Practical Relative-Value Volatility Trading

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21 January 2005

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Volatility modelling

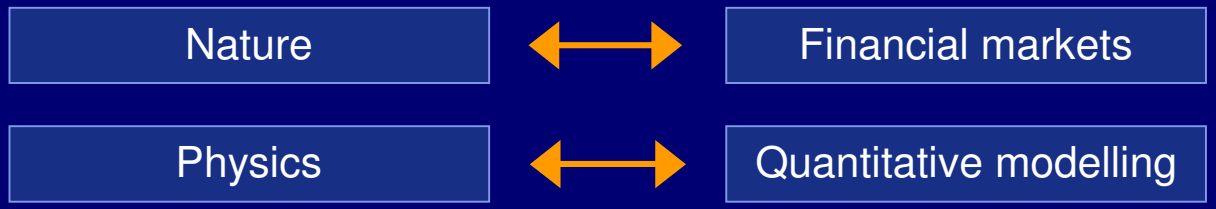
- Construct a consistent framework to identify and extract value from interest-rate options markets
- Use this framework to inform decisions concerning inception and management of proprietary options trading positions

Modelling paradigm

- Insist on consistent pricing and hedging framework: only one US dollar (or euro) libor yield curve, therefore only one process – random or otherwise – driving this curve
- Avoid pragmatic use of simplest model per product – can result in inconsistent dynamics
- Make sensible choices about objective features to incorporate into modelling formulation: e.g. multifactor, skew dynamics
- Require advanced analytics to develop tractable pricing and risk management tools – often an obstacle to successful implementation

Modelling paradigm (cont'd)

- Pricing inconsistencies between market and model can indicate:
  - market features
  - modelling errors
  - trading opportunities
- Use market experience and judgment to identify the latter
- Warning: Models have Limitations!
  - "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature" Niels Bohr



  - "A trader armed only with a clever model is soon removed from his capital"

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Modelling paradigm (cont'd)

- Significant advances in quantitative financial modelling over past ten years. Need judgment to harness these advances

- Tukey: judgment based on
  - mathematical knowledge of the particular techniques
  - experience of the particular field of subject matter
  - experience of how these techniques have worked out in practice

Forward volatility surfaces

- Translate universe of option prices into how each point on yield curve (e.g. each Euribor future) oscillates over time
- More precisely: strip swaptions, caps and other option products into a forward volatility surface, $\sigma(t,T)$
- For fixed T, $\sigma(t,T)$ represents the volatility of a particular Eurodollar (or Euribor) contract over its life. These “forward volatilities” are observable quantities about which can make subjective judgments

Forward volatility surfaces (cont'd)

- Framework adopts multifactor BGM Model

$$df(t, T) = a(t,T)\, dt + \sigma(t, T) \sum_{i=1}^{n} \rho_i (t, T)\, dW_i (t)$$

  - $f(t, T)$ 3m-forward rate from T at time t
  - Drift $a(t, T)$ determined from volatility $\sigma(t, T)$

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Forward volatility surfaces (cont'd)

- Implement sparsely-parametrized surface. Use market knowledge to impose reasonable functional forms ("subjective judgment about objective features")
- We employ parametric surfaces of form:

$$\lambda_1 + \exp\left(- \lambda_2 \tau \right) \left(\lambda_0 - \lambda_1 - \lambda_3 \tau \right)$$

τ : forward, calendar or relative time

- Discretize for fully non-parametric surface
- Can impose smoothing or linking on nonparametric surface

For further details see Blyth (2004)

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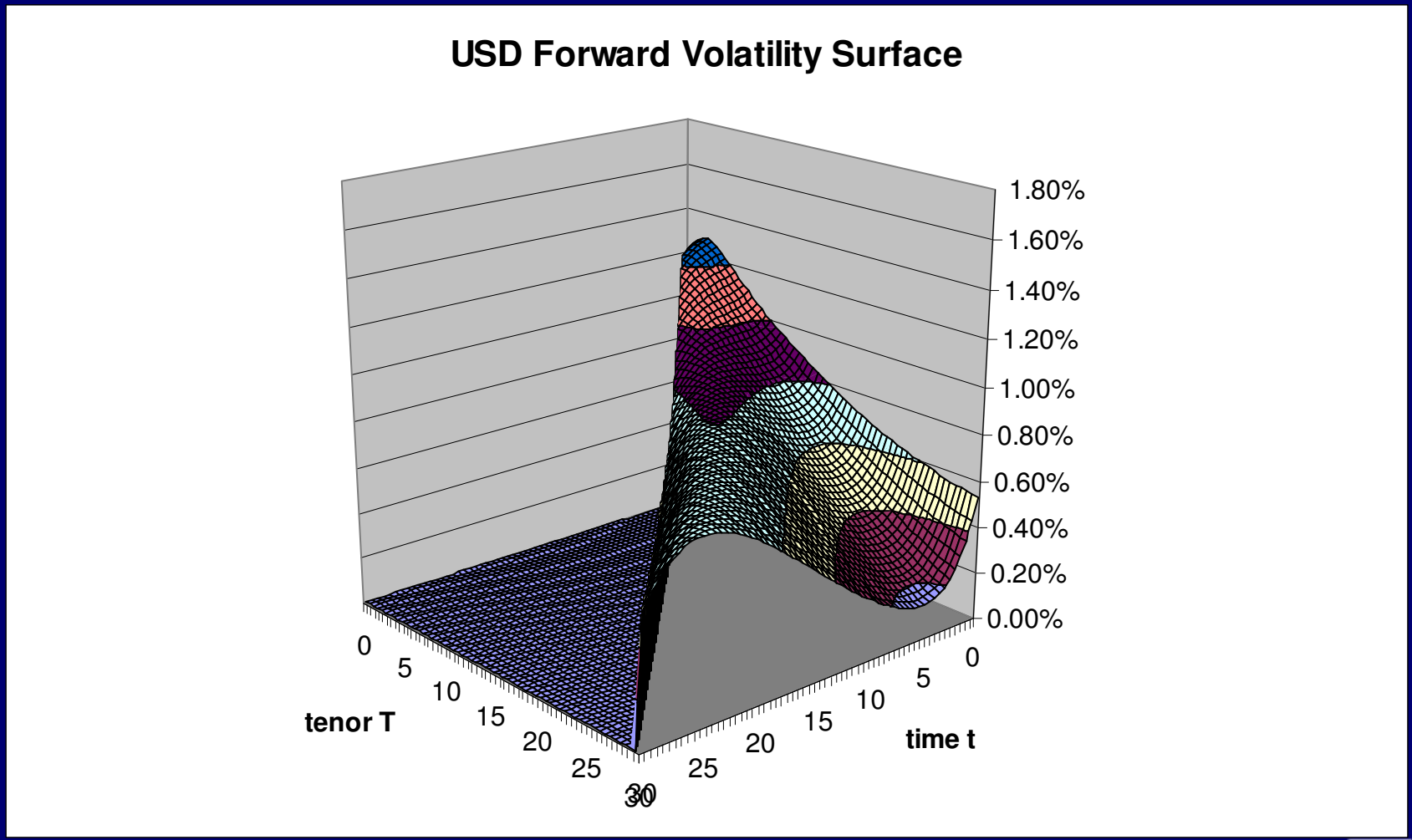
Parametric forward volatility surface for US dollar



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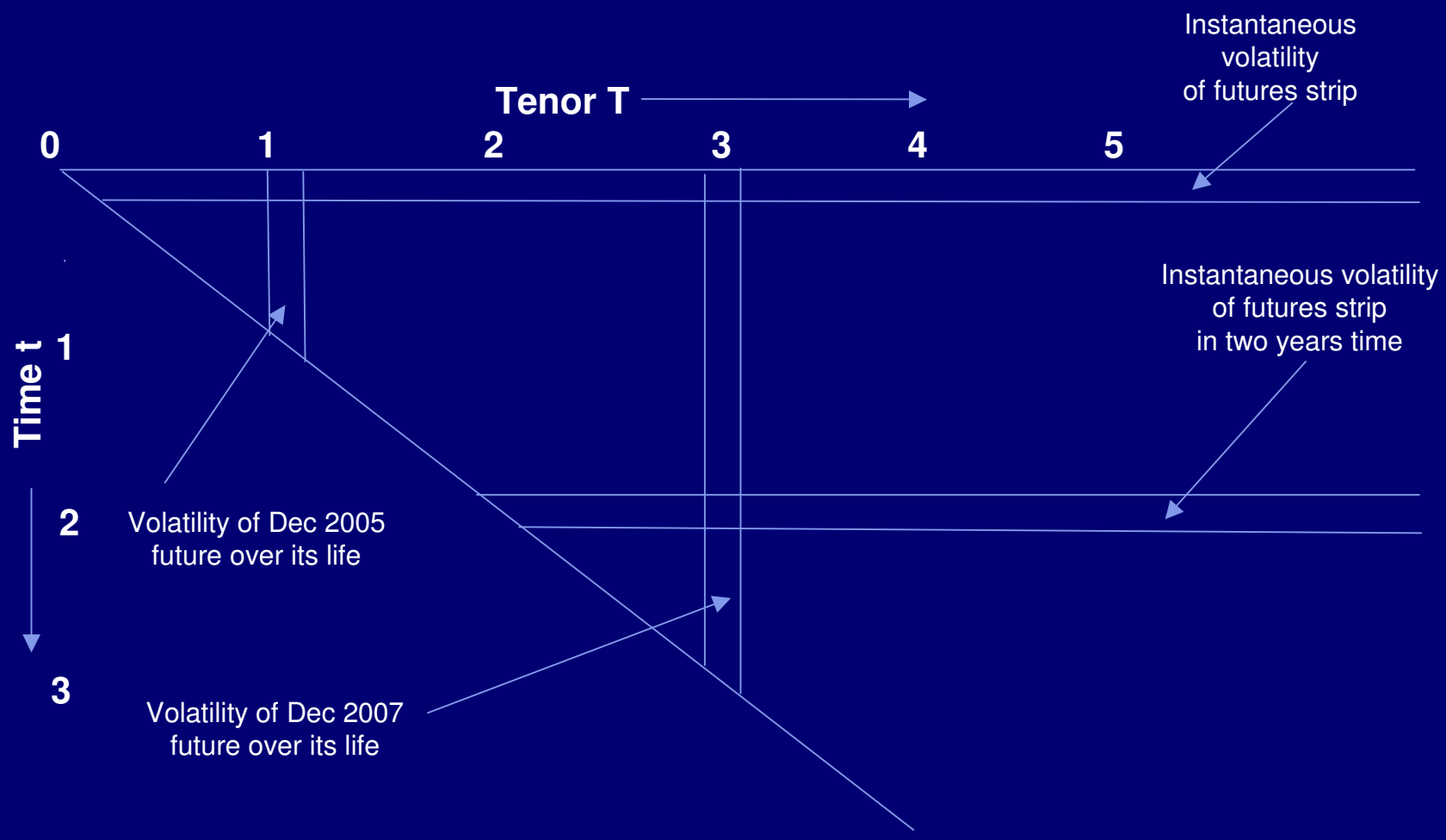
Map of a forward volatility surface



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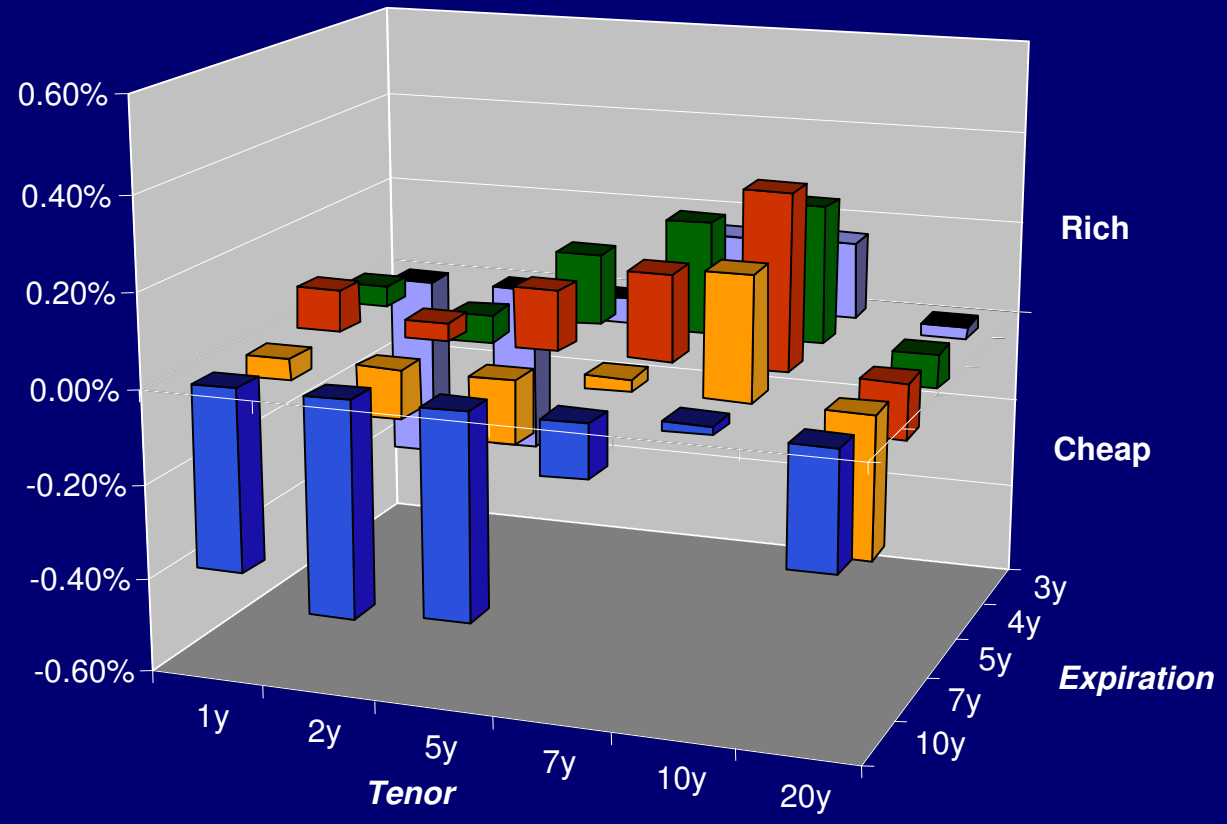
US dollar relative-value indicators

USD residuals



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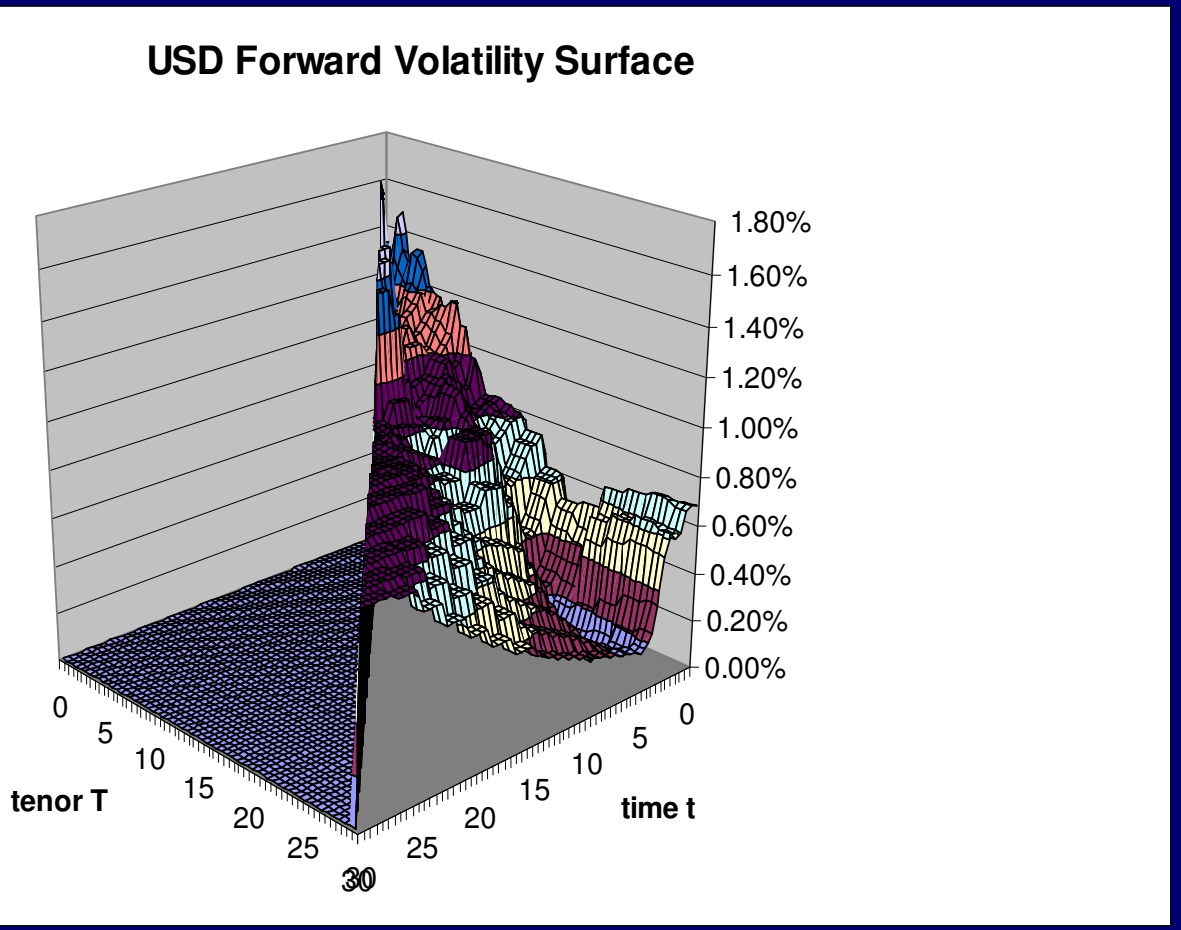
Exact fit to US dollar market prices



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Using forward volatility surfaces in practice

- Implement both parametric smooth surface and exact-fit surface
- Trading decisions informed by:
  - Residuals between market and smooth-fit prices
  - Shape of exact-fit forward volatility surface
- Hedge trades under consistent dynamic
- Hold to maturity or close out strategies when volatility levels revert to fair value. Holding periods can vary from days to years
- Approach predicated on power of consistent dynamic to offset inevitable modelling shortcomings

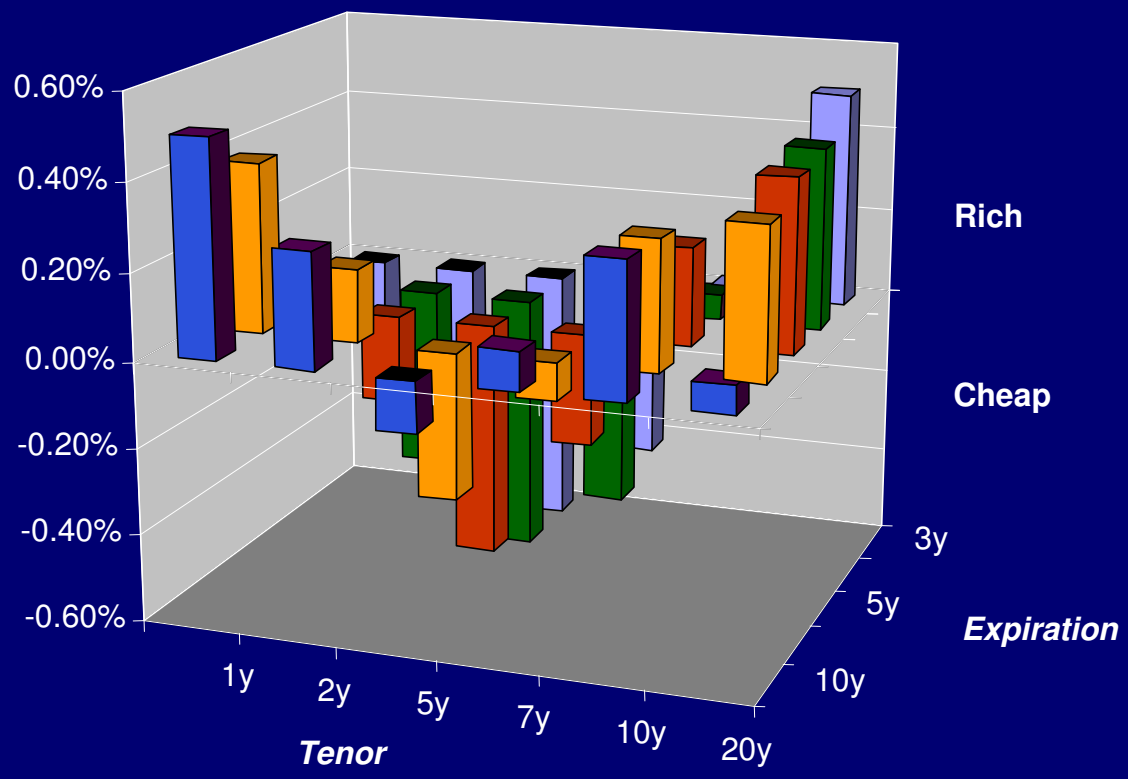
Further examples

1. Hedge 10yr CMS liabilities with 5yr-tailed swaptions

- 10yr-tailed swaptions are rich due to hedging of 10yr CMS product
- Portfolio of 5yr-tail options with spectrum of expirations captures similar volatility, but at cheaper levels
  - 10yr-5yr and 15yr-5yr payers are better value than 10yr-10yr payer

Euro relative-value indicators

EUR residuals



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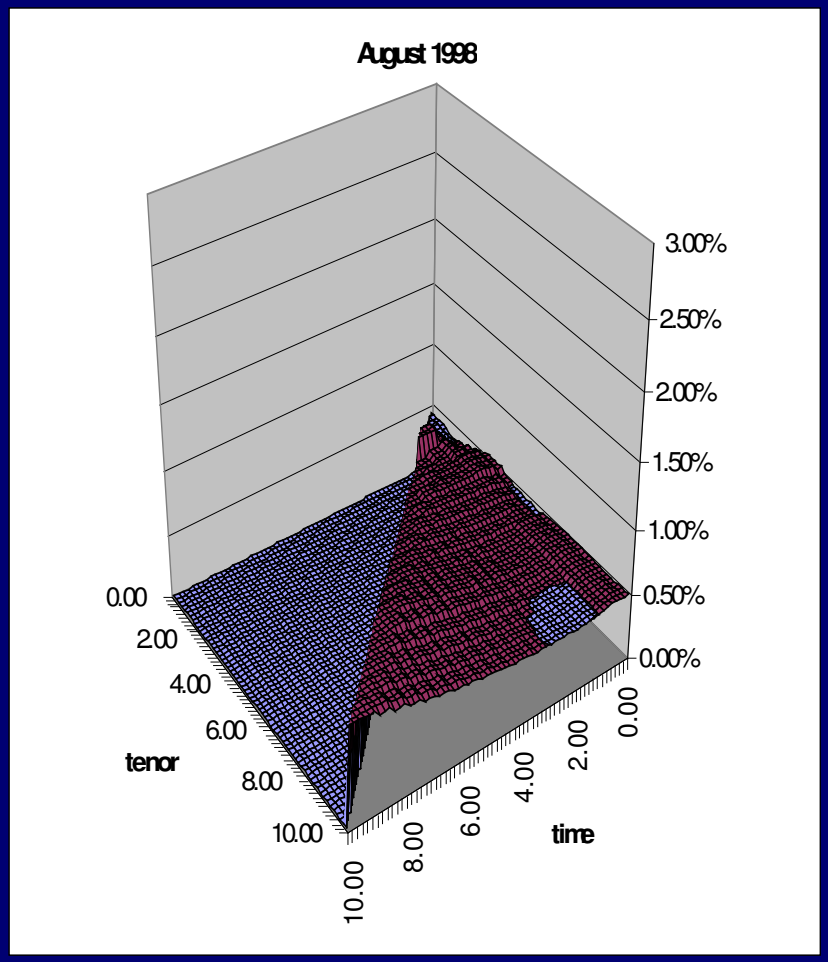
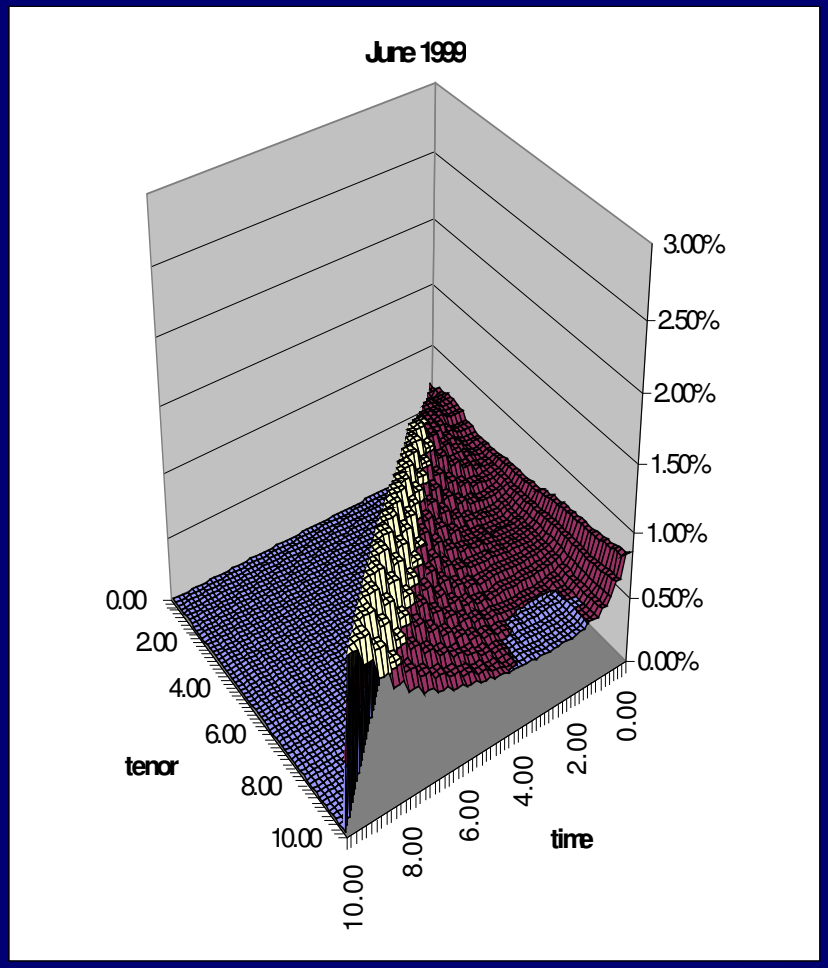
2. Caps versus swaptions, 1998-9

- Unwind of proprietary desks' short positions in caps versus long positions in swaptions widened volatility spread between products
- In forward volatility space, this resulted in large spikes in forward volatility of the front contracts
- Volatility dynamic implied by these prices absurd

Euro forward volatility surface, Aug 98-June 99:

The approach of the storm





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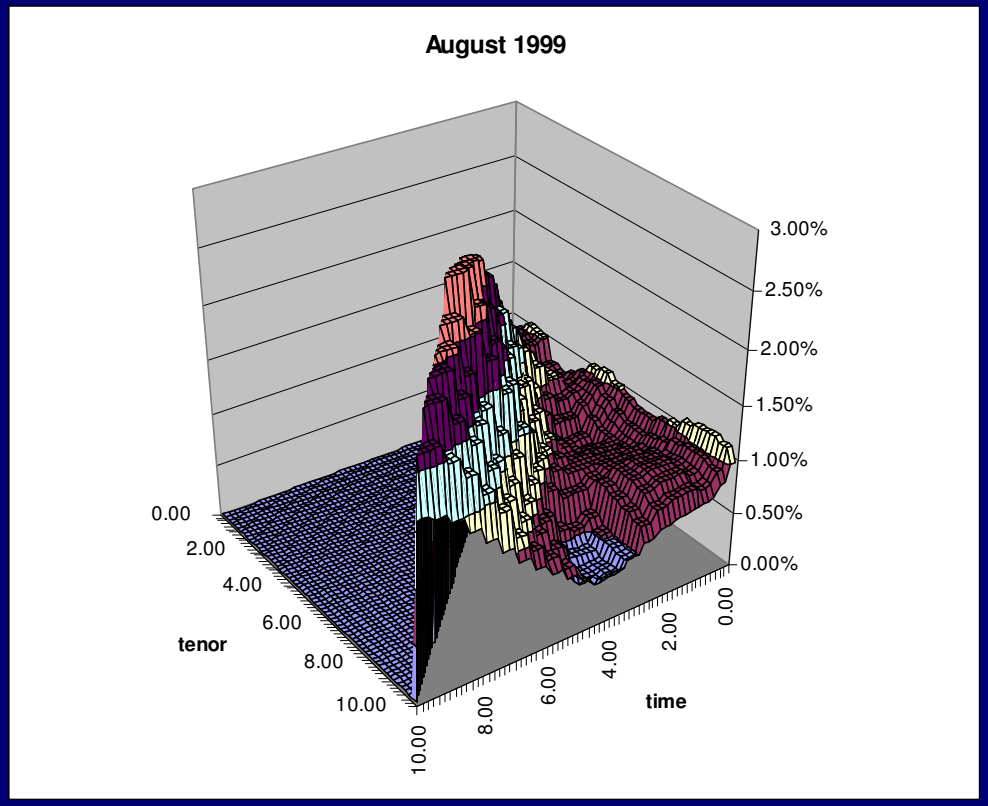
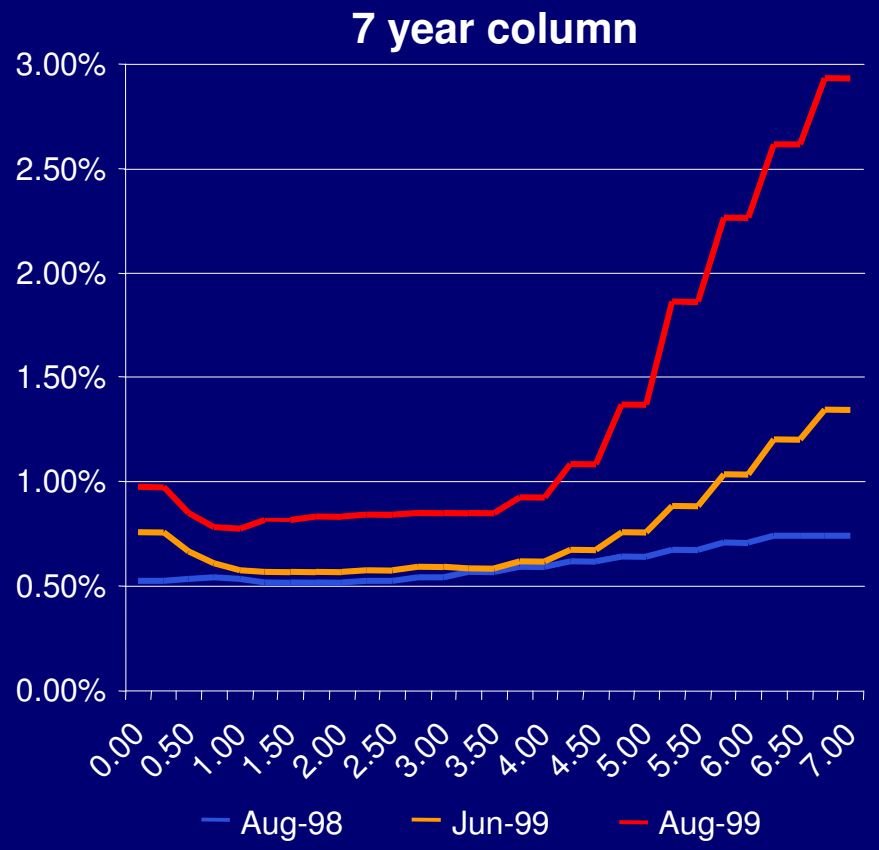
Euro forward volatility surface, Aug 99:
Inundation





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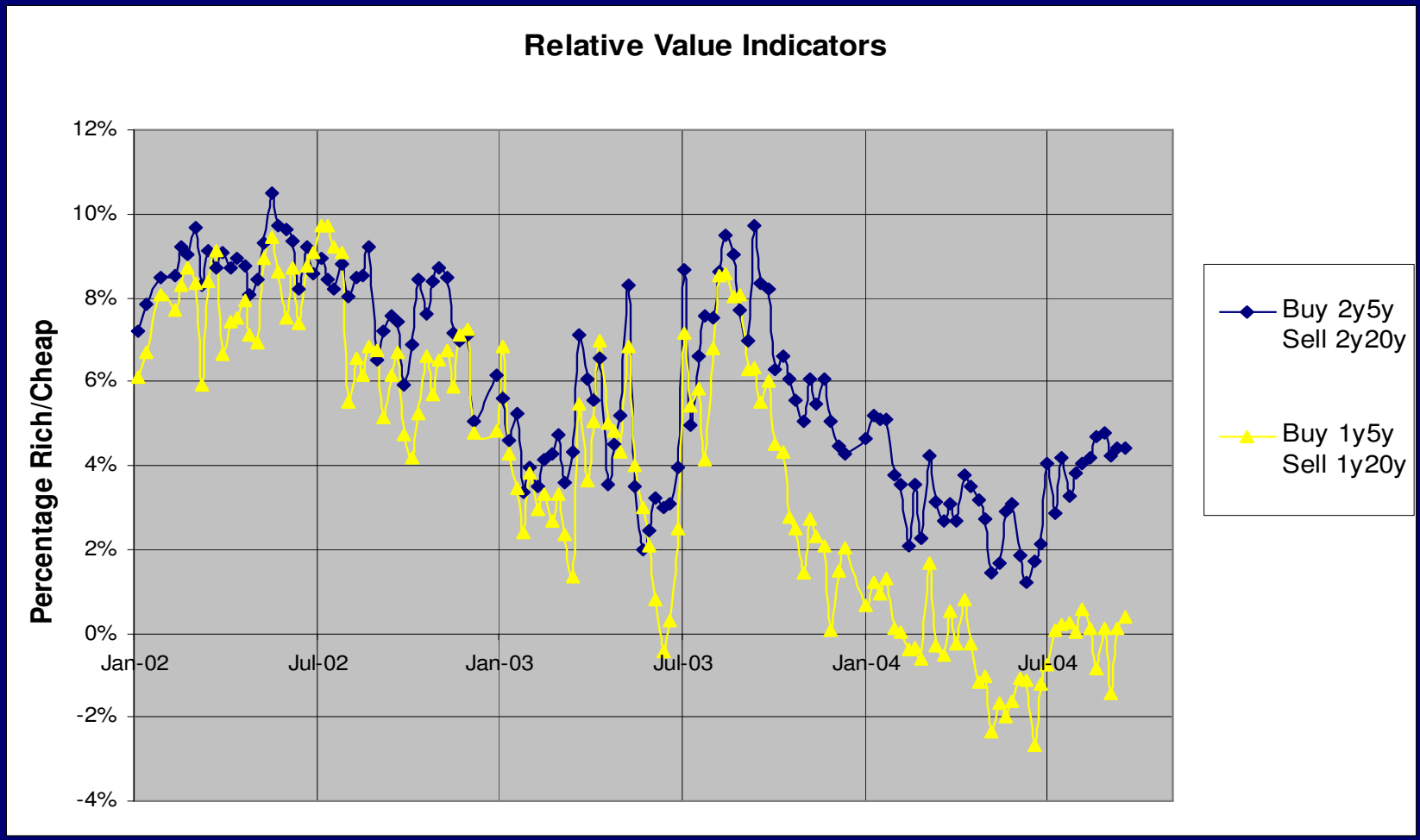
3. Euro swaption relative value: comparing indicators

- Consistent modelling framework can identify better trading strategies to simpler historical implied analysis - although often indicators concur.
- Consider performance of relative-value trades in euro swaption market.

Euro swaption relative value (cont'd)



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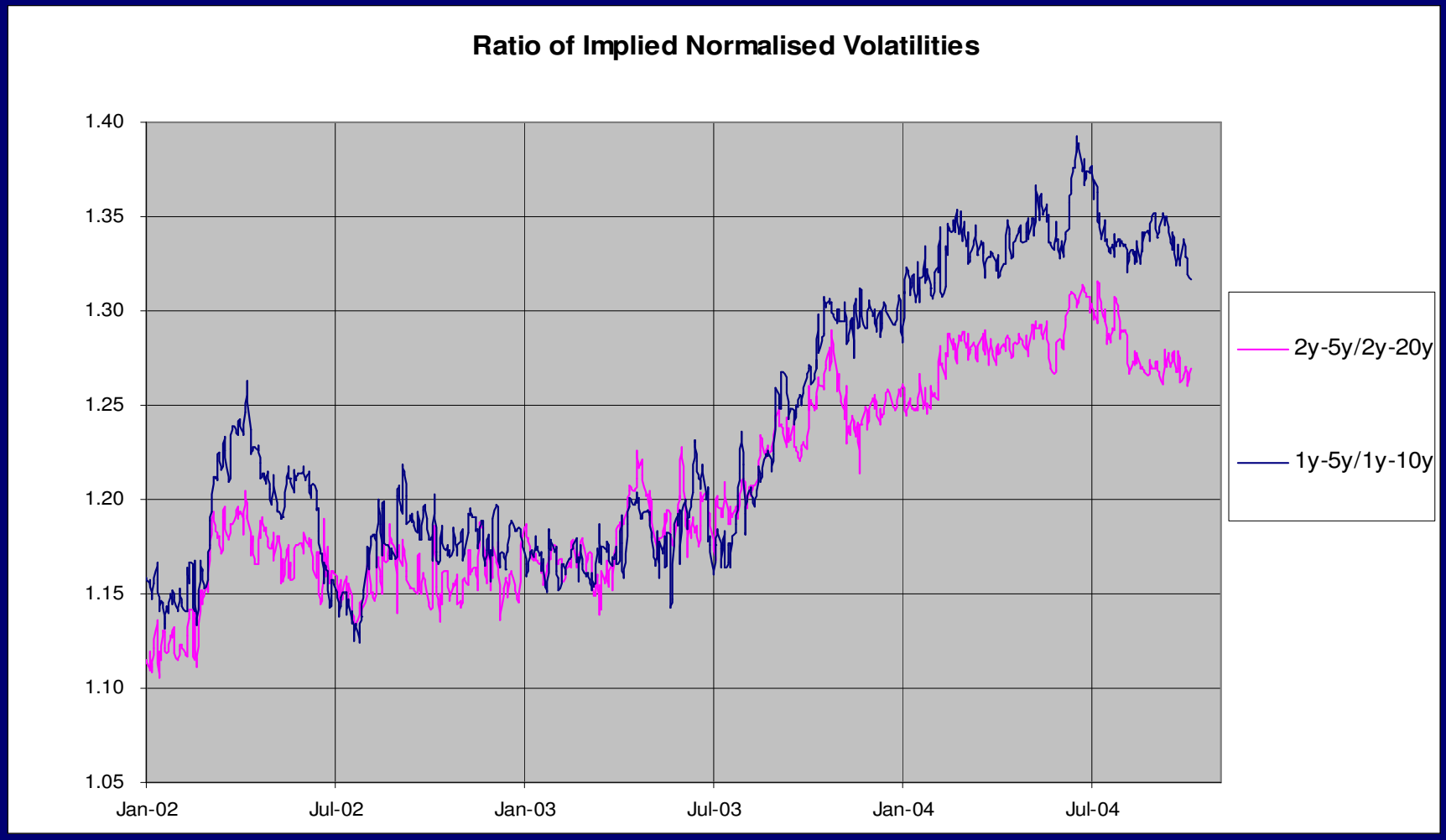
Euro swaption relative value (cont'd)



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Additional dimensions

- Skew Modelling: demand consistency between pricing of skew and dynamics of implied volatility under market movements
- Term Structure of Skew: skew structure for short-term rates coupled with specified dynamics may not be consistent with certain skew structure for longer-dated rates
- Stochastic Volatility: demand consistency between stochastic volatility overlays used to price option smile and prices of compound options

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