Volatility-Induced Financial Growth:
Modelling FX Market Makers

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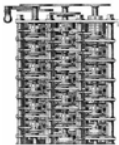**University of Cambridge**
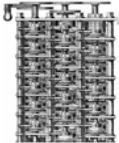
&

Cambridge Systems Associates Limited

Co-workers: **R G Bates & V Leemans**

Outline

1 Introduction

- Increasing evidence that markets are predictable
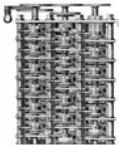 - Lo & McKinley (2000) state that rather than being a symptom of inefficiency predictability in the financial markets is the "oil that lubricates the gears of capitalism"

- Most technical traders are active in the FX markets and at high frequency
  - Daily *vs* high frequency Neeley (1999)
  - Equities *vs* FX Taylor & Allen (1992)
  - Asset allocation *vs* trading Dempster & Jones (2001)

Literature Review

- Macroeconomic fundamentals based models of FX timeseries do not fit empirical evidence at horizons of less than one year Meese & Rogoff (1983)

- Increasing interest in *microstructure* based approaches Lyons (2001)

- Published work on orders and transaction flows in equity markets Gabaix et al (2003)   Farmer & Lillo (2003)

- Much less published for FX due to lack of data Bates et al (2003)
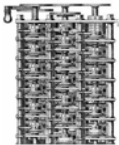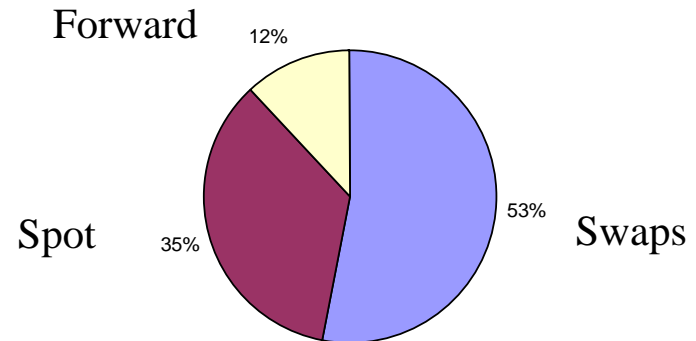
2   The Global FX Market Turnover 2004

Latest Bank for International Settlements FX market survey was conducted in April and June 2004

Average daily FX market turnover $1.9 trillion

Now estimated at $2.5 trillion

Swaps 53%, Spot 35%, Outright Forward 12%

Spot daily turnover about $620 billion


Forward 12%

Spot 35%

Swaps 53%

FX Market Turnover by Currency

EuroDollar easily the most traded currency pair with 28% of global turnover
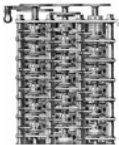
DollarYen next with 17%

SterlingDollar was 14%

Further 5% of turnover in the crosses between these currencies - mainly the EuroYen and SterlingEuro crosses

All other currencies and their crosses together accounted for about a third of global FX turnover
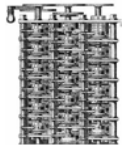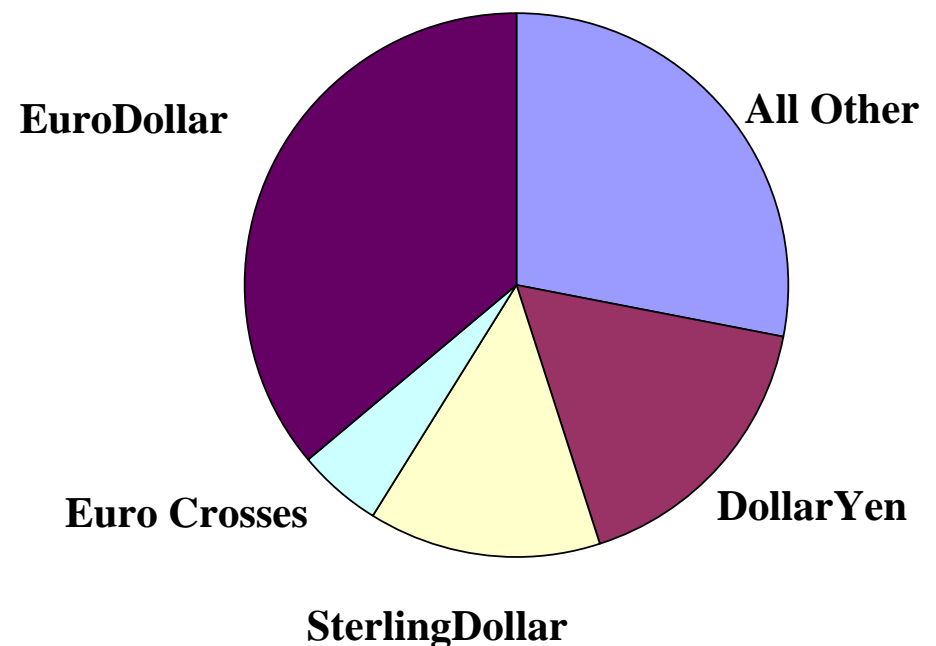
FX Market Concentration

- Global FX trading is highly concentrated

- In 2001 there were almost 2000 institutions active in the FX market

- But just 30 of those accounted for 35% of global turnover

- In the three years to 2005 this concentration into fewer banks continued until 20 banks accounted for 40% of turnover

- In 2005 the top 7 banks accounted for 60% of turnover of which Deutsche and UBS alone accounted for over 29% between them

Inter-Bank FX Market

- EBS and Reuters (D-2000/3000) are the interbank trading systems

- In 2001 they accounted for just under 40% of spot interbank trading. In 2004 they accounted for over 90% of all interbank FX

- Over 97% in the three major currency pairs: EuroDollar DollarYen and SterlingDollar

- About 70% of spot FX turnover is interdealer and 30% is customer trades -- in total currently about $ 800 billion per day of which $ 120 billion is carried by EBS

- Interdealer turnover has fallen as EBS and Reuters improve efficiency of interbank trading
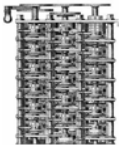
FX Market Structure

- General customers can interact with the global FX market only through market participants: the FX market makers

- Customers deal with more than one market maker

- Market makers deal with each other to clear excess inventory

- Proprietary traders are usually within banks - and so deal primarily with their own bank's market maker (often at reduced spreads) - or directly with EBS/Reuters
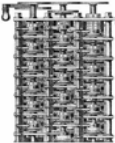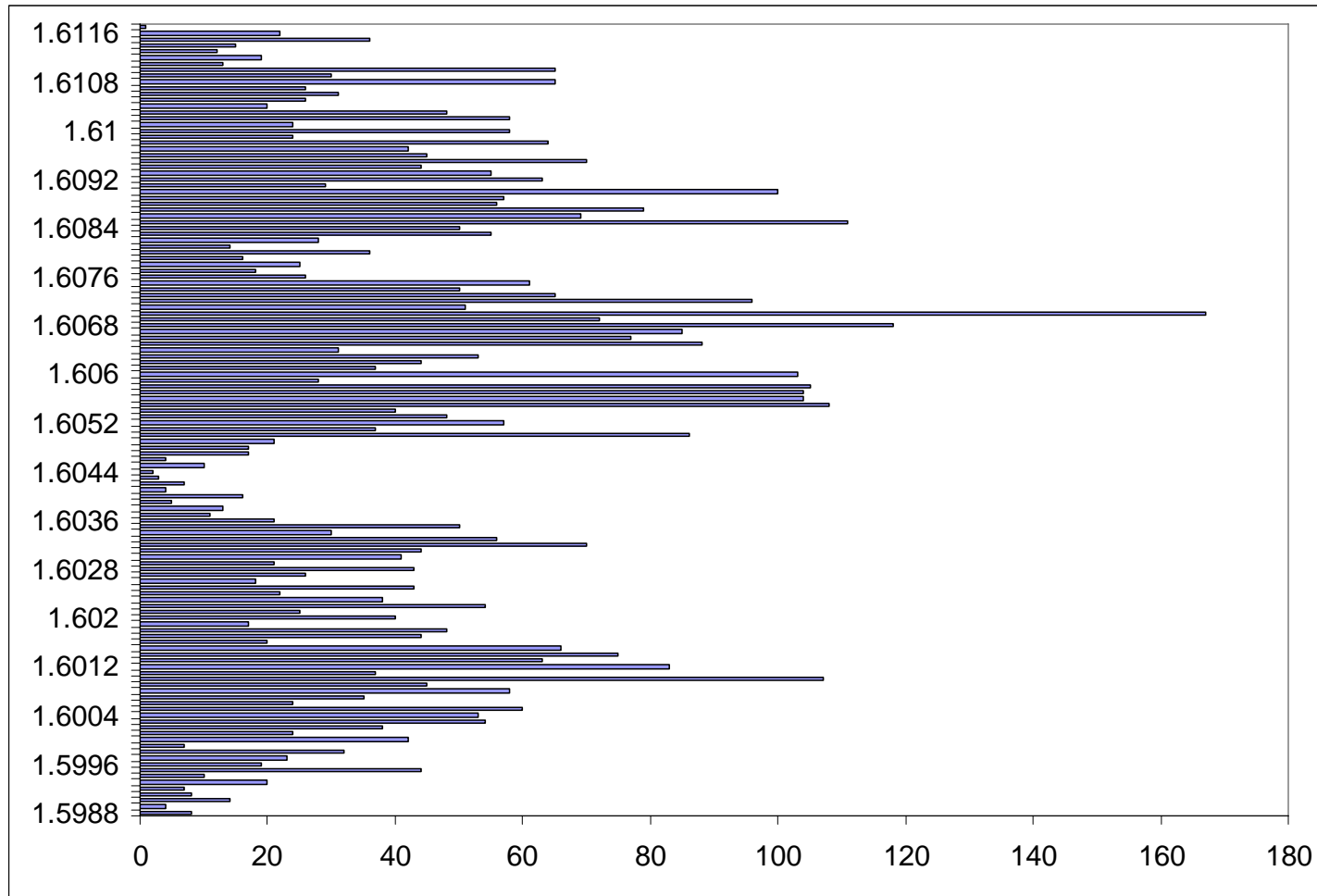
- Also deal with other banks' market makers
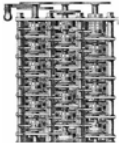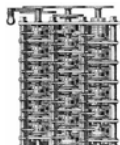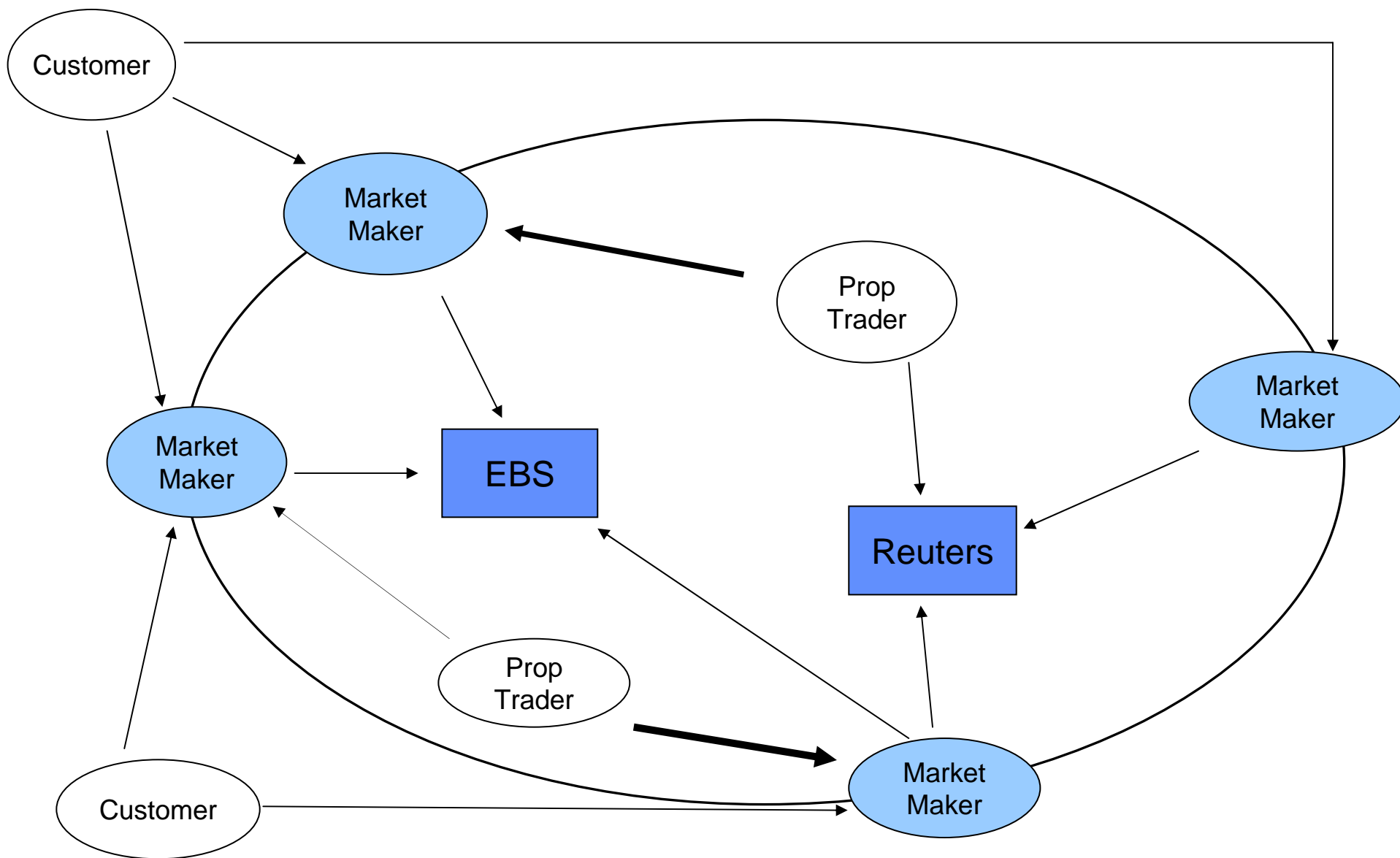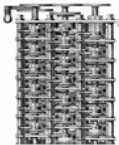
Global Market: EUR/USD Spread and Volume
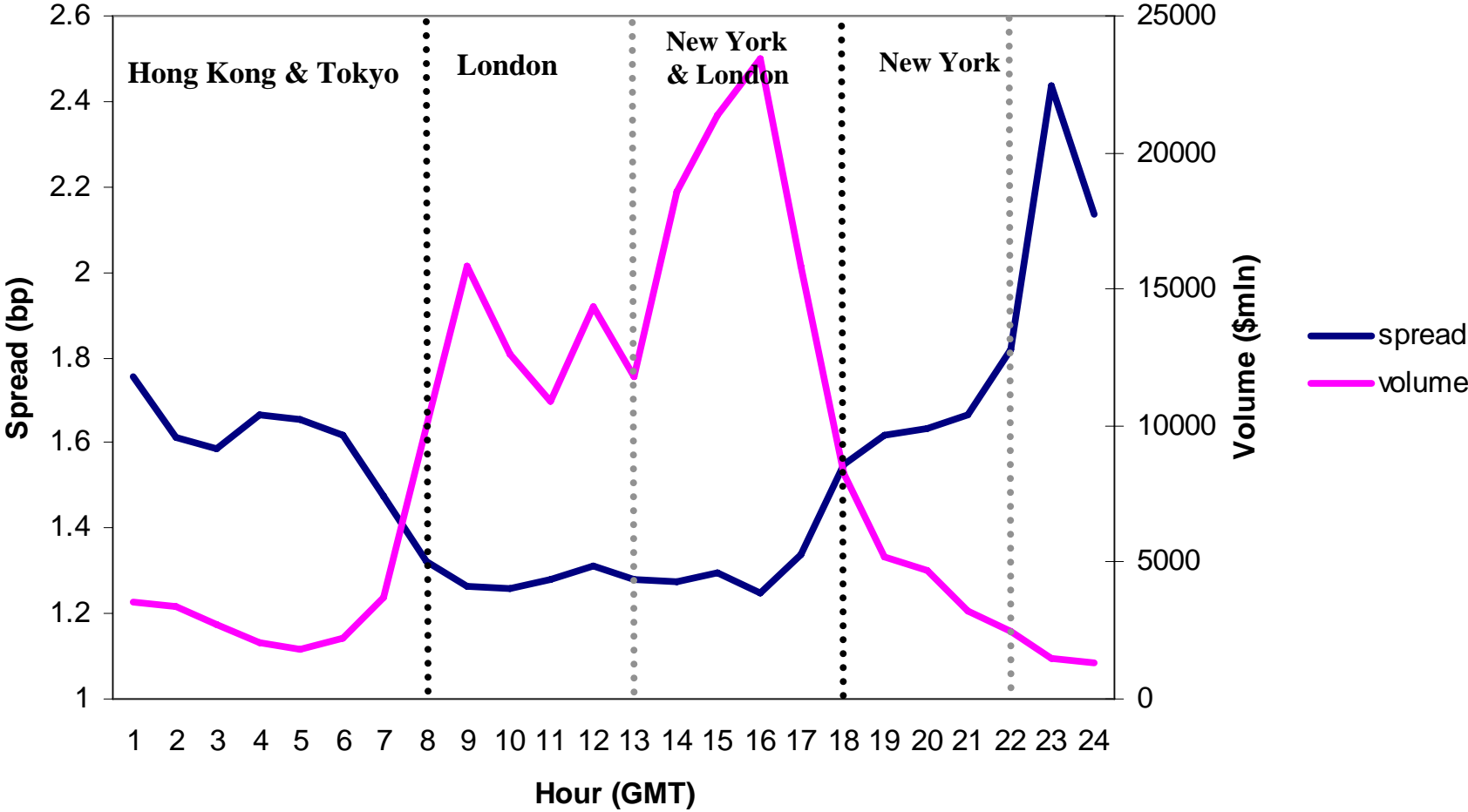
Source: Stacy Williams, HSBC Investment Bank

Global Market: Average GBPEUR Liquidity by Rate

Customer Terminology Example

- Customers leave limit orders with market makers:

  - Two types: *Take Profit* orders and *Stop Loss* orders

  - These have very different effects when they are activated by a price move in the market. A **take profit** order acts in the opposite direction to the market move that triggers it, a **stop loss** order acts in the same direction as the market move that triggers it

  - An example will make the difference clear. We will use the DollarYen exchange rate: the price of a dollar in terms of Yen
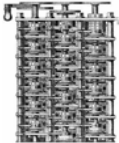
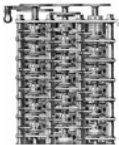# Customer Terminology Example

- A customer has bought dollars at a price of 100 Yen; if the price rises to 105 there will be a profit. If the customer leaves a limit order to sell the dollars at a price of 107 Yen this would be a **take profit** order. When there is a *rise* in the market price the order is to *sell*. The resulting transaction acts in the **opposite direction** to the move that triggered it

- If the customer had again bought dollars at a price of 100 Yen but the price was now 95 a limit order could be left to close the position – that is to sell the dollars – if the price fell to 92. This would stop the loss getting any bigger and is thus known as a **stop-loss** order. In this case a *fall* in the market price triggers a *sell* and so the order acts in the **same direction** as the move that triggered it

3   Structure of the FX Market

- Both EBS and Reuters provide double auction markets (buy and sell markets) in virtually all currency pairs

- However, the market has segmented: EBS has the greatest turnover and liquidity in EuroDollar and DollarYen while Reuters is the main market for SterlingDollar and for the Euro against non-Dollar currencies

FX Market Terminology

- Bid : Price market maker will pay to buy the currency

- Ask (or Offer): Price at which market maker will sell the currency

- Depth: Total amount available at a particular price

- Pip: Minimal price change e.g. 100th of a cent for dollar crosses

- Regular Amount: A characteristic size set for each currency pair - typically 20 million dollars

- Regular Price: The price nearest the best price at which the regular amount is available to deal

  - Note the depth profile is usually bimodal with peak depth at prices a few pips worse that the best prices
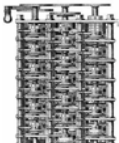
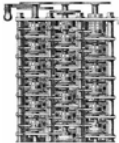# FX Market Makers Private Information

- Market makers have private information

    1. Order flow from their own customers:
        - Direction
        - Size
        - Type of customer

    2. Their own customer order book:
        - Type of order
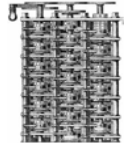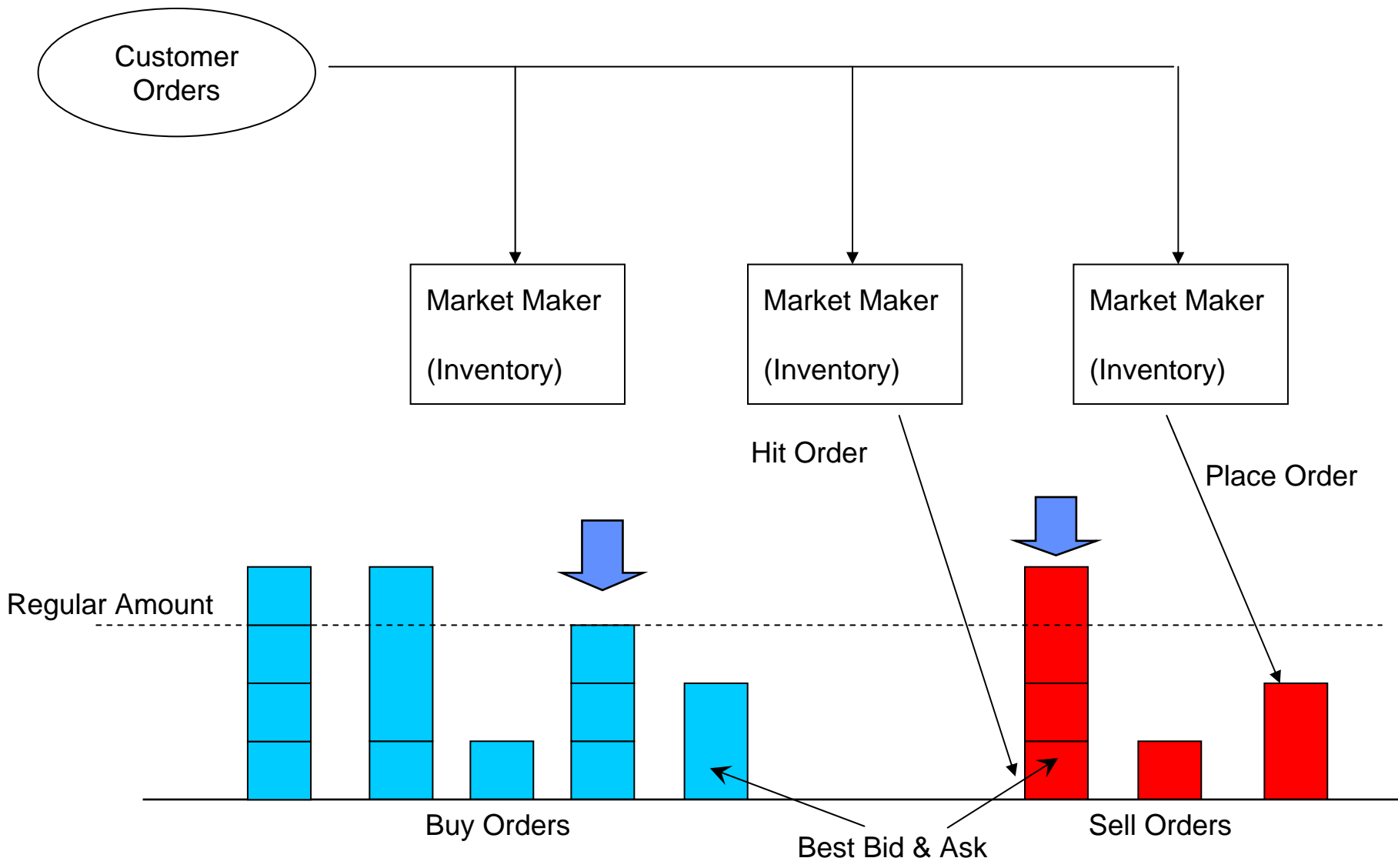   - Price
        - Size
        - Type of customer
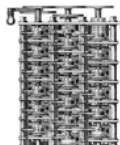
FX Market Makers Limited Information

- Market makers see limited information on EBS or Reuters

  1. Best bid and offer price

  2. Size at best bid (offer) but only if not regular

  3. Price where bid (offer) first goes regular (possibly best price)

  4. Every transaction showing only restricted information:

     1. Price

     2. If buyer or seller initiated (if hit ask or bid)

     Note, size of transaction is NOT shown

Customer Orders

Market Maker (Inventory)

Market Maker (Inventory)

Market Maker (Inventory)

Hit Order

Place Order

Regular Amount

Buy Orders

Best Bid & Ask

Sell Orders

EBS Screen

4 Modelling FX Investment

- Consider an investor holding a self financing currency portfolio and rebalancing this portfolio to a fixed mix at regular intervals – say daily or weekly

- This rebalancing strategy is determined by a square matrix $A = (\alpha_{kj})$ where α_{kj} is the proportion of the current holding g_{t-1}^j in currency j that is to be used to purchase currency k at exchange rate π_t^{kj} and proportional transaction cost δ_t^{kj}

- The resulting portfolio dynamics are given by

$$g_t^k = \sum_{j=1}^{K} \alpha_{kj}(1-\delta_t^{kj})\pi_t^{kj} g_{t-1}^j = \sum_{j=1}^{K} \alpha_{kj}(1-\delta_t^{kj})\frac{p_t^j}{p_t^k} g_{t-1}^j$$

Portfolio Dynamics

- Assume that exchange rates and transactions costs are driven by a stationary ergodic process and the latter are sufficiently small

- Then the portfolio growth dynamics are given in vector form by

$$\boldsymbol{g}_t = \boldsymbol{B}_t \boldsymbol{g}_{t-1} \quad t = 1,2,\ldots \quad g_0 \; fixed$$

- As a consequence of a stochastic version of the Perron-Frobenius theorem -- or alternatively the Birkhoff ergodic theorem -- for any fixed rebalance strategy A the resulting portfolio holding grows exponentially fast almost surely Dempster, Evstigneev & Schenk Hoppe (2003, 2004)
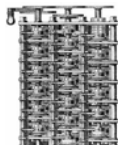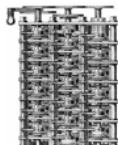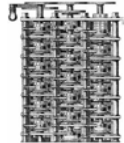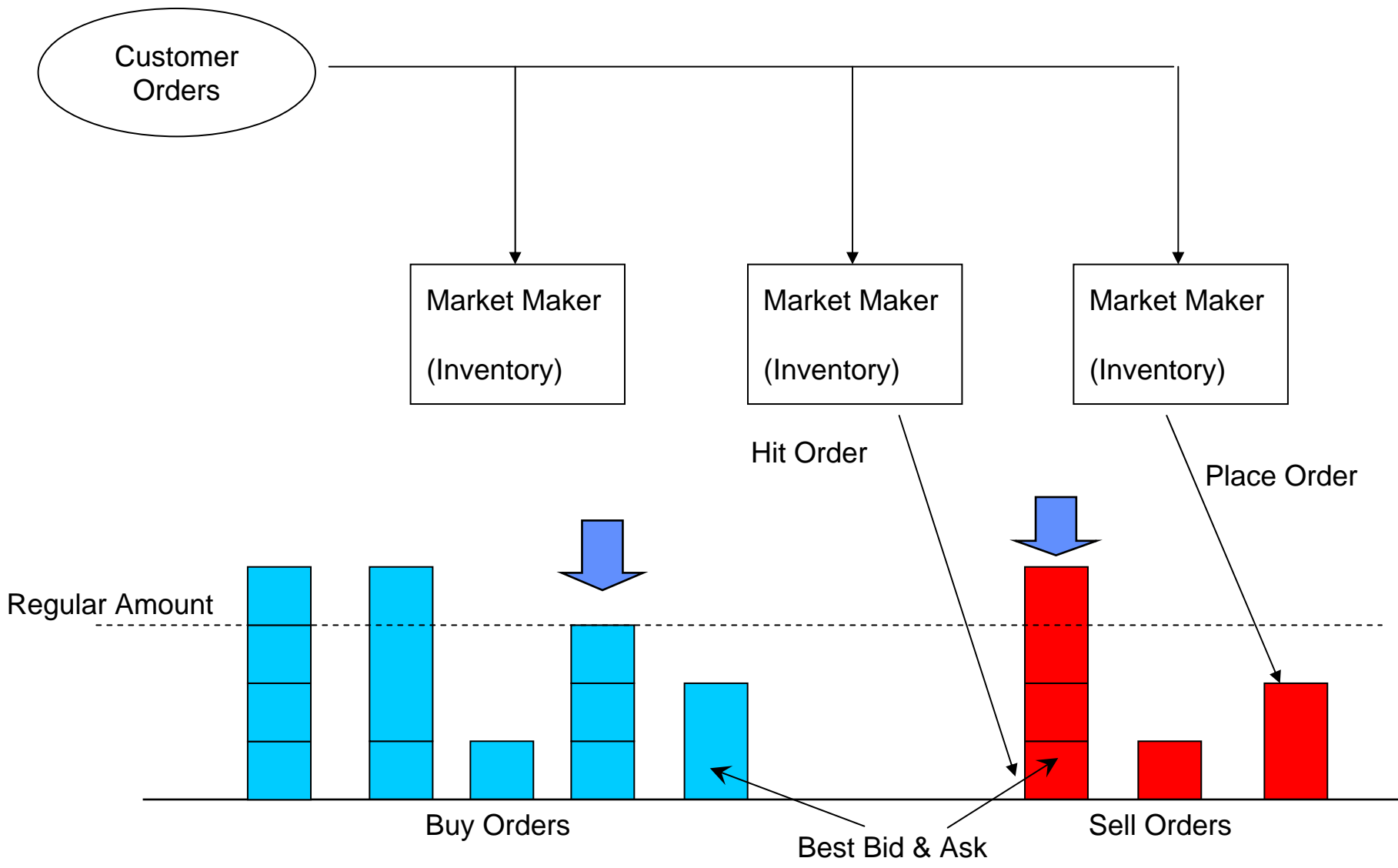
Balanced Growth and Risk Management

- Moreover for each rebalance strategy *A* the initial portfolio currency holdings g_0 can be chosen so that the resulting portfolio growth is balanced – all currency holdings rise or fall together almost surely after each rebalance with maximal expected positive growth rate – the stochastic turnpike Arkin & Evstigneev (1986)

- Using global optimization we may theoretically choose a different initial set of portfolio currency holdings so as to optimize final portfolio value over a finite horizon subject to a probabilistic maximum drawdown constraint – an NP-hard problem

- By stochastic turnpike theory the resulting portfolio holdings will converge to balanced growth at the (maximal) turnpike rate
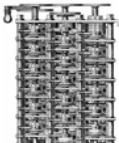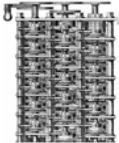
4   Modelling Market Makers

- The model of the FX market shown earlier provides a structure for simulation

- Treat market makers as the active agents with customer order flow exogenously determined

- Treat customers initially as generating a fixed sequence of orders

- How should market makers respond?

Customer Orders

Market Maker (Inventory)

Market Maker (Inventory)

Market Maker (Inventory)

Hit Order

Place Order

Regular Amount

Buy Orders

Best Bid & Ask

Sell Orders

Market Maker Behaviour

- What is optimal market maker behaviour?

- Model market maker actions – conditional on:

    - 'Public' market information

    - 'Public' events
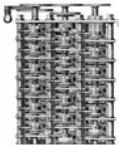
    - Private market information

    - Private events

Market Maker Actions

- **Take** a **bid/ask price** by hitting limit order of size S

- **Place** a **limit buy/sell order** at price P of size S

- **Cancel** some or all (size S) of an existing buy/sell order

- **Change** existing buy/sell order to new price and size P and S

- **Do nothing** at this time

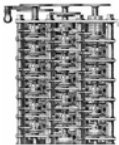- Modelling market reaction in terms of **spread** as a **random walk** between two **partially absorbing barriers**
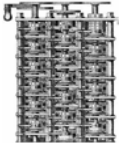
Market Maker Public Information

- Best bid and ask prices (and so spread)

- Size at best bid and best ask

- Regular bid distance from best bid

- Regular ask distance from best ask

- Volatility of best prices

- Short-term technical indicators (trend, etc.)

- Inter-dealer activity in number (not size) of trades and buy/sell balance

Market Maker Public Events

- **Best bid** price changes

- **Best ask** price changes

- **Liquidity at best bid** changes

- **Liquidity at best ask** changes

- **Regular bid** price changes

- **Regular ask** price changes

EBS Screen

Market Maker Private Information

- Inventory

- Customer deals: net buy/sell balance

- Customers deals: volume (both number and size)

- Structure of customer limit orders

- Own limit orders with inter-dealer market
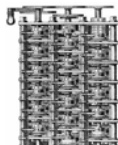
Market Maker Private Events

- Buy (from us) order executed with customer

- Sell (to us) order executed with customer

- Other market maker hits our limit buy/sell order (event also known to the other MM)

Market Maker Stylized Behaviour

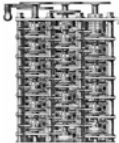- Market makers are risk averse but must deal continuously to generate revenues

- Continuous small profits on small trades – e.g. 10 million dollars – are preferable to occasional large profits on big trades with related losses!

- Large customer orders in inventory are dealt as many small trades Lyons (2001)

- Prices and amounts of bid and ask quotes are asymmetric depending on both the current market and customer order inventory

- Gaming behaviour as for example bidding when selling off inventory is standard

- The interdealer market is mean reverting on very short time scales creating opportunities for proprietary traders as a result
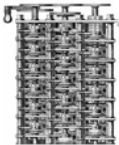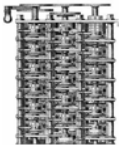
What is 'Optimal' Market Maker Behaviour?

- Market makers seek to benefit from the random fluctuations of prices: buy low, sell high!

- MMs are **not profit maximizers** but they are profit satisfiers with bounded rationality   Simon (1955)

- Continual small profits are taken rather than risking large losses

- MMs may be considered to be utility maximizers with high risk aversion captured by high curvature of a concave utility function
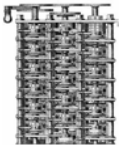
Simple Model

- Random walk
 - Discrete time
  - Integer price in pips
  - Unbiased: P(up) = P(down)
  - Extension to biased case

- Two partially absorbing barriers represented by limit orders in the market book
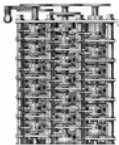
- Our limit order may not be cleared at best bid / ask

Customer Order Treatment

- Consider trying to dispose of positive inventory (from customers) by selling into the interdealer market
  - Where should the absorbing barriers be placed?
  - Upper barrier is a limit sell order in the market book
  - Lower barrier is a stop-loss (trigger) level – hit the best market bid
- Customer was quoted lower bid than market best bid

- Guaranteed profit? Possibly not depending on lower barrier placement

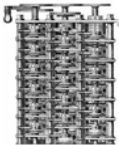- Disposing of negative customer inventory by buying in the interdealer market is similar
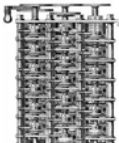
Asymmetric Barriers & Market Prices
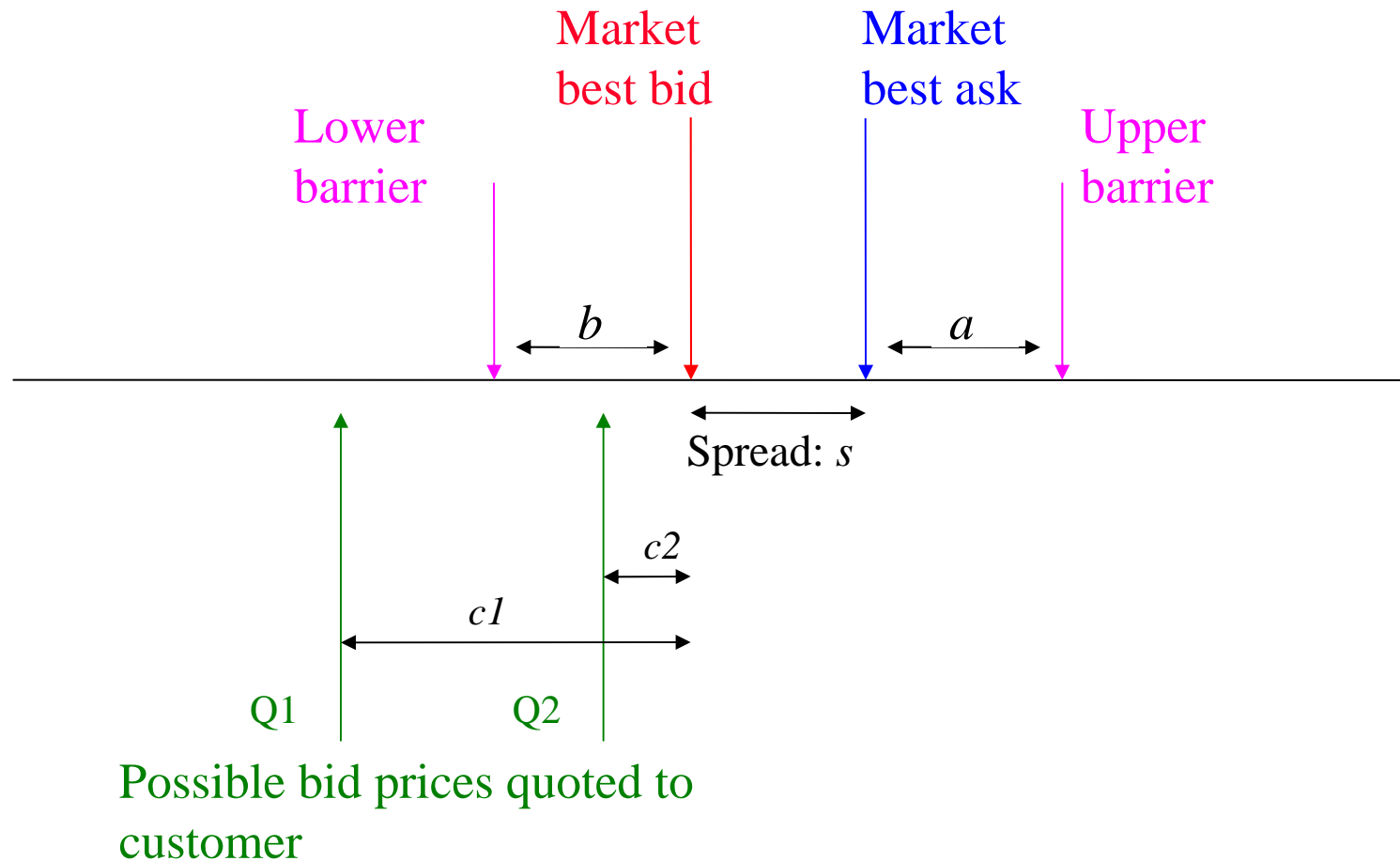
Placing the Barriers

- Consider symmetric barrier placement ($a = b$)

- For risk-averse market makers with a concave utility of return symmetric placement is never optimal

- We find $a = b = 0$ i.e. immediately close the position at the market best bid

- What about asymmetric barrier placement?
  - Maximize CARA (exponential) utility function

CARA Utility

$$U := \frac{-1}{\lambda}\left(P_U e^{-\lambda r_U} + P_L e^{-\lambda r_L}\right)$$

- r_U and r_L denote the returns at the upper and lower barriers respectively

- P_U and P_L denote the probabilities of absorption at the upper and lower barriers respectively
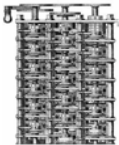
- λ is the risk aversion parameter
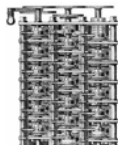
Barrier Placement and Utility

- Find the unique utility maximum in a and b from the first order conditions

$$\frac{\partial U}{\partial a} = \frac{be^{-\lambda(c+a+s-1)}}{\lambda(a+b)^2}\left(1 - e^{\lambda(a+b+s-1)} + \lambda(a+b)\right) = 0$$

$$\frac{\partial U}{\partial b} = \frac{ae^{-\lambda(c-b)}}{\lambda(a+b)^2}\left(1 - e^{-\lambda(a+b+s-1)} - \lambda(a+b)\right) = 0$$
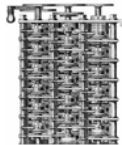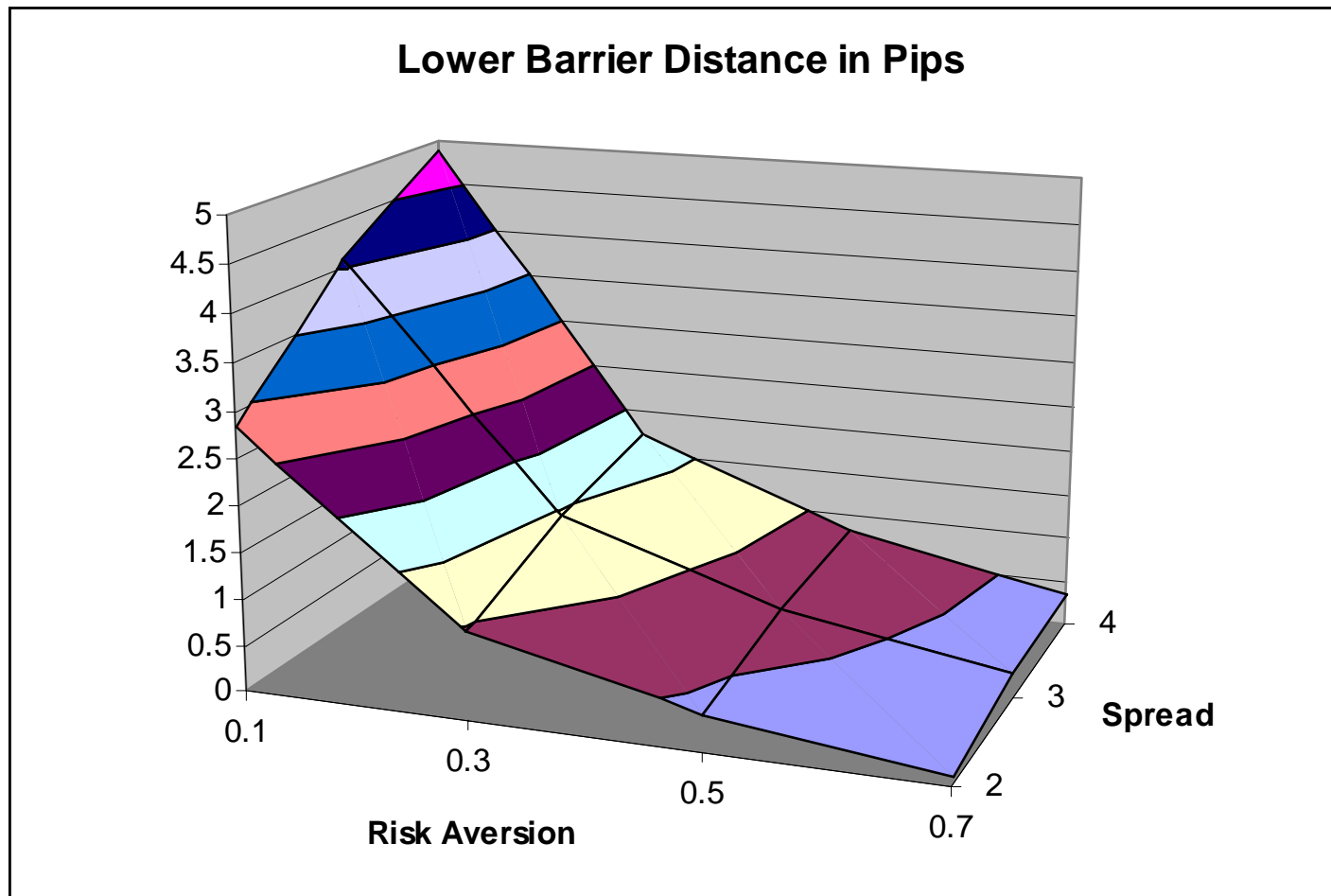
Asymmetric Barrier Placement

- **Utility decreases** as the upper barrier moves **above the** market **best ask** (for all spreads and values of risk aversion)

- So always **place sell order** at market **best ask**

- **Lower barrier** placement depends on spread and risk aversion

- Place **below** market **best bid**

CARA Utility

Lower Barrier Distance in Pips
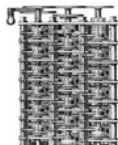
Conclusions & Directions for Future Work

- Have shown increasingly profitable FX investment strategy with transaction costs

- Choosing a symmetric fixed mix trading strategy with minimal inventory levels at days end for diagonal entries and balanced positions off diagonal entries this model applies to market makers providing the theoretical results can be extended to allow customer order flows into and out of the portfolio

- Detailed simulation modelling of the customers – market maker – interbank trading system is in progress

- In particular modelling of the market makers actions using Markov chain models and corresponding simulations of the interbank global FX market is in progress

- Investigation of the two layer market and other agent based alternatives in the future