Advanced Financial Models Michael Tehranchi

Sample question 4

(a) By Itô's formula,

$$\begin{aligned} d\xi_t =& dV(t, S_t) \\ =& \left(\frac{\partial}{\partial t} V(t, S_t) + \frac{a(S)^2}{2} \frac{\partial^2}{\partial S^2} V(t, S_t) \right) dt \\ &+ \frac{\partial}{\partial S} V(t, S_t) a(S_t) dW_t \end{aligned}$$

so ξ is a local martingale as it is the stochastic integral with respect to Brownian motion. Since V is bounded, so is ξ, and bounded local martingales are true martingales.

(b) By part (a) we have

$$d\xi_t = \frac{\partial}{\partial t} V(t, S_t) dS_t$$

and hence

$$\pi_t = \frac{\partial}{\partial t} V(t, S_t).$$

Now differentiate the PDE for V to arrive at the given PDE for U.