Advanced Financial Models Michael Tehranchi

More on 2019 exam 1(d)

By part (a), there exists no vector $H \in \mathbb{R}^n$ such that $H \cdot X \geq 0$ almost surely and $\mathbb{P}(H \cdot X > 0) > 0$. By part (c), there must exist a bounded minimising sequence.

Looking at the solution of part (b) (or the proof of the hard direction of the one-period 1FTAP) for ever $c \in \mathbb{R}$, the random variable

$$\rho_c = \frac{e^{-h_c \cdot X} \zeta_c}{F_c(h_c)}$$

where ζ_c is given by the hint, is such that

$$\mathbb{E}(\rho_c X) = 0.$$

But by the assumption of the question, we have $\rho_0 = \rho_1$ and hence

$$Y = \log(F_1(h_1)/F_0(h_0)) + (h_1 - h_0) \cdot X$$

as desired.