**Advanced Financial Models** Michael Tehranchi

More on example sheet 2.6(g)(6)

Let $(Y_t)_{t\geq 0}$ be a martingale deflator. Since $YP^T$ is a martingale, we have for $0 \leq t \leq T$ that

$$Y_t P_t^T = \mathbb{E} Y_T P_T^T | \mathcal{F}_t]$$

Using the fact that $P_T^T = 1$ (i.e. at maturity, the price of a bond is its face value) we have

$$P_t^T = \frac{1}{Y_t} \mathbb{E}[Y_T | \mathcal{F}_t].$$

Now, the spot rate is

$$\begin{aligned} r_T &= \frac{1}{P_{T-1}^T} - 1 \\ &= \frac{Y_{T-1}}{\mathbb{E}[Y_T | \mathcal{F}_{T-1}]} - 1 \end{aligned}$$