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Dynamic Learning, Herding and Guru Effects in Networks

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Outline

- 1. Motivation
- 2. The model
- 3. Results
- 4. Conclusion

- Simulations

1. Motivation

- Herding arises from :

(i) Directly copying others – presupposes a communication channel between agents
Who are we connected to ? Who influences us ?

(ii) Spontaneous convergence of beliefs based on publicly available information (no communication network structure)

- Herding behaviour in stock markets meant to cause positive and negative bubbles ie. deviations from the fundamental value

Size of the fat tail index is a function of 'herding' , Cont and Bouchaud (2000)
Periodic collapses and reversion to the mean done by anti-herd 'fundamentalists' (Lux; Kirman; bubbles and extreme events from endogenously forming herd behaviour in markets)

- Herding from mimetic behaviour or herding for other reasons reflects bounded rationality

Herding and Bounded Rationality

- Behavioural models of herding Bandwagons; fads etc (Bikchandani, Hirschleifer, Welch (1992,1998); Sharfstein and Stein(1990) ; Optimism and pessimism Thaler (1992,1993)
- *Why is there anything less than perfect rational expectations of stock prices ?*
- Classic conundrum of no trade

Lack of a unique effective decision procedure :endogenous to the logic of the decision problem

- Spear (1987) and Markose (2004, 2007)
  Non-computability of rational expectations equilibrium : RE as fixed point of market price function, g

*In a rational expectations equilibrium (**REE**) there exists some computable forecast function :*

f ^= fa *such that* fg(a) ≅ fa , (i)

Then a is a fixed point of the market price function g. Note, a is the algorithm or program that computes the output of the market game when the price function g that determines the outcome is consistent with agents' prediction functions for Pt+1.

An agent has to find a meta forecast rule *f ^= fa* that satisfies (I) .

That is, the agent has to identify a proper subset of the set of all partial computable functions { f0, f1, f2, ...}, such that only the fixed points of the total computable function g are identified, viz.

{ m | fg(m) = fm }. (II)

By Rice's Theorem no uniform recursive/ algorithmic procedure to identify set of indices in (ii). There is no systematic way of forming REs of the market price function g.

Only inductive trial and error learning that begins search in an arbitrary subset of diverse forecast rules.

Canonical Example of Self-Reflexive Systems and Contrarian Structures which have no computable fixed points

- First example developed by Santa Fe Institute is the Artificial Stock Market (ASM)
- Brian Arthur gave a powerful rebuttal of why traditional economic analysis will fail to understand stock markets and why ACE modelling is needed
- In stock market an investor makes money if he/she can sell when everybody else is buying and buy when everybody else is selling. In other words, one needs to be in the minority or contrarian
- Arthur called this the El Farol Bar problem. You want to go to the pub when it is not crowded. Assume everybody else wants to do the same. How can you rationally decide/strategize to succeed in this objective of being in the minority ?
- If all of us have the same forecasting model to work out how many people will turn up – say our model says it will be 80% full – then as all of us do not want to be there when it is crowded – none of us will go.
- This contradicts the prediction of our model and in fact we should go. If all reasoned this way – once again we will fail etc. So there is no **Homogenous Rational Expectations** and no rational way in which we can decide to go. Traditional economics cannot deal with this
- Hence, Brian Arthur said we must use ACE models and see how the system dynamically self-organizes

- Peyton Young (with Dean Foster, 2003) has now introduced the notion of *radical decoupling* . Unlike, traditional games where agents know the rules of the game, here and in most real world situations, one can learn to win only by having the ‘right’ connections or advisors. One knows only one’s own realized payoffs. Games which have winning strategies – but no effective procedures – hence you copy those who are successful
- Local interaction, communication and learning from others

- Starting from a random graph we study how <u>star formations</u> can take place by dynamically updating the links. This type of study would be very difficult to carry out with traditional economic models. Kirman (1997), Kirman and Vignes (1991) suggest dynamic link formation: reinforced by good experience and broken by bad ones.

Simple Stock Market Model With Agents Relying on Investment 'Tips' From Others

- http://privatewww.essex.ac.uk/~aalent/herding/herding.htm

- Agents have to buy or sell one unit of an asset; they take advise from their neighbours; they act on the basis of the majority view amongst their neighbours;
- Neighbours who give bad advise are eventually cut off and new advisers are found

- All agents are identical except for how far back they can remember;

Some have zero memory and they give random advise; others with memory give the average trend of the market

Who will give best advise in a minority winning structure ?

Eventually what does the communication network look like?

- Hence, paper is called Dynamic Learning, Herding and Guru Effects

Godel Centennial: Logical Approaches to Computational Barriers (CIE,2006)

Gödelian Foundations of Non-Computability and Heterogeneity In Economic Forecasting and Strategic Innovation (Markose, 2006)

Features of Herding Simulator

- The aim is to model a network that has the properties of a <u>real world network</u>

The main feature of real world networks is:

- High clustering coefficient (Internet example)
- Star formations

- The paper contrasts clustering which represents the network topology of the underlying communication network with herding which represents aggregate behaviour with regard to a binary decision problem.

Properties of Networks

Diagonal Elements Characterize Small World Networks

Watts and Strogatz (1998), Watts (2002)

Properties / Networks	Clustering Coefficient	Average Path Length	Degree Distribution
Regular	*High*	High	Equal and fixed In-degrees to each node
Random	Low	*Low*	Exponential/ Poisson
Scale Free/Power Law	Low	Variable	*Fat Tail Distribution*

2. The model

- A number of agents N are initially placed on the nodes of a random graph. Probability of a link between i,j is p.
- The links between agents i and j are directed and have an weight $w_{i,j}$, which represents the strength of the advice that agent i will take from agent j.
- The set of agent i's neighbours is denoted by $\Xi_{i,}$ and contains all out-links from i to j.
- Each agent i is assigned a memory value M_i from a uniform distribution on $[0, M_{max}]$.

The game

- The agents participate in a market
- At each time period *t*, the agents have to decide whether to buy or sell one unit of an asset.
- There are 3 reward schemes:
  - 1. Random rewards Krause (2003/4)
  - 2.Reward scheme: Minority Game (causes endogenous volatility)
    - If there are more buyers than sellers, sellers win
    - If there are more sellers than buyers, buyers win

    -3 Majority Game causes one way markets

The Decision Rule

- Step 1: Individual forecast
  - Each agent calculates its own forecast for the next period based on its own past
- Step 2: Decision
  - The decision of an agent $r_{t,i}$ is based on a weighted sum of forecasts that its neighbours give it and its own

Step 1: Individual forecast

- Each agent *i* calculates a forecast $f_{i,t+1}$ for the next period t+1 based on its own past M_i number of decisions and outcomes as follows:

$$f_{i,t+1} = \sum_{\tau=0}^{M_i} \lambda^{\tau} \cdot u\left(r_{t-\tau,i}\right) \cdot r_{t-\tau,i}$$

The forecast $f_{i,t+1}$ can take a value in the range [-1,+1],
where $f_{i,t+1} > 0$ recommendation to buy,
$f_{i,t+1} < 0$ recommendation to sell, and
$f_{i,t+1} = 0$ random recommendation.

Step 2: Decision

- The decision of an agent $r_{t,i}$ is based on a weighted sum of forecasts that its neighbours give it, based on their own memory and past experience.

$$r_{t,i} = \begin{cases} +1 \quad (buy) & if \quad \sum_{j \in \Xi} f_{t+1,j} \cdot w_{i,j} > 0 \\ -1 \quad (sell) & if \quad \sum_{j \in \Xi} f_{t+1,j} \cdot w_{i,j} < 0 \end{cases}$$

- Zero-memory agents give advice based on random basis.

Dynamic Updating of Links

- The weights w_{ij} to the neighbours who give correct advise are reinforced by a rate of increment R_i^+, up to a maximum threshold Γ_{max}
- And weights to neighbours who give incorrect advise are reduced by a rate of reduction R_r^-
- There is a Minimum threshold Γ_{min}, after which the agent breaks the link to the neighbour, and randomly selects another agent in the network to take advice from.

Some Graph Theoretic Measures

- Degree of a node: is the number of first order neighbours. In our context, the degree of a agent is the number agents that are taking advice from it.

- Degree distribution: the distribution of the degrees for all agents in the network.

Clustering coefficient

- Clustering coefficient: average probability that two neighbours of a given node (agent) are also neighbours of one another. The clustering coefficient Ci for agent i is given by:

$$C_i = \frac{E_i}{k_i(k_i - 1)} \qquad E_i = \sum_{j \in \Xi_i} \sum_{m \in \Xi_i} a_{jm}{}^{1}$$

- The clustering coefficient of the network as a whole is the average of all Ci's and is given by

$$C = \frac{\sum_{i=1}^{N} C_i}{N} \qquad ; C_{rand} = p$$

Herding coefficient

The herding phenomenon in both classes of experiments is captured at each t by a time varying simple herding function $\frac{N_{bt}}{N} \in [0, 1]$. Here, N_{bt} is the number of agents who have bought at time t and N is the total number of agents.

The average measure of herding in the system over the length of time T which is irrespective of the direction of herding is given by a herding coefficient

$$\sigma = 2/N\sqrt{\left(\frac{1}{T}\sum_{t=1}^{T}\left(N_{bt}-\frac{N}{2}\right)^2\right)}\ ,\ \sigma \in (0, 1).$$

3. Results

Summary of Results: For different reward functions (*With and Without memory; Static and Dynamic Links* ;R^-=-0.4 , R^+=0.2) ; λ = 0.9; N=100; T=1000);

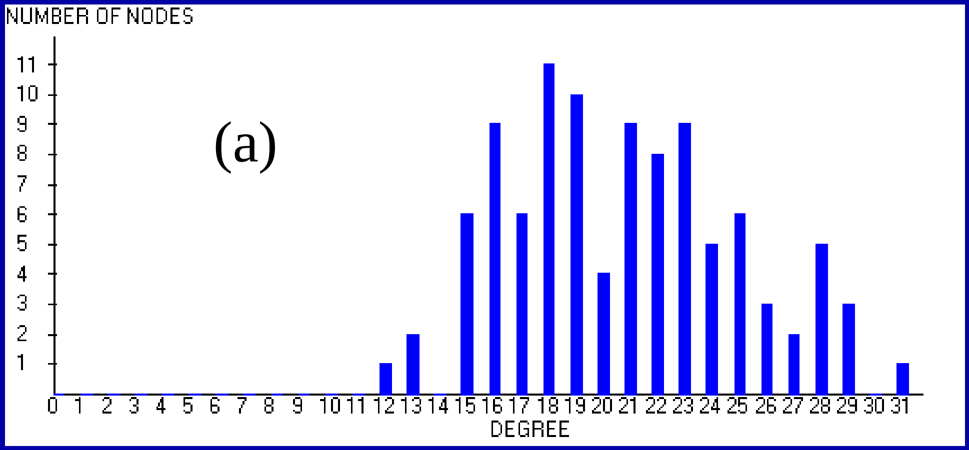
SIMULATION PARAMETERS				**RESULTS**			
Rewards function	**Memory Range**	**Erdös-Rényi *P***	**Weights**	**Herding coefficient σ**	**Clustering Coefficient C**	**No. Gurus**	ℓ
Random Rewards	[0]	0	-	0.09	-	0	N.A
		0.2	Static	0.34	0.2	0	≈1.5
	[0,10]	0	-	0.74	-	0	N.A
		0.2	Static	0.93	0.2	0	≈1.5
		0.2	$R_r < R_i$	0.93	0.2	0	≈1.5
		0.2	$R_r > R_i$	0.93	0.2	0	≈1.5
Minority Game	[0]	0	-	0.09	-	0	N.A
		0.2	Static	0.34	0.2	0	≈1.5
	[0,10]	0	-	0.70	-	0	N.A
		0.2	Static	0.97	0.2	0	≈1.5
		0.2	$\|R^-\| < \|R^+\|$	0.96	0.21	0	≈1.5
		0.2	$\|R^-\| > \|R^+\|$	0.91	0.57	10 (M=0)	≈1.25
		0.1	$\|R^-\| > \|R^+\|$	0.66	0.84	10 (M=0)	≈1

Note: ℓ_{rand} = 1.5 for p=0.2 and ℓ_{rand} =2.09 for p= 0.1

Highly connected agents

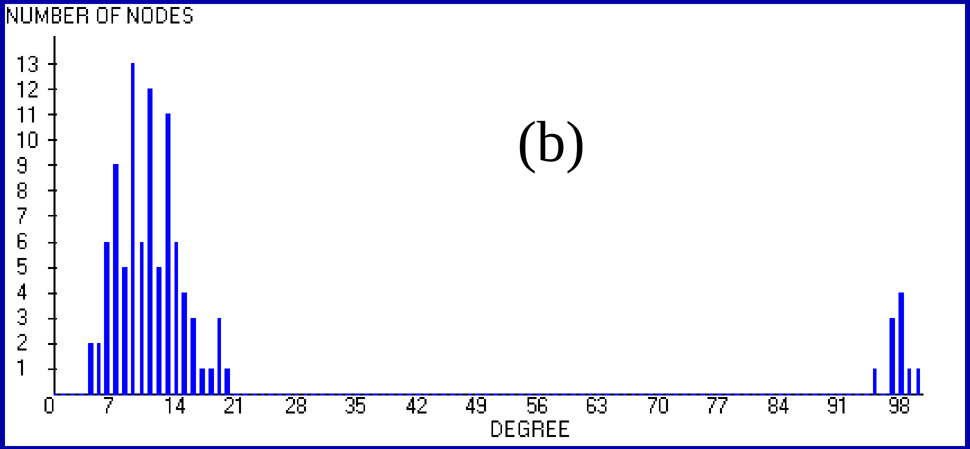
- We find that agents with <u>zero-memory</u> become highly connected.
- Why? Because playing the Minority game in isolation, zero-memory agents perform best, while other agents become trend-followers.
- These highly connected nodes can be seen as "gurus":
  - Many agents take advice from them

Degree distributions

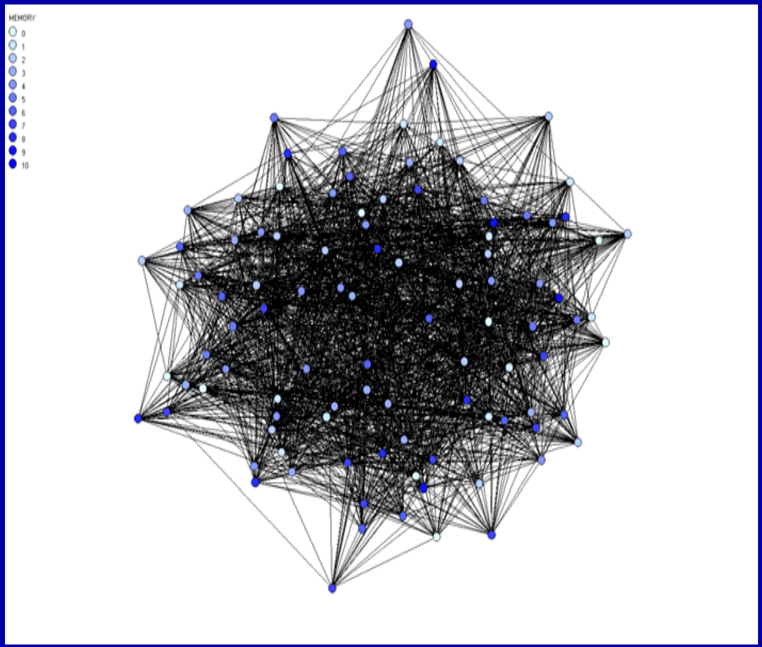


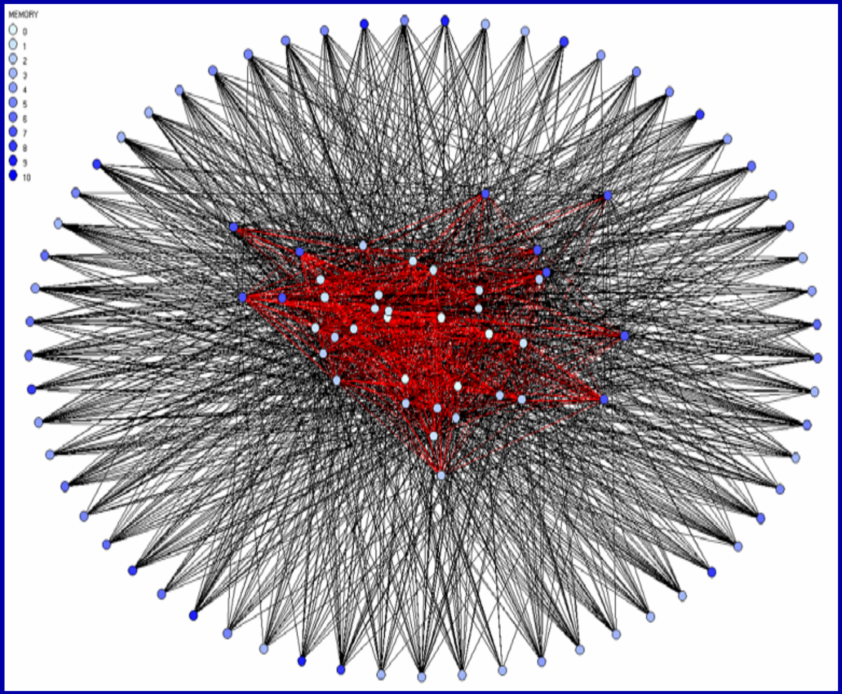
Degree distribution of the initial random network

Degree distribution of the network after the dynamic updating of links



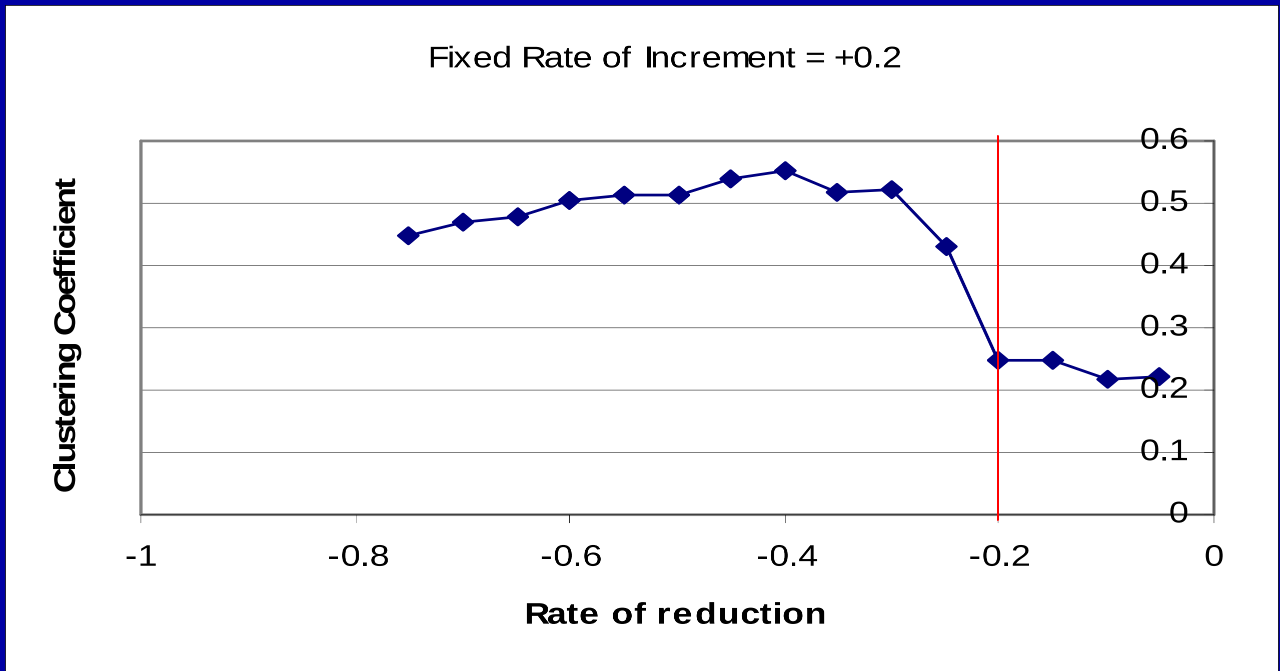
A graphical representation



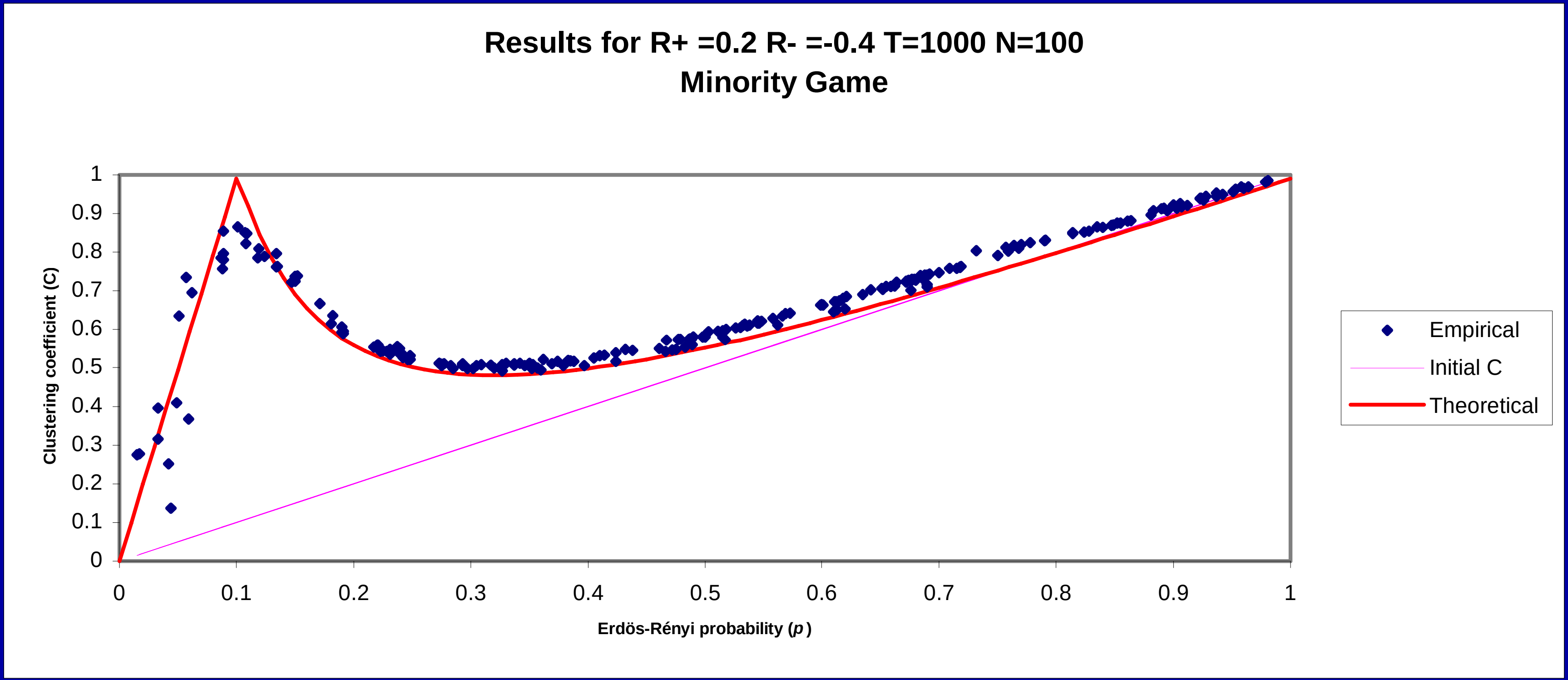


Rates of adjustment

- We find that a necessary condition for the agents to find the "gurus" is that: $R_r > R_i$
- But too much inertia (R_r >>) cause instability



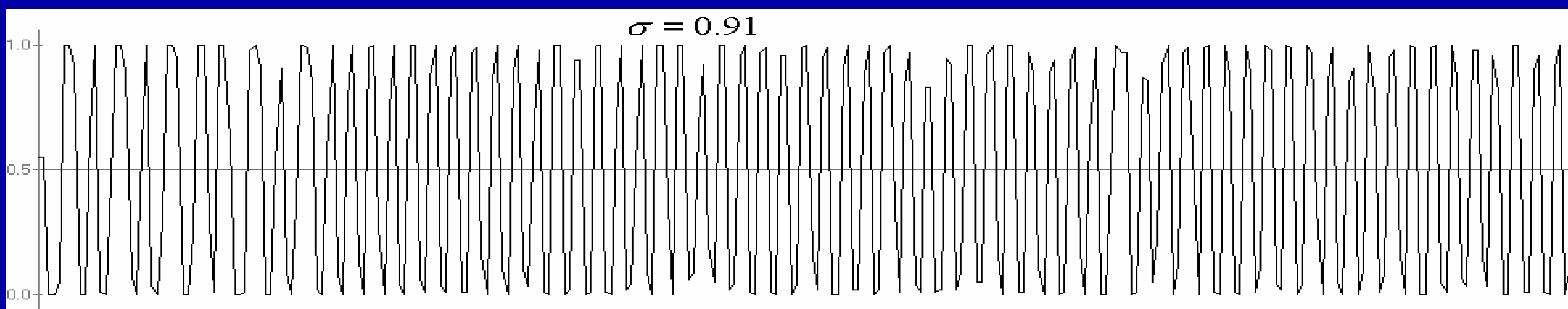
Maximum impact of gurus on clustering



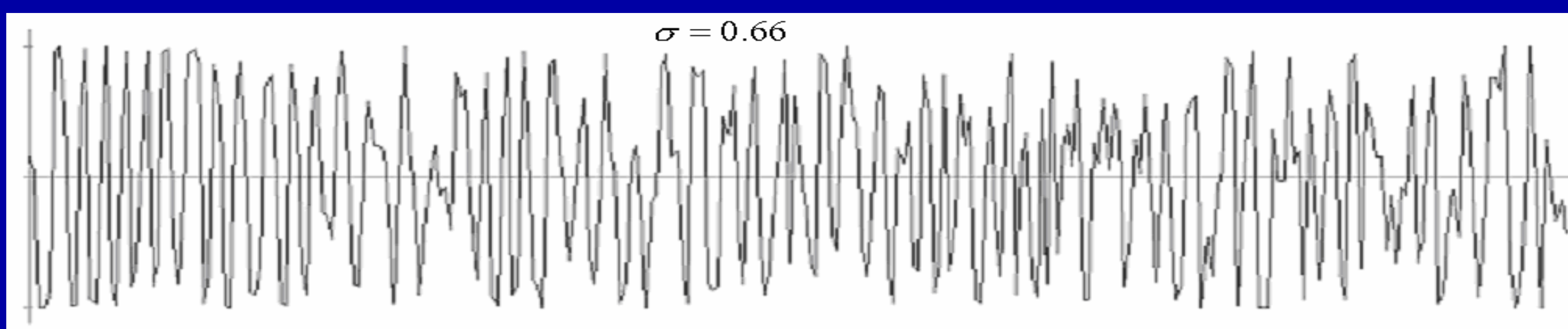
Clustering coefficient vs. p for empirical and theoretical results

Influence of gurus on herding

Dynamic Learning in Minority Game : Herding With Clustering C= 0.57
(p= 0.2; R- =-0.4, R+ =0.2 ;T= 1000)



Dynamic Learning in Minority Game : Herding With Clustering C= 0.84
(p= 0.1; R- =-0.4, R+ =0.2 ;T= 1000)



4. Conclusion

- Agents discover the gurus in the system, by simple adaptive threshold behaviour and random sampling.
- The dynamic process of link formation produces the star/hub formations in the network topology often found in real world networks.
- When updating the links, the rate of reduction has to be greater than the rate of increment.
- We succeed in producing small world network properties of $C > C_{rand}$ and shorter average path length than random graphs.