Let \((Y_t)_{t \geq 0}\) be a martingale deflator. Since \(YP^T\) is a martingale, we have for \(0 \leq t \leq T\) that

\[ Y_t P_t^T = \mathbb{E} Y_T P_T^T | \mathcal{F}_t \]  

Using the fact that \(P_T^T = 1\) (i.e. at maturity, the price of a bond is its face value) we have

\[ P_t^T = \frac{1}{Y_t} \mathbb{E} [Y_T | \mathcal{F}_t]. \]

Now, the spot rate is

\[ r_T = \frac{1}{P_{T-1}^T} - 1 = \frac{Y_{T-1}}{\mathbb{E} [Y_T | \mathcal{F}_{T-1}]} - 1 \]