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Quantifying Extreme Operational Risk

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This presentation

- Extreme risk
- Why quantify extreme operational risk?
- Quantile models
- Modelling severity
- A small sample simulation workshop
- Severity and frequency aggregation

Mad Cows

- Extreme risk events are rare
- Rare events can have extreme consequences
  - Chernobyl, CJD, lottery jackpot, bank failure
- Balance of risk
  - DDT and inoculations
    - These save lives despite some damaging consequences

Operational Risk

- Losses from business activity other than market (price variability) and credit (non-repayment) and from external events

Why model?

- OR disasters such as at Barings 1995 and Daiwa Securities 1996 alerted regulators and financial institutions to the absence of pricing guidelines
- Pricing OR can highlight risky business units, assets, activities
- Objectivity, Predictability, Quantifiability

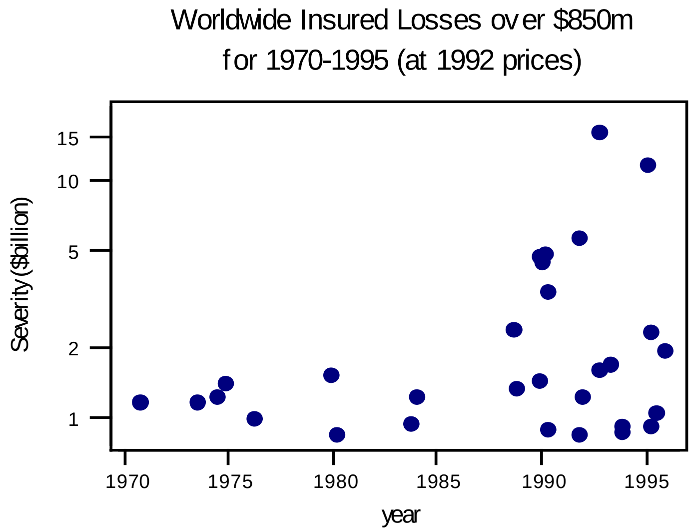
Basle Committee on Banking Supervision

- Capital allocation
  - internal or regulatory pricing
- Supervision and control
  - risk management
  - these absorb potential risk capital
- Transparent and consistent management
  - reporting, external pricing (scoring), hedging

US banking regulations

- The US banking regulators are set to impose a capital allocation to reserves to cover extreme loss due to operational risk
- This will be at the same rate for all banks regardless of the quality of management, control, and supervision
- This is because there is no agreed quantification process

Worldwide insured losses



Return value models

- Building regulations require
  - P(catastrophic loss in any year) < 1/50
- Nuclear plants, dams, bridges, sea dykes
  - P(catastrophic loss in any year) < 1/10,000
- These are return values
  - defined via quantiles

Return value models

- P(An extreme loss (> x) in a year) < p
- P(No loss > x in any year) > 1 - p gives
- P(No loss > x over k years) > $(1 - p)^k$
- The 100p% quantile gives the loss x exceeded in 100(1-p)% of the years
  - $$Q(p) = x$$

Nuclear Safety (USA)

- P(Meltdown in any given plant in a year) = 1/20000
- P(No meltdown in any of 600 plants in any year)
  = $(1 - 0.00005)^{600}$ = 0.970
- P(Meltdown in a plant in next 5 years) = 14%
- P(Meltdown in a plant in next 10 years) = 26%
- P(Meltdown in a plant in next 20 years) = 44%
- P(Meltdown in a plant in next 50 years) = 78%

Quantile models

$$Q(p) = \lambda + \eta\, Q_0(p)$$

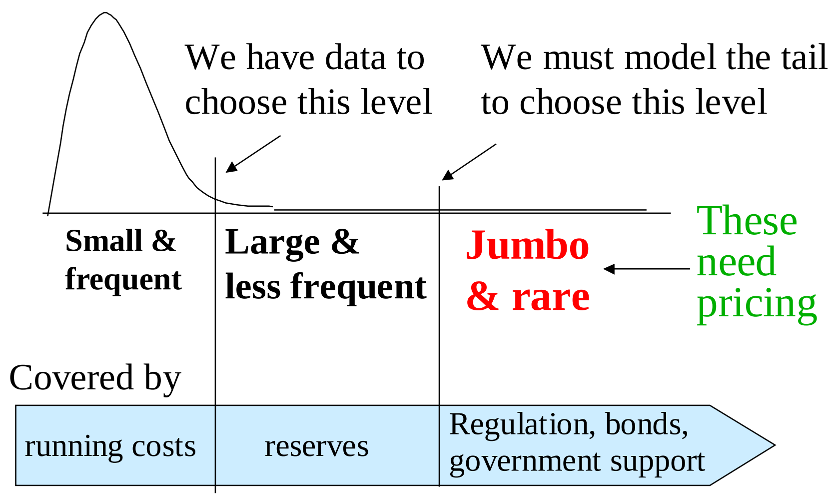
λ = location, η = scale, ξ = shape

Model	$Q_0(p)$
Normal	$\Phi^{-1}(p)$
Gumbel	$-\ln(-\ln p)$
Frechet	$(-\ln p)^{-\xi}$
Weibull	$-(-\ln p)^{\xi}$
GEV	$\frac{1}{\xi}\left\{1-(-\ln p)^{\xi}\right\}$

Quantile measures

- Lower quartile, Q(.5)
- Median, Q(.25)
- Upper quartile, Q(.75)
- Inter-quartile range, Q(.75) - Q(.25)
- Quartile difference, Q(.75) - 2Q(.5) + Q(.25)
- Galton skewness, G = QD/IQR
- and so on

Loss Severity



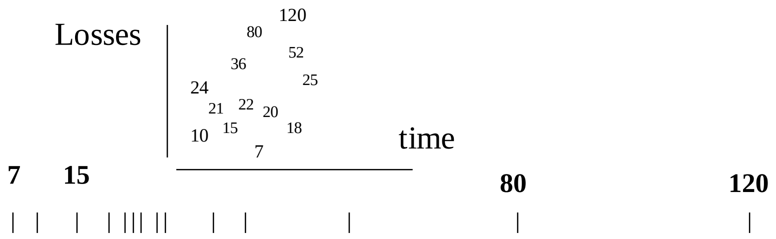
Estimating Quantiles

- The objective is to estimate the quantiles of the loss distribution or aggregate loss distribution (a Maximum-at-Risk measure)
- High quantiles like 99.99% can be too conservative, overpricing the risk

Modelling Severity

Losses in order of size:

7, 10, 15, 18, 20, 21, 22, 24, 25, 32, 36, 52, 80, 120



Estimated Location (Sample Mean) = **34.4**
Estimated Scale (Sample Standard Deviation) = **31.0**
Estimated Shape (pdf) ?? — non-normal

Probability plots: two parameter distributions

Probability plots

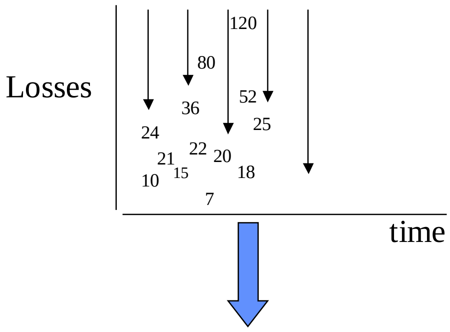
Data: 7 10 15 18 20 21 22 24 25 32 36 52 80 120

Distribution	95%	95% CI		width	99%	95% CI		width	120
Normal	83	60	107	47	104	74	134	60	99.7
Lognormal	86	48	154	106	141	68	295	227	97.9
Gumbel	94	73	116	43	111	87	135	48	>99
Weibull	87	56	134	78	121	74	198	124	99
Logistic	70	45	95	50	94	59	129	70	>99
Loglogistic	83	43	161	118	162	64	411	347	98

Modelling Severity

- With many small losses and very few very large, the distribution shows
  - skewness (asymmetry with a long right tail)
  - kurtosis (leptokurtic, peakedness)
- We fit the Generalised Extreme Value Distribution (GEV) model taken from Extreme Value Theory:
  - a three-parameter model

Generalised Extreme Value Distribution



The theoretical limiting case for maxima over equal time periods gives the GEV for severity

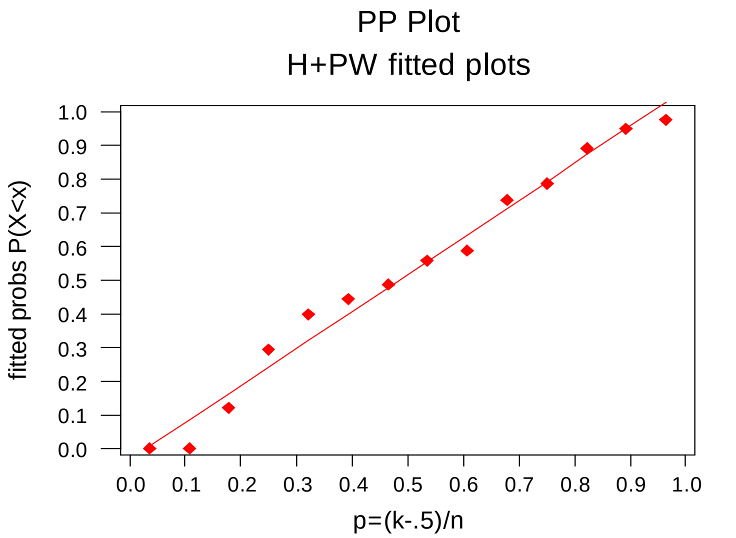
Fitting a GEV

7
10
15
18
20
21
22
24
25
32
36
52
80
120

- Hill estimates shape xi
- Probability Weighted Moments estimates location mu and scale psi and/or xi

	Hill+PWM	**PWM fit**
mu	**19.361**	**20.131**
psi	**7.270**	**13.087**
xi	**0.607**	**0.347**
r	**0.991**	**0.986**
90%	**54.319**	**64.762**
95%	**80.029**	**88.127**
99%	**202.711**	**168.520**

GEV P-P probability plot



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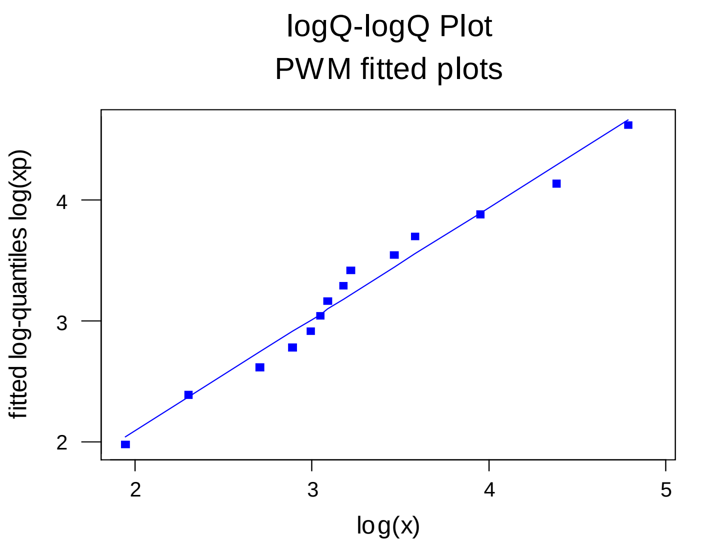
GEV logQ-logQ probability plot



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80
120

Testing model fit

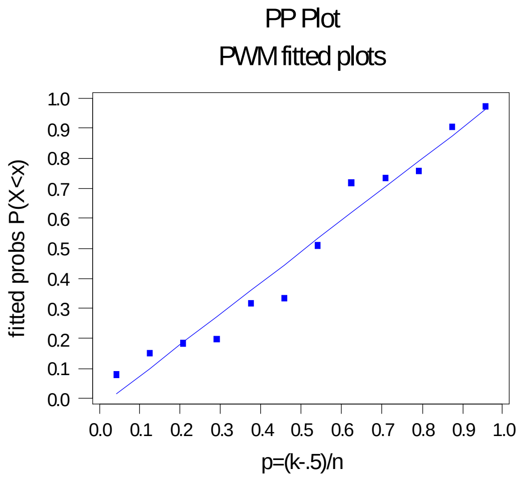
- There is as yet no formal test for fit for three-parameter models such as the GEV
  - The Kolmogorov-Smirnov test lacks power
  - Probability plots (P-P, Q-Q) and other diagnostic plots require skilled interpretation

Example: Frauds in a UK retail bank

The largest fraud (or attempted fraud) of the month

		1994	1995	1996	1997	1998
Largest maximum	1	*907,077*	*1,100,000*	*6,600,000*	*600,000*	*1,820,000*
	2	*845,000*	*650,000*	*3,950,000*	*394,672*	*750,000*
	3	*734,900*	*556,000*	*1,300,000*	*260,000*	*426,000*
	4	*550,000*	*214,635*	*410,061*	*248,342*	*423,320*
	5	*406,001*	*200,000*	*350,000*	*239,103*	*332,000*
	6	*360,000*	*160,000*	*200,000*	*165,000*	*294,835*
	7	*360,000*	*157,083*	*176,000*	*120,000*	*230,000*
	8	*350,000*	*120,000*	*129,754*	*116,000*	*229,369*
	9	*220,357*	*78,375*	*109,543*	*86,878*	*210,537*
	10	*182,435*	*52,049*	*107,031*	*83,614*	*128,412*
	11	*68,000*	*51,908*	*107,000*	*75,177*	*122,650*
smallest maximum	12	*50,000*	*47,500*	*64,600*	*52,700*	*89,540*

Probability plot for 1997



	Hill+PW	PWM
mu	119.56	128.190
psi	42.41	89.510
xi	0.59	0.213
r	0.97	0.982
90%	319.20	386.630
95%	463.15	499.087
99%	1136.49	827.465

Case Study

- Maximum loss selected for each month
- Rolling 12 maxima to fit GEV by Hill's estimator for shape, and PWM (probability weighted moments) for location and scale

Year to December	1992	1993	1994	1995	1996
95% quantile (£m)	2.9	5.9	14.0	1.1	5.1
99% quantile (£m)	12.9	29.2	122.9	3.3	28.1

Cruz, Coleman & Salkin, 1998, 2000

Limitations

- A simulation study using data from 12-samples shows that Hill plus PWM give unstable estimation of quantiles
  - Sample sizes of about 100 seem to be required
  - Other estimation methods fare worse
- From 30 independent GEV(0,1,0.5) samples

Quantile	95%	99%
True value	6.83	17.95
Average of 30 independent ests	5.81	15.85
Estimated standard error	3.06	12.95

Small sample estimation

- Sample sizes are small
- Historical data are unreliable
- An extreme loss in a small sample is over-represented (EL case)
- No extreme loss in a small sample under-represents extreme losses (NEL case)
- Fitting a small sample will not model the true situation

Does it matter?

- In the case study, the rolling data allowed the impact of extreme loss to be seen, and its effect to decline in time
- It provided a price to be set which reflected the occurrence of extreme loss and which could be compared with hedge prices

Simulation workshop

- The workshop sought a compromise using ideas from **influence analysis**
- The influence of the largest value in each case would be used to make it **just influential**
- **This does not solve the small sample problem**

Simulation workshop ...

- Assume censoring at the just influential point and estimate using maximum likelihood
- Unlikely to solve the small sample problem

Simulation workshop ...

- Bayesian statistics:
  - Impose probabilistic constraints which restrict the model and its parameter values to “acceptable” ranges
  - Data are often from time-varying processes, so
    - For rolling data, we update parameter estimates, either by Bayes’ rules, or by centring at the most recent estimated values
  - Use Bayes hierarchical modelling of parameter values (Medova 2000)

Simulation workshop ...

$$F(x) = (1-\alpha)F_{NEL}(x) + \alpha F_{EL}(x) \ (\alpha \text{ small})$$

- Treat EL as contamination
  - Model as a mixture of an NEL distribution and an EL distribution
  - This needs prior experience
  - And extensive trials
- No simple quantile function

Statistical modelling of loss data

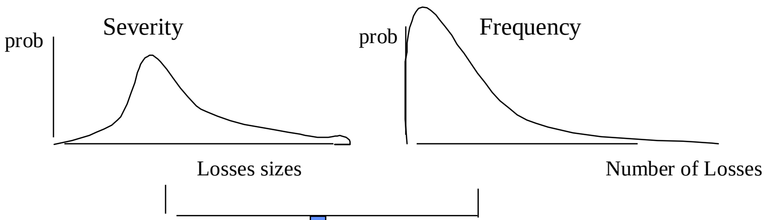
- Estimating severity (loss size distribution)
  - Choosing a distribution, and testing its fit
  - Estimating its parameters and quantiles
- Estimating frequency (loss rate distribution)
  - Model the process of loss events
- Aggregating severity and frequency
  - Simulation, validation and testing

Loss Frequency : Poisson Process

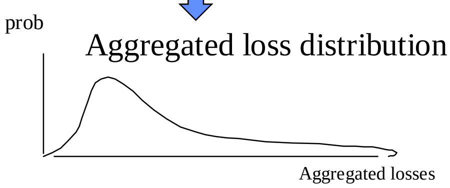
- The Poisson Point Process is the model generally used for the time points at which a random sequence of events occur
- The points occur at random at a constant rate

Aggregating Severity/Frequency Models for the joint process

Aggregated Loss → **risk transfer premiums**



Alternatives:
- FFT
- Panjer Algorithm
- Recursion



Aggregated loss distribution $\sum_{n=0}^{\infty} p_n F_X^{*n}(x)$

Needs simulation

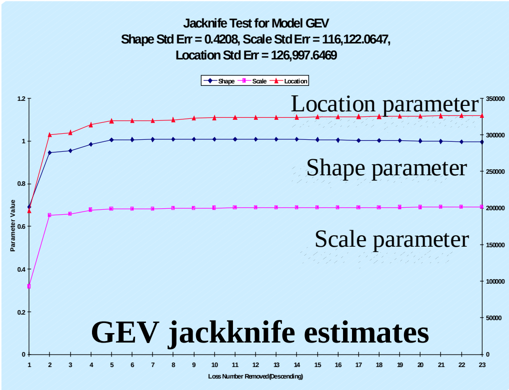
No analytic solution

Marked point processes

- The points are the times of loss events
- The marks show the loss
  - and cause, activity, realised loss/near-miss, etc
- Allows relationships between loss and severity and correlations with other loss predictors
- Allows time-varying behaviour modelling

Resampling Techniques

- The **Jackknife** and **Bootstrap** are techniques that can reduce estimator bias and create confidence intervals
- The **jackknife** resamples to give estimates each with one observation omitted
- The **bootstrap** resamples (with replacement) to give a set of parameter estimates that can be used to obtain confidence intervals



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