Advanced Financial Models Michael Tehranchi

Self-financing condition

In a discrete-time market with prices $(P_t)_{t\geq 0}$ and dividends $(\delta_t)_{t\geq 1}$ a previsible process $(H_t)_{t\geq 1}$ is *self-financing* iff

$$H_{t+1}\cdot P_t = H_t\cdot(P_t+\delta_t) \text{ for all } t\geq 1.$$

This simply means it is a pure invesment strategy, there is neither consumption nor external income.

If there are no dividends, the discrete-time self-financing condition becomes

$$(*) \qquad H_{t+1}\cdot P_{t+1} - H_t\cdot P_t = H_{t+1}\cdot(P_{t+1}-P_t)$$

For a continuous-time market with no-dividends, we take the analog of equation (*) as the definition of a self-financing strategy: a previsible process H such that

$$d(H_t\cdot P_t) = H_t\cdot dP_t$$

with the additional technical condition that

$$\int_0^t \sum_{i,j} H_s^i H_s^j \; d\langle P^i, P^j\rangle_s < \infty \text{ almost surely for all } t\geq 0$$

so that the stochastic integral $\int_0^t H_s dP_s$ is defined.