Modern Portfolio Theory:

Should Active Portfolio Managers Be Using It?

David J Buckle

Director of Research, Lee Overlay Partners

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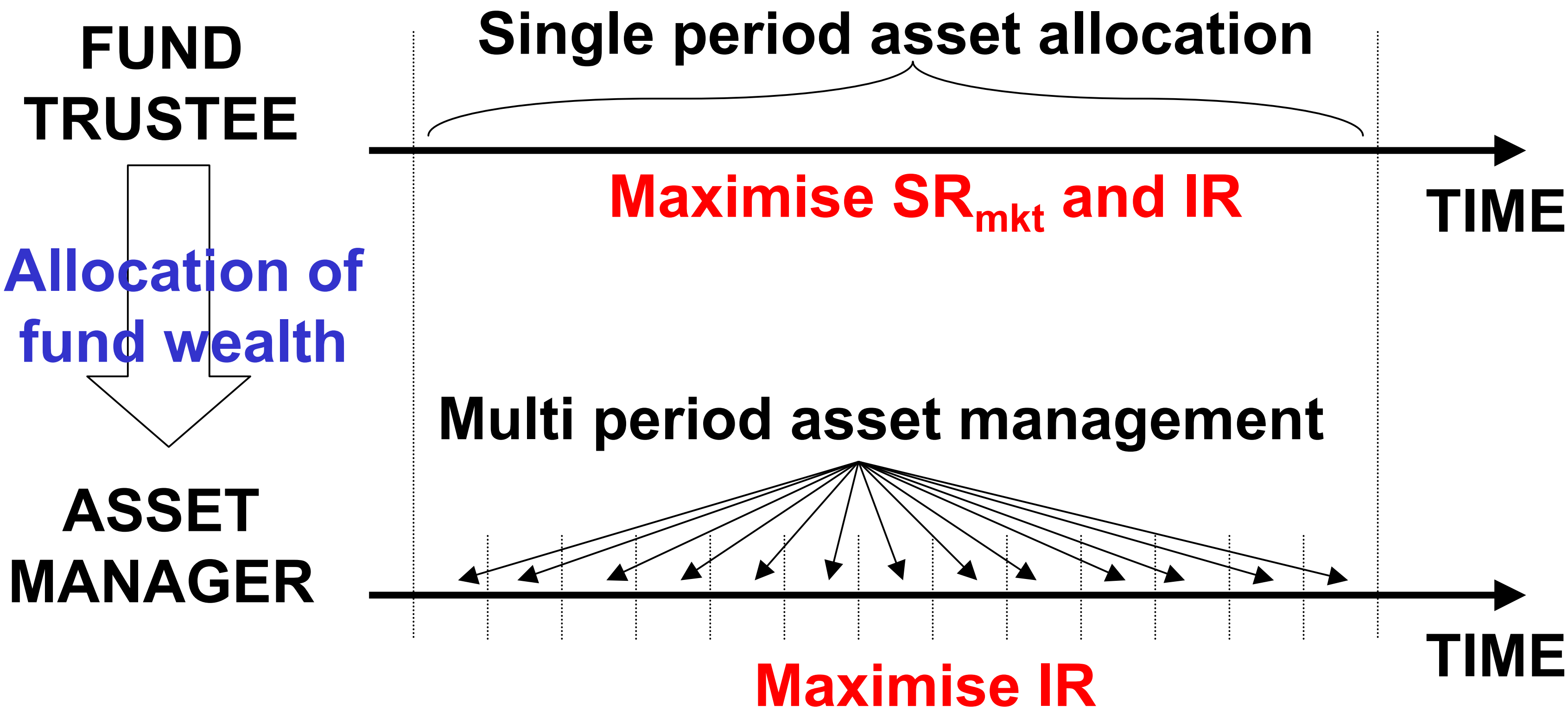
Conceptual Framework for active portfolio management



CAPM

$\mathbf{r}_t = \mu_t + \theta_t$ μ, θ **indep** $V[\theta] = \Sigma$

ASSUMPTION 1: $E[\theta] = 0$

Treynor & Black (but multiperiod and no independence assumptions)

$\theta_t = \alpha_t + \phi_t$

ASSUMPTION 2: $E[\alpha] = 0$

Forecasting equation

$\alpha_t = \mathbf{m}_t + \mathbf{e}_t$ $V[\mathbf{m}] = \Omega$

ASSUMPTION 3: $E[\mathbf{e}] = 0$

Active Management equation

$\theta_t = \mathbf{m}_t + \varepsilon_t$ $V[\varepsilon] = \mathbf{V}$

ASSUMPTION 4: $\Sigma = \Omega + \mathbf{V}$ **ASSUMPTION 5:** $\theta, \mathbf{m} \sim$ **MVN**

Definition of information coefficient

$$IC = \frac{E[m\theta]}{\sqrt{V[m]V[\theta]}}$$

Using IC to identify input parameters

$$\omega = IC\ \sigma$$

$$\nu = \sqrt{1 - IC^2}\ \sigma$$

$$\rho_\Sigma = \rho_\Omega\ IC_1 IC_2 + \rho_V \sqrt{(1-IC_1^2)(1-IC_2^2)}$$

Portfolio weights

Total portfolio weights: $\mathbf{w}_t$ such that $\mathbf{w}_t^T\mathbf{1} = 1$

Market portfolio weights: $\mathbf{x}_t$ such that $\mathbf{x}_t^T\mathbf{1} = 1$, $\mathbf{x}_t^T\theta_t = 0$

Active portfolio weights: $\mathbf{y}_t$

$$\mathbf{w}_t = \mathbf{x}_t + \mathbf{y}_t$$

Portfolio returns

$$\mathbf{w}_t^T\mathbf{r}_t = \mathbf{w}_t^T\mu_t + \mathbf{y}_t^T\theta_t$$

ASSUMPTION 6: y is indep of μ and x

CAUTION: $\mathbf{w}_t^T\mu_t = \mathbf{x}_t^T\mu_t + \mathbf{y}_t^T\mu_t$

ASSUMPTION 7: active manager can take an active position of any size in the market portfolio

Definition of information ratio

$$\mathrm{IR} = \frac{\mathrm{E}[\mathbf{y}^{\mathrm{T}}\theta]}{\sqrt{\mathrm{V}[\mathbf{y}^{\mathrm{T}}\theta]}}$$

A useful related expression

$$\mathrm{IR} = \frac{1}{\sqrt{h - 1}}$$

$$h = \frac{\mathrm{E}[\mathbf{y}^{\mathrm{T}}(\mathbf{m}\mathbf{m}^{\mathrm{T}} + \mathbf{V})\mathbf{y}]}{\mathrm{E}[\mathbf{y}^{\mathrm{T}}\mathbf{m}]^2}$$

<u>MPT Approach</u>: $\mathbf{y}_t = c\,\mathbf{W}^{-1}\mathbf{m}_t$

Aside: If W is diagonal then $\mathbf{y}_t \propto \mathbf{m}_t$

Generalised Fundamental Law of Active Management

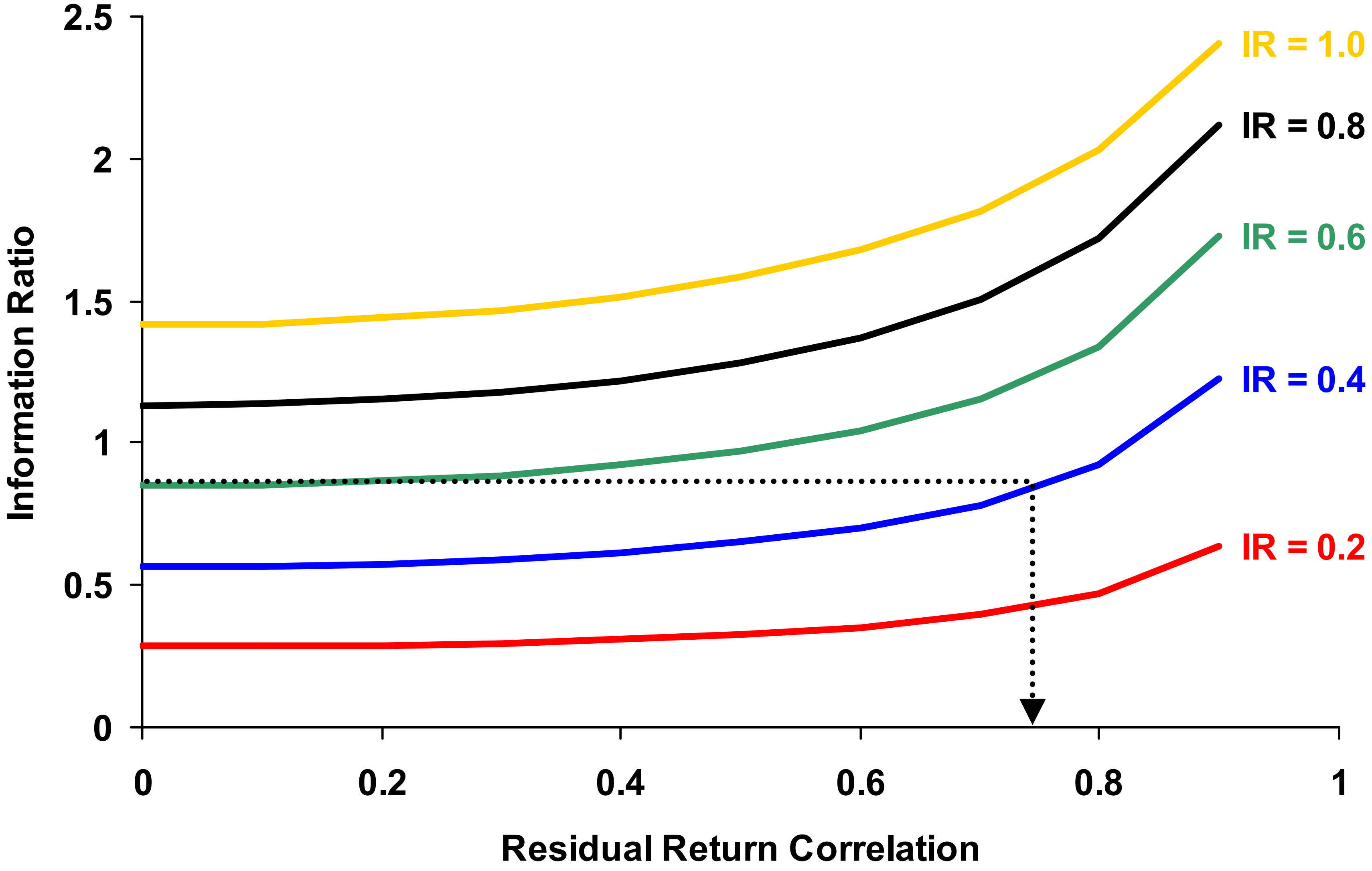
$$IR = \frac{tr(\Omega \mathbf{W}^{-1})}{\sqrt{tr(\Omega \mathbf{W}^{-1} \Sigma \mathbf{W}^{-1}) + tr((\Omega \mathbf{W}^{-1})^2)}}$$

If Σ, Ω, W are diagonal and $\sigma_1 = \ldots = \sigma_{BR}$, $IC_1 = \ldots = IC_{BR}$

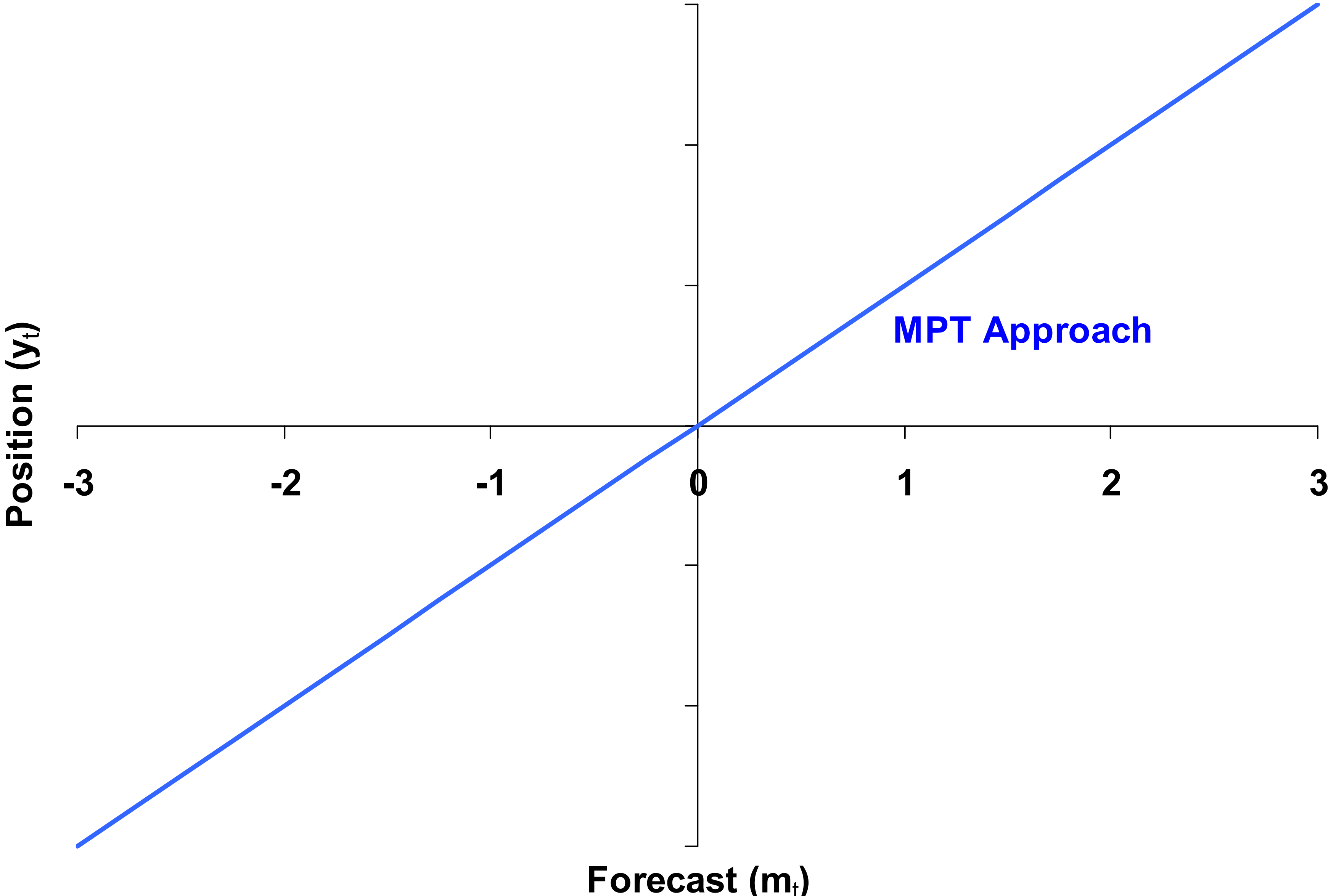
then $IR \approx \sqrt{BR}\, IC$

Univariate MPT: $IR = \dfrac{IC}{\sqrt{(1 + IC^2)}}$

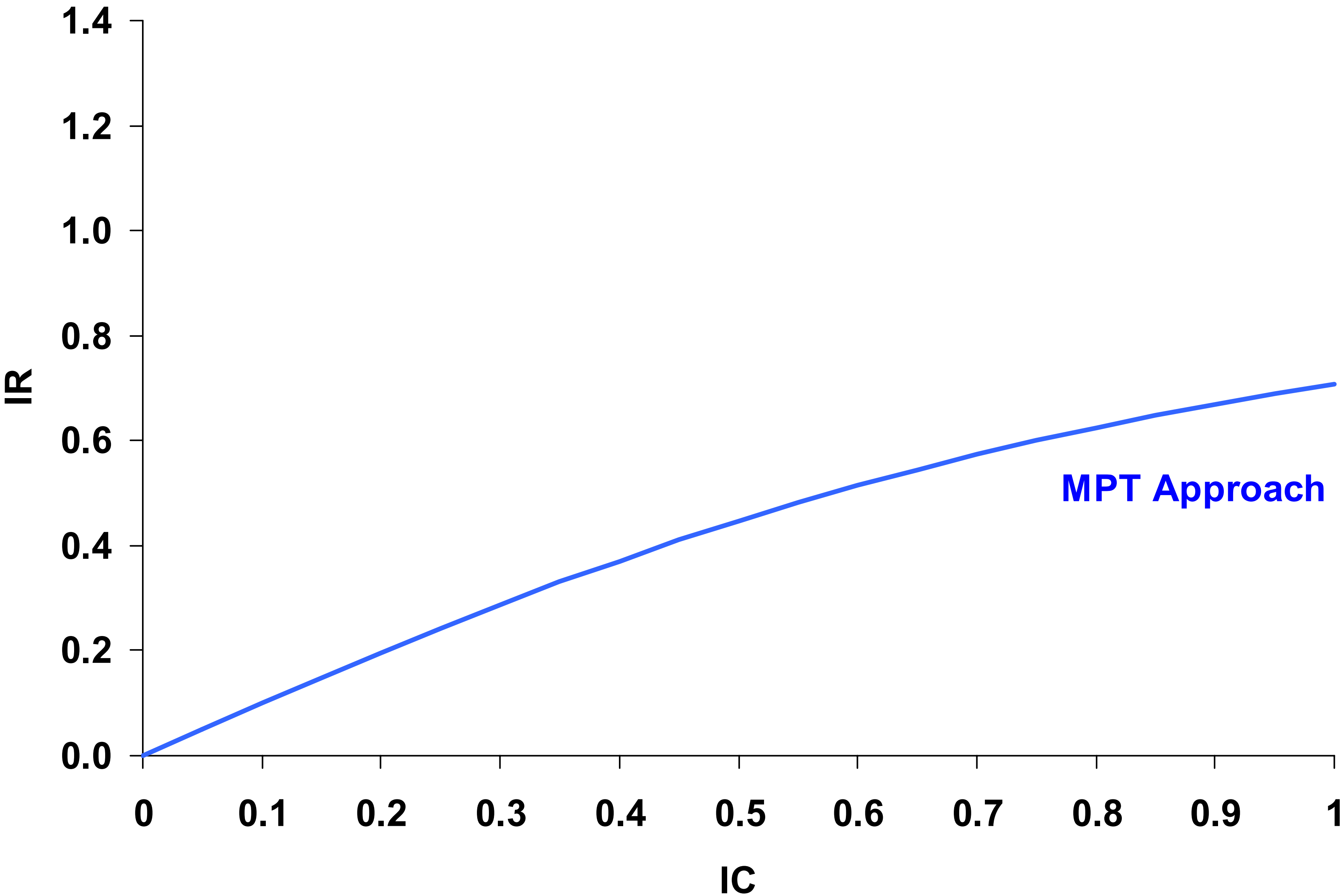
Application: Two specialists or one generalist?



Positions in the univariate case



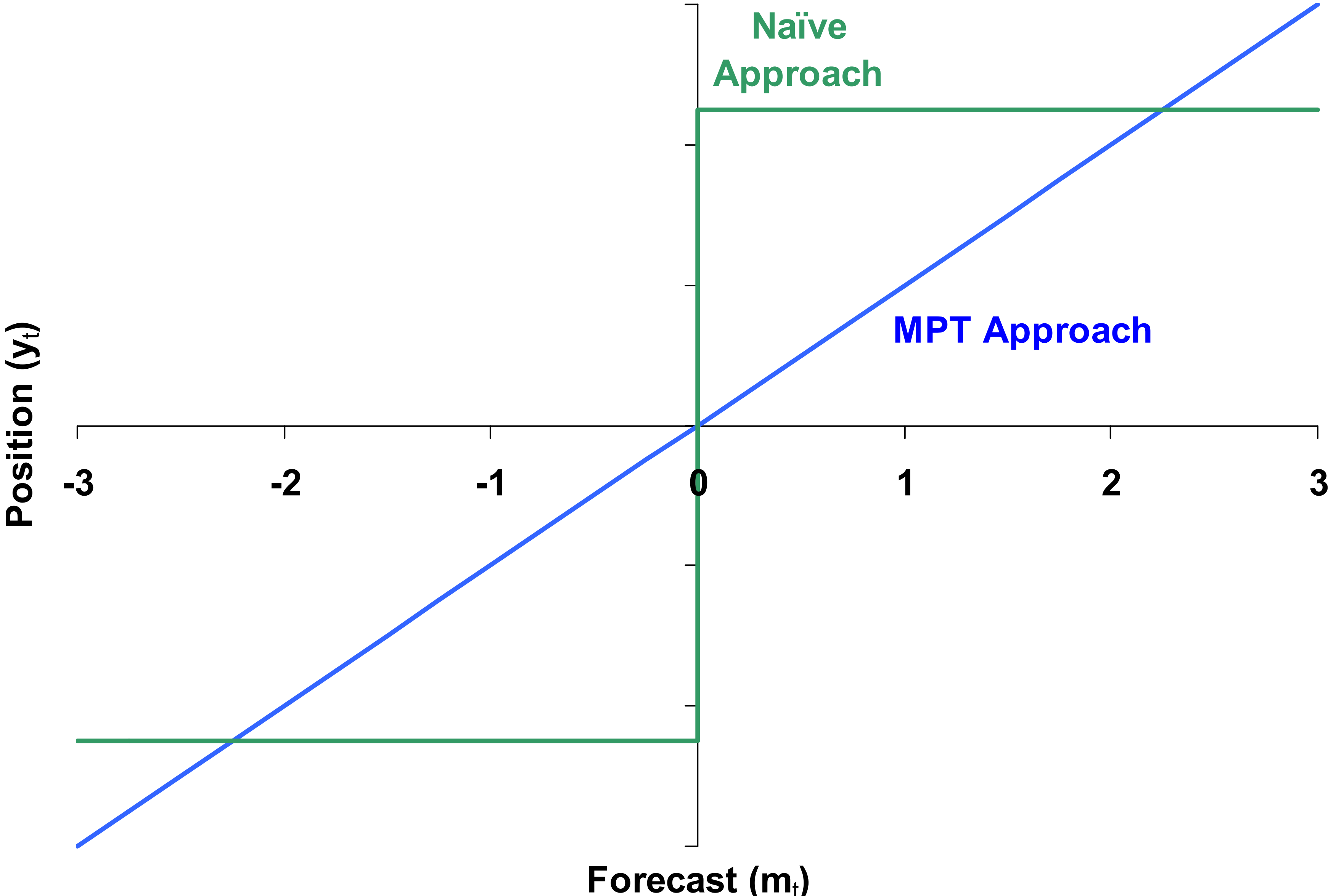
IR versus IC in the univariate case



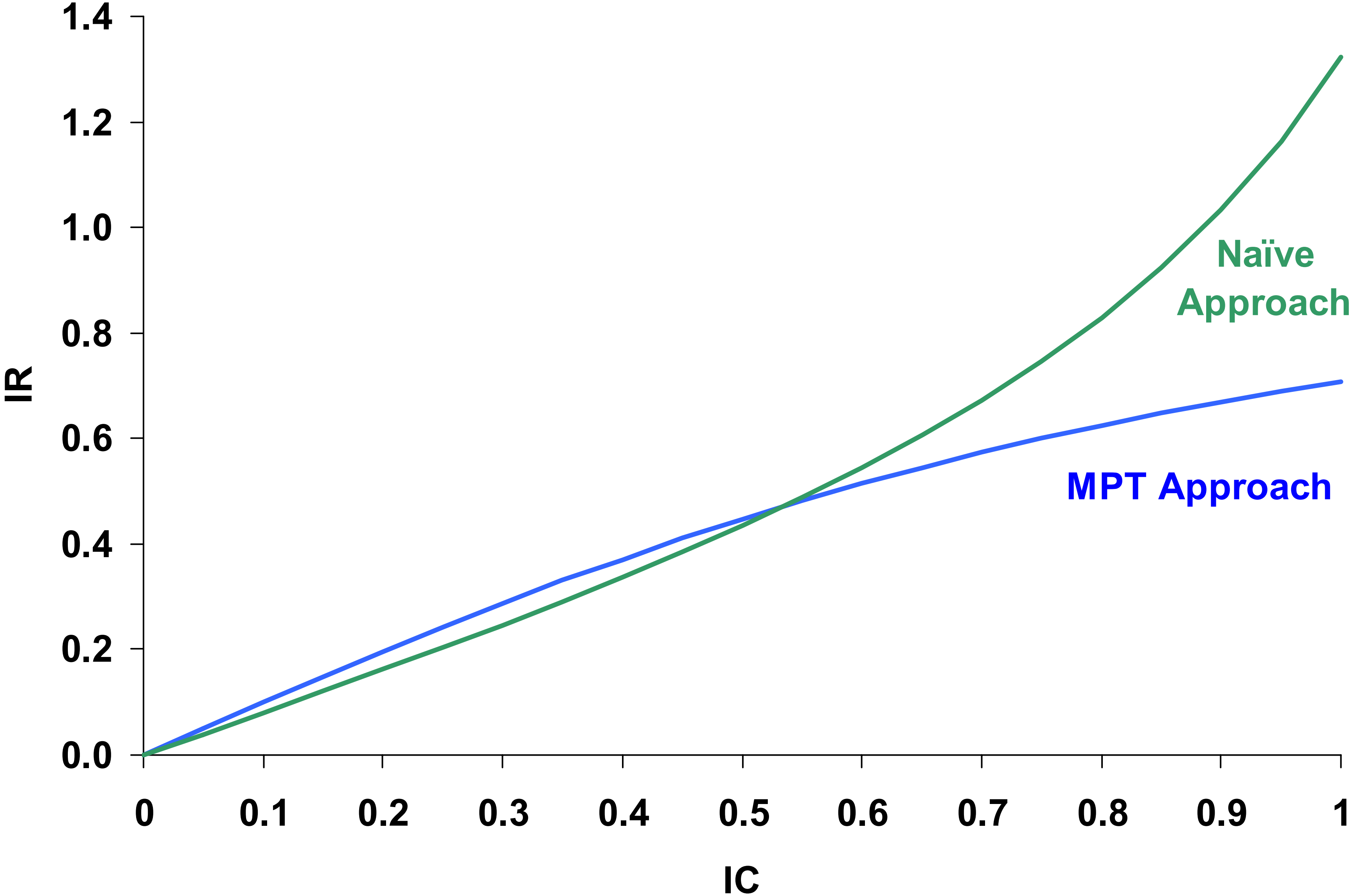
<u>Naïve Approach</u>: $\mathbf{y}_t = k \text{ sign}(\mathbf{m}_t)$

Univariate Naïve: $$IR = \frac{IC}{\sqrt{(\pi/2 - IC^2)}}$$

Positions in the univariate case



IR versus IC in the univariate case



THEOREM OF ACTIVE PORTFOLIO MANAGEMENT

The single period investment position that maximises the multiperiod information ratio is the result of a mean/second-moment optimisation

$$\mathbf{y}_t = c\,(\mathbf{m}_t\mathbf{m}_t^T + \mathrm{V})^{-1}\mathbf{m}_t$$

PROOF:

Schwarz ineq: For any random vectors U & S,

$$\mathrm{E}[\mathbf{U}^T\mathbf{U}] \,/\, \mathrm{E}[\mathbf{U}^T\mathbf{S}]^2 \geq \mathrm{E}[\mathbf{S}^T\mathbf{S}]^{-1}$$

with equality holding if and only if U = c**S**

Recall $h = \mathrm{E}[(\mathbf{y}^T\theta)^2] \,/\, \mathrm{E}[\mathbf{y}^T\theta]^2$

Set $\mathbf{U} = \mathbf{L}^T\mathbf{y}$ **and** $\mathbf{S} = \mathbf{L}^{-1}\mathbf{m}$ $\quad$ $(\mathbf{L}\mathbf{L}^T = \mathbf{m}\mathbf{m}^T + \mathrm{V})$

Then $h \geq \mathrm{E}[\mathbf{m}^T(\mathbf{m}\mathbf{m}^T + \mathbf{V})^{-1}\mathbf{m}]^{-1}$

with equality holding if and only if $\mathbf{y} = c(\mathbf{L}\mathbf{L}^T)^{-1}\mathbf{m}$

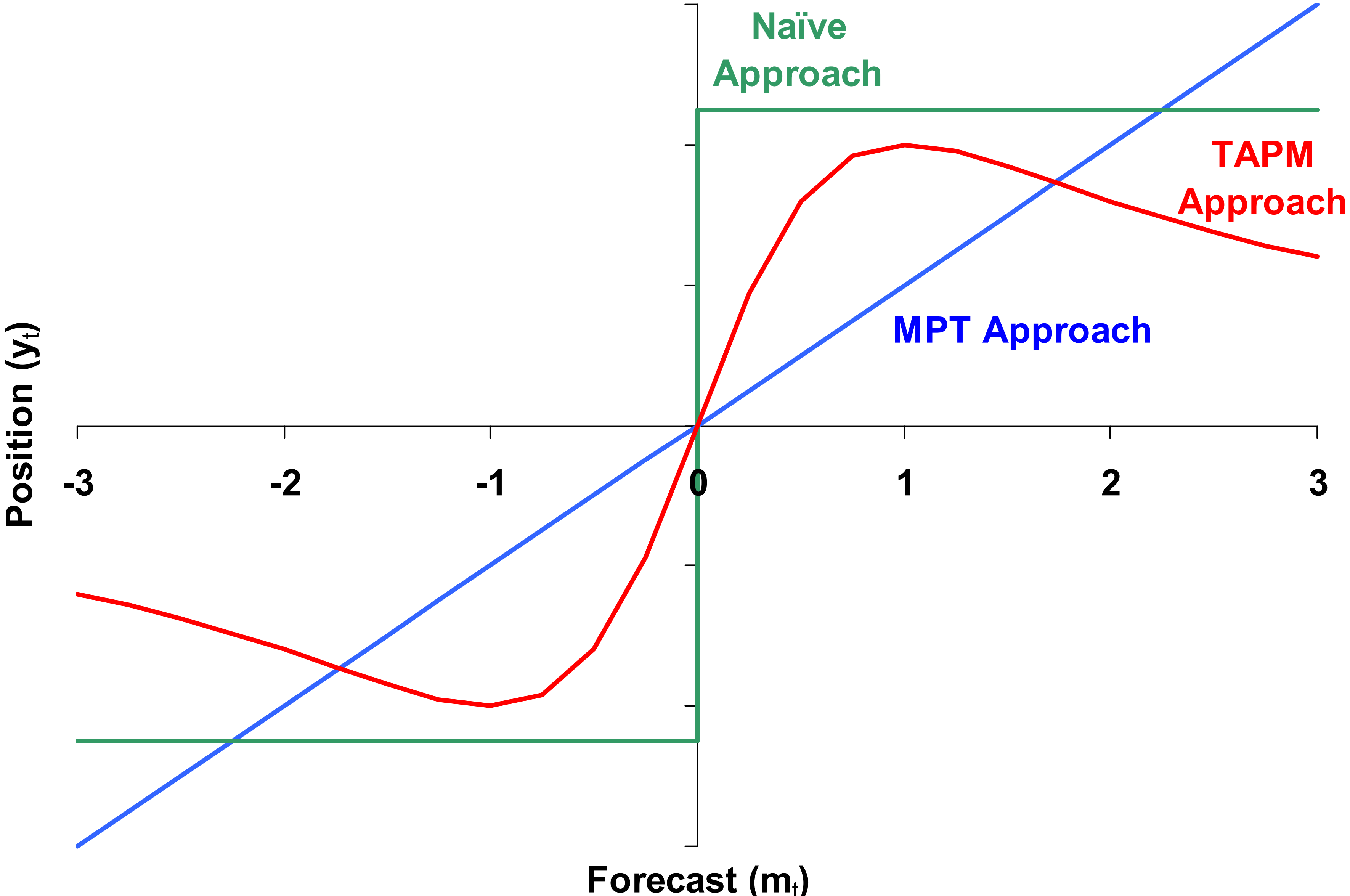
Univariate TAPM approach

$$y_t = \frac{c\, m_t}{m_t^2 + \nu^2}$$

$$IR = \sqrt{\frac{\phi(\xi)}{\xi\Phi(-\xi)} - 1}$$

where $\xi = \frac{\sqrt{1 - IC^2}}{IC}$

Positions in the univariate case



IR versus IC in the univariate case