual risk type parameters is made via the parameters of aggregated data -- the successive conditioning of Bayesian modeling

Results of analysis of aggregated loss data

Running the **MCMC simulator** on the **full** sample:

meanxi:0.5224937 medianxi:0.518592

meanbeta:201.2993 medianbeta:196.448

meanintensity:0.2057822
meanintensity per year:75

meanexp.excess:734.397
medianexp.excess:621.6149

No.of exceedances:86

Excess capital required to support bank
734.39x75 =55079.25

Predicting the crisis

- Take three 'event' dates 17th Aug '98, 21st Aug '98 and 28th Aug '98 -- two before and one after GKO default date

- Using the EV parameters estimated from data prior to these event dates we predict the next 'big' event(s) and compare their size with the second half of the data. The analysis is based on fixed threshold u=200

Data Split date: (a) 17th August 1998 one week before GKO default

Summary Statistics

- Observations: 174

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.00204	38.67	90.53	166	227.2	891.3

- Span: start end
  01/10/97 - 17/08/98

POT Model Results

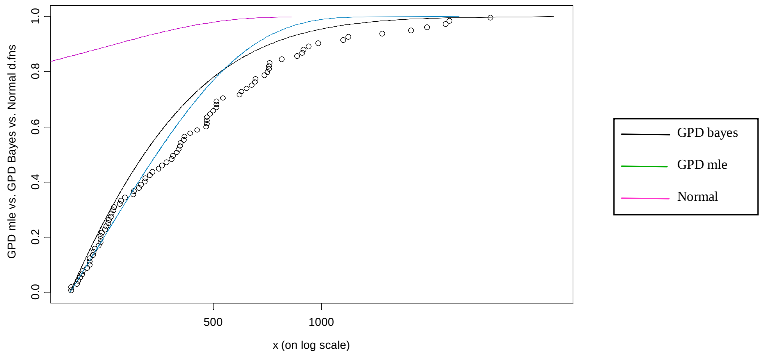
Expected excess 446.0

Average number of losses per year exceeding 200 approximately 60

Six days on average between excessive losses

Risk capital 25,635

GPD Severity (ML and Bayesian estimates) vs. Normal



GPD tail fit looks much better using the Bayesian estimates
Normal distribution does not capture far-end tail losses which are material for risk management purposes

Data Split date: (b) 21st August 1998
3 days before GKO default

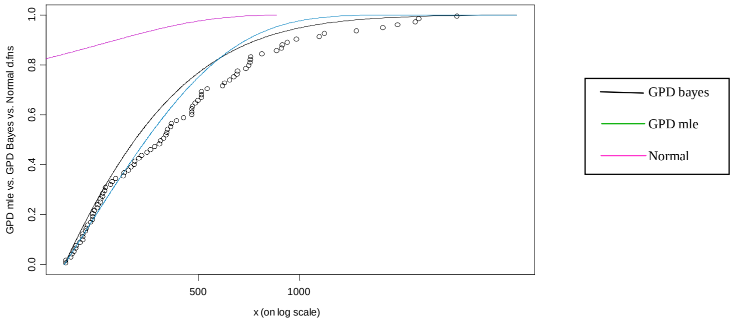
Summary Statistics

- Observations: 178

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.00204	40.55	95.06	173.9	230.5	1148

- Span: start end
  01/10/97 - 21/08/98

GPD Severity (ML and Bayesian estimates) vs. Normal



•Better GPD fit at the tail, particularly with the Bayesian approach

Data Split date: (c) 28th August 1998
4 days after GKO default

Summary Statistics

- Observations: 183

Min.	1st Qu.	Median	Mean	3rd Qu	Max
0.00204	40.66	97.35	207	237.5	2266

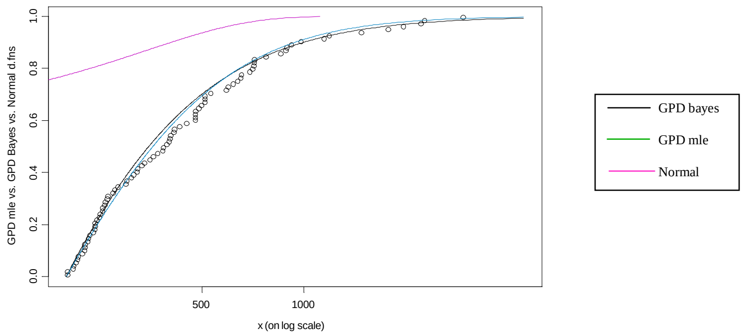
- Span: start end
  01/10/97 - 21/08

POT Model Results

Expected excess 844.43 on average every 5.71 days, or 63events per year

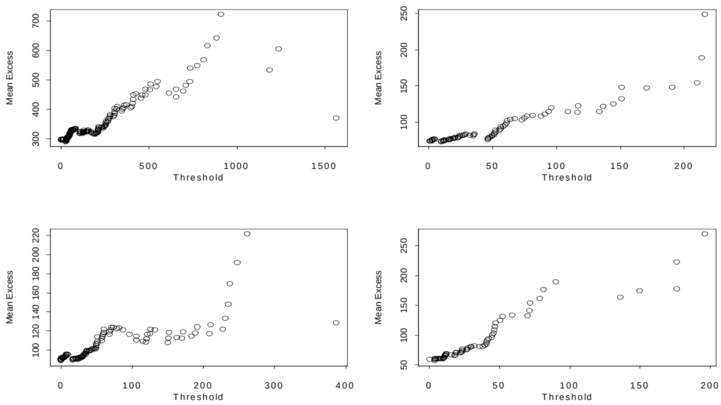
Risk capital : 54178.45

GPD (based on ML and Bayesian estimates) vs. Normal



- ML and Bayesian estimates yield comparable results, excellent tail fit

Breaking the loss data into the four types



Mean excess plot of the first sub-sample show a loss distribution more heavily- tailed (i.e. riskier) than the rest which do not have such long tails

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Firm-level	ξ	β	Quantile .9	Quantile .95	Quantile .99	Expected Excess	Expected number
MLE	.4	219.8	570.7	864.7	1961.9	499.7	75
Post Med	.5	196.4	564.8	888.1	2268.5	621.6	75
Trading desks							
One	.34	205.2	632.7	919.9	1901.5	365.9	72
Two	.25	108.1	188.6	282.0	576.4	190.4	11
Three	.24	118.6	233.2	343.2	693.1	206.5	19
Four	.26	106.1	147.4	231.1	502.3	192.8	7

Loss Estimates

	95% GPD Quantile	Risk Capital
Firm-level	888.1	46620
Trading desks		
One	919.9	26344
Two	282.0	2086
Three	343.2	3857
Four	231.1	1407
		34494

Conclusions

- In total risk measurement framework threshold u would be greater than the capital allocated for market and credit risks under normal market conditions

- expected excess loss would be the extra capital required to support large operational losses

- Risk capital allocation

$$\textit{Expected Excess} \cdot \hat{\Lambda} + \mathrm{VaR}$$

where $\hat{\Lambda}$ is the estimated expected number of events per annum and VaR is an appropriate total p.a. value at risk

Conclusions

- Using expected excess loss allows aggregation of the business lines to obtain the risk capital at the firm-level
- Due to small samples breaking down the risks can only be achieved via a Bayesian approach where the posteriors are computed using MCMC simulation procedures
- Decentralization of Risk Management allows one to identify the 'real' risks and more efficient capital allocation, but
- Total capital estimated for the four trading desks is less than that calculated at the firm level due to dependent tail events -- super not subadditivity of capital allocation - because of losses only

Extreme Operational Risk System Implementation Issues

- P&L data, volatility of returns and other factors should be constantly analyzed for identification of extremes
- Losses similar to LTCM belongs to category of *extreme operational* and could be prevented through *control*
- Availability of statistical software, e.g. S-plus, should be utilized for operational risk control
- Larger sample sizes are needed for operational risk than for market or credit risk, but smaller are still useful