Advanced Financial Models Michael Tehranchi

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Given an n-dimensional Itô process R, let H be an n-dimensional previsible process such that

$$d(H_t \cdot R_t) = H_t \cdot dR_t$$

Let Z be a scalar Itô process. Claim:

$$(*) \qquad d(H_t \cdot R_t \; Z_t) = H_t \cdot d(R_t Z_t)$$

Indeed, by Itô's formula in the form

$$d(U_t V_t) = U_t dV_t + U_t dV_t + d\langle U, V\rangle_t$$

we have

$$\begin{aligned} d(H_t \cdot R_t \; Z_t) &= Z_t d(H_t \cdot R_t) + H_t \cdot R_t dZ_t + d\langle H \cdot R, Z\rangle_t \\ &= H_t \cdot Z_t \; dR_t + H_t \cdot R_t dZ_t + \sum_{i=1}^{n} H_t^i \langle R^i, Z\rangle_t \\ &= H_t \cdot d(R_t Z_t) \end{aligned}$$

In particular, if P is a price process and Y is a martingale deflator, if the process H satisfies

$$d(H_t \cdot P_t Y_t) = H_t \cdot d(P_t Y_t)$$

then we also have

$$d(H_t \cdot P_t) = H_t \cdot dP_t.$$

Proof: use equation (*) with $R_t = P_t Y_t$ and $Z_t = 1/Y_t$.