Let $Y$ and $N$ be positive adapted processes. Assume that $NY$ is a martingale. Fix $T > 0$, and let define a measure $Q$ on $(\Omega, \mathcal{F}_T)$ by

$$\frac{dQ}{dP} = \frac{N_T Y_T}{N_0 Y_0}.$$ 

Now suppose the adapted process $X$ is such that $XY$ is a $P$-martingale. Claim: $X/N$ is a $Q$-martingale.

**Proof:** By Bayes’ formula we have

$$E^Q \left( \frac{X_T}{N_T} \big| \mathcal{F}_t \right) = \frac{E^P (X_T Y_T \big| \mathcal{F}_t)}{E^P (N_T Y_T \big| \mathcal{F}_t)} = \frac{X_t Y_t}{N_t Y_t} = \frac{X_t}{N_t}.$$

**Remark 1.** Let $Y$ be a local martingale deflator, $N$ the price of a numéraire asset and $X$ the wealth from a self-financing strategy. In all cases we know that both $NY$ and $XY$ are local martingales. However, suppose that $NY$ happens is a true martingale. Then we can define an equivalent martingale measure with respect to the numéraire. If $XY$ also happens to be a true martingale for $P$, then the above calculation shows that the discounted wealth $X/N$ is a true martingale for $Q$.

We used this idea around pages 80 of the notes, when when finding minimal cost replicating strategies via the martingale representation theorem. Indeed, the first step was to define a true martingale $M$ via $M_t = E^P (\xi_T Y_T \big| \mathcal{F}_t)$ and then show that there exists a self-financing strategy whose wealth process satisfies $X_t Y_t = M_t$. So the claim says $X_t = N_t E^Q (\xi_T / N_T \big| \mathcal{F}_t)$. 
