Advanced Financial Models

More on the definition of numéraire

Given a price process $P = (P_t)_{t \ge 0}$ and assuming no dividends, a numéraire strategy $\eta = (\eta_t)_{t \ge 1}$ is such that

- $\eta_t \cdot P_t = \eta_{t+1} \cdot P_t$ for all $t \ge 1$
- $\eta_{t+1} \cdot P_t > 0$ almost surely for all $t \ge 0$.

Since the above definition looks a bit asymmetrical, we can include a period (-1, 0] numéraire portfolio by defining $\eta_0 = \eta_1$. This is okay, since we always assume \mathcal{F}_0 is trivial, so the period (0, 1] portfolio η_1 is not random. By including η_0 , the defining properties are now

- $\eta_t \cdot P_t = \eta_{t+1} \cdot P_t$ for all $t \ge 0$
- $\eta_t \cdot P_t > 0$ almost surely for all $t \ge 0$.

Recall that a numéraire asset is an asset with strictly positive price for all time. Suppose asset *i* is a numéraire asset. Then the the constant strategy $\eta_t = (0, \ldots, 0, 1, 0, \ldots, 0)$ of holding one share of asset *i* for all $t \ge 1$ is a numéraire strategy.