By the second fundamental theorem of asset pricing, for fixed $T > 0$, there exists a unique equivalent martingale measure $Q$ on $(\Omega, \mathcal{F}_T)$ for the numéraire asset with price $N$, so that $(S_t/N_t)_{0 \leq t \leq T}$ is a $Q$-martingale.

Claim: If $\xi_t$ is the payout of a replicable contingent claim with maturity $t$, then the initial cost of replication of this claim is $N_0 \mathbb{E}_Q(\xi_t/N_t)$.

Proof. Suppose that $(H_s)_{1 \leq s \leq t}$ is self-financing, i.e. such that $H_s \cdot P_s = H_{s+1} \cdot P_s$ for all $1 \leq s \leq t - 1$. Let $X_0 = H_1 \cdot P_0$ and $X_s = H_s \cdot P_s$ for $1 \leq s \leq t$. Then $X/N$ is a $Q$-martingale. Indeed, for $1 \leq s \leq t$ we have

$$\mathbb{E}_Q(X_s/N_s | \mathcal{F}_{s-1}) = \mathbb{E}_Q(H_s \cdot P_s/N_s | \mathcal{F}_{s-1})$$

$$= H_s \cdot \mathbb{E}_Q(P_s/N_s | \mathcal{F}_{s-1})$$

$$= H_s \cdot P_{s-1}/N_{s-1}$$

$$= X_{s-1}$$

[Since the market is complete, all random variables are bounded, so there is no concern about integrability in the conditional expectations.] In particular, if $\xi_t = X_t$, then $N_0 \mathbb{E}_Q(\xi_t/N_t) = X_0 = H_1 \cdot P_0$, the initial cost of the replicating strategy.

Now, the initial cost of a replicating strategy for a call option with payout $\xi_t = (S_t - K)^+$ for $0 \leq t \leq T$ is given by

$$C(t, K) = N_0 \mathbb{E}_Q\left[\frac{1}{N_t}(S_t - K)^+\right]$$

Now

$$C(t + 1, K) = N_0 \mathbb{E}_Q\left[\frac{S_{t+1}}{N_{t+1}} - \frac{K}{N_{t+1}}\right]^+$$

$$\geq N_0 \mathbb{E}_Q\left[\frac{S_{t+1}}{N_{t+1}} - \frac{K}{N_t}\right]^+$$

since $N_{t+1} \geq N_t > 0$ a.s.

$$= N_0 \mathbb{E}_Q\left\{\mathbb{E}_Q\left[\frac{S_{t+1}}{N_{t+1}} - \frac{K}{N_t}\right] | \mathcal{F}_t\right\}$$

tower

$$\geq N_0 \mathbb{E}_Q\left\{\mathbb{E}_Q\left[\frac{S_{t+1}}{N_{t+1}} | \mathcal{F}_t\right] - \frac{K}{N_t}\right\}$$

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$$= N_0 \mathbb{E}_Q\left[\left(\frac{S_t}{N_t} - \frac{K}{N_t}\right)^+\right]$$

martingale

$$= C(t, K)$$

where we have used the $\mathcal{F}_t$-measurability of $N_t$ and the convexity of $x \mapsto x^+$. 

Remark 1. One could, in principle, discuss completeness without discussing arbitrage. For instance, consider the one-period case, with no dividends. The market is complete iff for
every contingent claim with payout $\xi_1$ there exists a portfolio $H_1$ such that $\xi_1 = H_1 \cdot P_1$ almost surely. Note that this definition does not even mention the initial prices $P_0$. So one could, in principle, even discuss one-period completeness without even assuming that the market has time-0 prices.

However, question 2(b) discussed above asks about the initial replication cost $H_1 \cdot P_0$, so without some assumption on the initial prices, this problem would be impossible.

(By the way, we were able to prove that completeness implies that there are only a finite number of disjoint events of positive probability without assuming no-arbitrage. But in general, there is little point discussing completeness for a market that has arbitrage.)