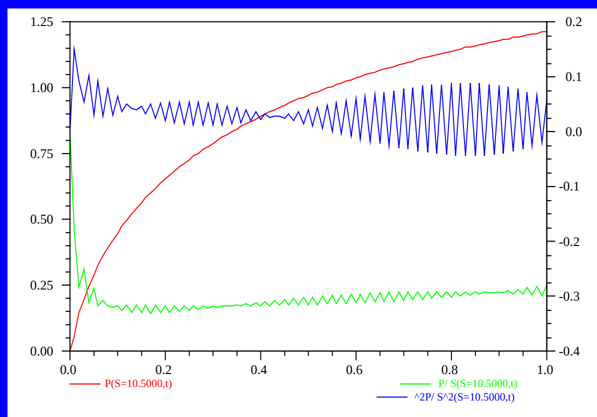
Pricing and Hedging Options

Some cautionary notes on the Emperor's tailoring

Centre for Financial Research JIMS



Pricing and Hedging Options

Some cautionary notes on the Emperor's tailoring

- Quantitative finance is NOT science.
- Nevertheless, it owes its influence to the numbers its mathematical models produce.
- It should be judged on the basis of good engineering practice.

- It is reasonable to ask how well have we done in the *30 years* since Black, Scholes and Merton's results.

How are we doing?

- Not very well by engineering standards.

Does this really matter?

- Yes, and it is going to matter a great deal more in the near future.
- The status quo is an unstable equilibrium and there are substantial perturbations on the way.

De-stabilising perturbations

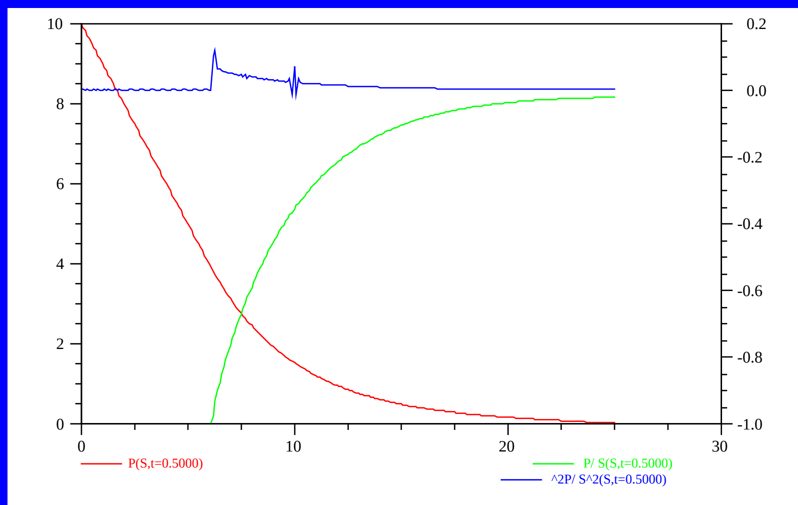
- FAS 133
- Sale of quantitative services by major banks or subsidiaries.
- Natural business growth depending on good pricing - e.g. clearing of OTC derivatives.
- Regulatory and/or client pressure.

Symptoms of the underlying problems

- Widespread experience of substantial pricing errors in “the new vanillas”
- Widespread experience of unreliable sensitivities even when prices are correct - even in the case of an American put.

How bad is this? An illustrative example.

- A widely used trading system allows the user to set a parameter value between 0 (an explicit finite difference scheme) and 1 (fully implicit) to price American puts. A value of 0.5 is the commonly recommended Crank Nicolson method.
- The following slides illustrate the unreliability of the resulting gammas for a simple American put with parameters in line with the current Brazilian market.
- Note: *Like price and delta, gamma is a smooth function- the oscillations are artefacts of the numerical algorithm.*



American put: vol 60%, risk free 30%, strike 10, duration 1 year Slice at t = 6 months. Crank Nicolson

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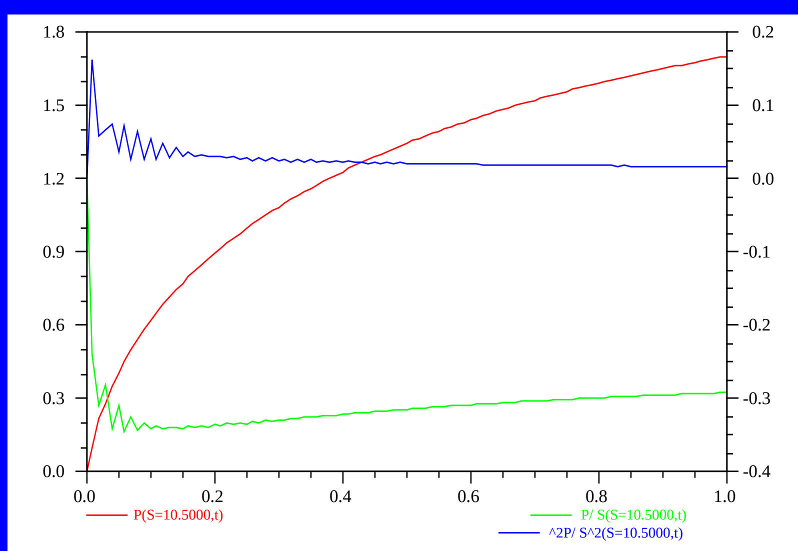

American put: vol 60%, risk free 30%, strike 10, duration 1 year. Slice through S =10.5. Crank Nicolson

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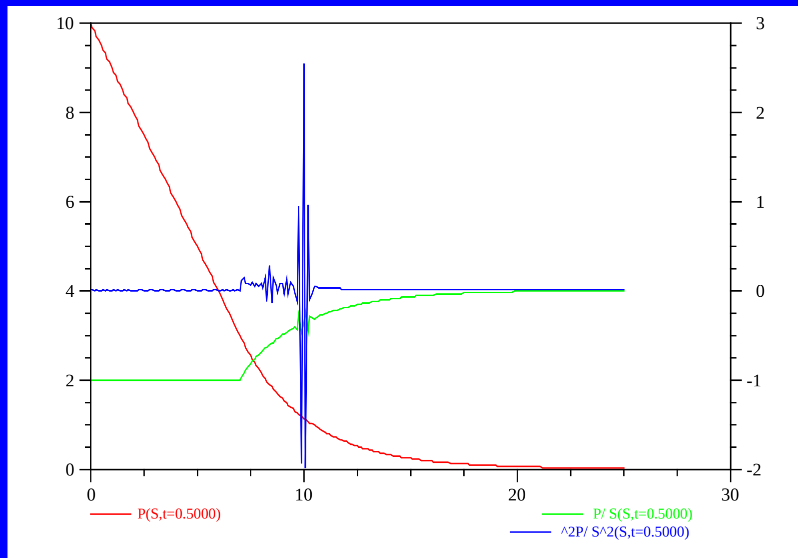
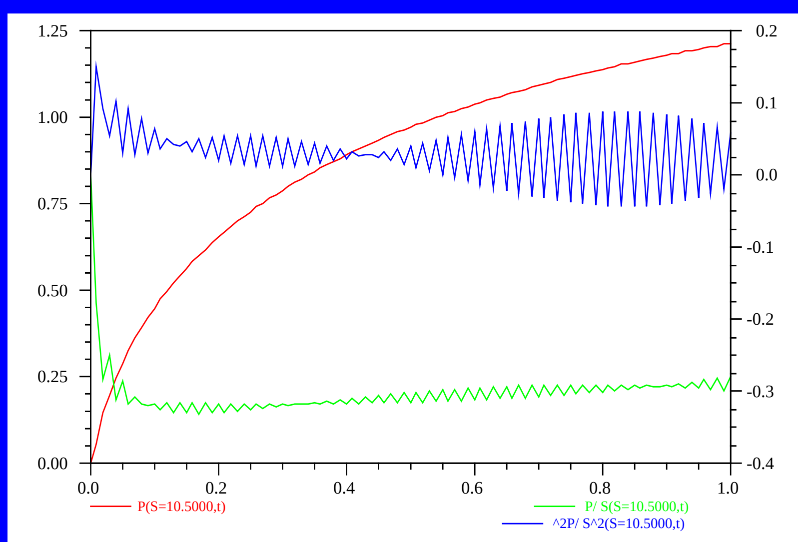


American put: vol 60%, risk free 30%, strike 10, duration 1 year. Slice at t = 6 months. Calculated at 0.48 where 0 is fully explicit, 0.5 is Crank Nicolson and 1.0 is implicit.



American put: vol 60%, risk free 30%, strike 10, duration 1 year. Slice through S =10.5
Calculated at 0.48 where 0 is fully explicit, 0.5 is Crank Nicolson and 1.0 is implicit.

Does it really matter?

- The system which allows the user this choice calculates prices more rapidly as the parameter is reduced from 1 to 0.
- These problems are not isolated. They are robust -i.e. they occur in open sets in the parameter space.
- Methods which produce such errors are extremely dangerous and require careful controls to avoid unsafe parameter regions. They will inevitably be applied in such regions through error or intent.
- An American put is the simplest path dependent option. If gamma is unreliable in this case what are the chances of it being better for barriers or more complex options?

Consequences of the status quo

Serious errors are being made in hedge costs.

Serious pricing errors are undoubtedly occurring in the valuation of ‘vanilla’ exotics such as lookbacks, Asians and barriers.

The consequent risks are large losses both financial and reputational.

How did we get here?

- The history of equity option pricing contains a series of mixed blessings.
- The numerical problems are surprisingly difficult.
- In spite of the enormity of the options business and the computational infrastructure which has been created for it, not enough attention has been paid to basic engineering principles.
- There has been too little meaningful dialogue between academia and industry.

Mixed Blessing 1

- Solvability in closed form of the Black Scholes Merton equation for European options with constant volatility and interest rates.

Upside:
• Instantly usable by everybody with a hand calculator.
• Encouraged extensions to more complex instruments.
• Allowed a plausible computation of 'implied volatility'.

Downside:
• Allowed both industry users and academics to side-step the issues of numerical computation in the simplest case.
• Led to countless wasted efforts to find closed form solutions for prices where none should ever have been expected.

Mixed Blessing 2

- The martingale formulation of the BSM option price and its associated numerical algorithms.

Upside:
• A beautiful and intuitively natural derivation of the price.
• Instantly usable by everybody with a computer.
• Allowed rapid expansion of the methods to more complex instruments.

Downside:
•Biased the choice of numerical technology in the pessimal direction.
• Encouraged indifference to a feature that standard mathematical practice tells us is always important.

Mixed Blessing 3

- The coincidence between the boom in demand for quantitative analysts and the bust in demand for theoretical physicists.

Upside:
• A pool of intelligent personnel with strong mathematical skills.

Downside:
• A pool of intelligent personnel with strong mathematical skills.

What was lacking was ENGINEERING skills.

Mixed Blessing 4

- Survivor bias and poor management .

Upside:
• The fact that survival was interpreted as evidence of skill combined with managers awed by ‘rocket science’ allowed rapid expansion–some of which was very successful.

Downside:
• Managers awed by ‘rocket science’ cannot carry out prudential risk assessment. This remark has been elevated to the status of a trading strategy on more than one occasion in more than one institution.

Numerical Difficulties

- The partial differential equations for equity option prices are simple but their boundary conditions complicate the problem to a surprising degree. Careful numerical analysis is essential, especially in computing the sensitivities.
- The apparently natural choices can fail miserably (e.g. explicit methods or the Crank Nicolson method)
- This is not a job for amateurs, however gifted. Serious expertise is required for correct, scalable techniques.

Basic engineering principles too often ignored

- If you can't solve the simple problems you can't solve the more general ones.
- It is critically important to distinguish between a mathematical model and the approximations, numerical algorithms and computer code which implement it in software.
- All models and implementations fail somewhere. It is essential to discover the boundaries of the safe operating environments of a model, as well as the approximations, numerical algorithms and software implementations. Error analysis is IMPORTANT.
- If there are two independent ways of doing a calculation always use one to check the other.
- Software that only fails some of the time is broken. A patch is not a repair.

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An example of the engineering approach

The usual presentation of the Black Scholes Merton model makes the unnecessary restriction to constant volatility and interest rates:

$$P_t + \tfrac{1}{2}\sigma^2 S^2 P_{SS} + r(SP_S - P) = 0$$

The derivation (including dividends which I ignore here for brevity) extends without modification to the (vastly) more general case where the stock price evolves according to:

$$\frac{dS}{S} = r(t)\,dt + \sigma(S,t)\,dw$$

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The result is: $P_t + \frac{1}{2}\sigma^2(S,t)S^2 P_{SS} + r(t)(SP_S - P) = 0$

This equation, with boundary conditions determined by the option details, is *the full Black Scholes Merton model for price*. Given this we should naturally ask and answer the following questions:

- What does this equation have to tell us about the market's volatility surface implicit in prices of exchange traded options?
- How well can we price exotics using this volatility surface and our best estimate of r(t)?
- What are the true sensitivities and how good are prices and hedges for given error bands in inputs?
- What are the optimal (measured by accuracy, speed, scalability and extendibility) numerical solution methods?

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With these answers in hand we can proceed to the critical question:

- What are the natural limitations of the BSM model?

This is the only rational approach to the question of the need expensive extensions such as stochastic volatility pricing models.

This program is an essential step for the extension to Asian options and other multifactor models, to real options and other of the myriad extensions of the original BSM model.

This is a program which can only be carried out in cooperation between academic research and industry.

I leave you with a prediction:

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This approach will be very profitable for academics and for the finance industry.