1/I/7D **Statistics**

The fast-food chain McGonagles have three sizes of their takeaway haggis, Large, Jumbo and Soopersize. A survey of 300 randomly selected customers at one branch choose 92 Large, 89 Jumbo and 119 Soopersize haggises.

Is there sufficient evidence to reject the hypothesis that all three sizes are equally popular? Explain your answer carefully.

$$\left[\begin{array}{lcccccccc} \textit{Distribution} & t_1 & t_2 & t_3 & \chi^2_1 & \chi^2_2 & \chi^2_3 & F_{1,2} & F_{2,3} \\ \textit{95\% percentile} & 6{\cdot}31 & 2{\cdot}92 & 2{\cdot}35 & 3{\cdot}84 & 5{\cdot}99 & 7{\cdot}82 & 18{\cdot}51 & 9{\cdot}55 \end{array}\right]$$

1/II/18D **Statistics**

In the context of hypothesis testing define the following terms: (i) simple hypothesis; (ii) critical region; (iii) size; (iv) power; and (v) type II error probability.

State, without proof, the Neyman–Pearson lemma.

Let X be a single observation from a probability density function f. It is desired to test the hypothesis

$$H_0 : f = f_0 \qquad \text{against} \qquad H_1 : f = f_1,$$

with $f_0(x) = \frac{1}{2}|x|\, e^{-x^2/2}$ and $f_1(x) = \Phi'(x)$, $-\infty < x < \infty$, where $\Phi(x)$ is the distribution function of the standard normal, $N(0,1)$.

Determine the best test of size α, where $0 < \alpha < 1$, and express its power in terms of Φ and α.

Find the size of the test that minimizes the sum of the error probabilities. Explain your reasoning carefully.

2/II/19D **Statistics**

Let $X_1, \ldots, X_n$ be a random sample from a probability density function $f(x \mid \theta)$, where θ is an unknown real-valued parameter which is assumed to have a prior density $\pi(\theta)$. Determine the optimal Bayes point estimate $a(X_1, \ldots, X_n)$ of θ, in terms of the posterior distribution of θ given $X_1, \ldots, X_n$, when the loss function is

$$L(\theta, a) = \begin{cases} \gamma(\theta - a) & \text{when } \theta \geqslant a, \\ \delta(a - \theta) & \text{when } \theta \leqslant a, \end{cases}$$

where γ and δ are given positive constants.

Calculate the estimate explicitly in the case when $f(x \mid \theta)$ is the density of the uniform distribution on $(0, \theta)$ and $\pi(\theta) = e^{-\theta}\theta^n/n!$, $\theta > 0$.

3/I/8D **Statistics**

Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, where μ and σ^2 are unknown. Derive the form of the size-α generalized likelihood-ratio test of the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, and show that it is equivalent to the standard t-test of size α.

[*You should state, but need not derive, the distribution of the test statistic.*]

4/II/19D **Statistics**

Let $Y_1, \ldots, Y_n$ be observations satisfying

$$Y_i = \beta x_i + \epsilon_i, \quad 1 \leqslant i \leqslant n,$$

where $\epsilon_1, \ldots, \epsilon_n$ are independent random variables each with the $N(0, \sigma^2)$ distribution. Here $x_1, \ldots, x_n$ are known but β and σ^2 are unknown.

(i) Determine the maximum-likelihood estimates $(\widehat{\beta}, \widehat{\sigma}^2)$ of (β, σ^2).

(ii) Find the distribution of $\widehat{\beta}$.

(iii) By showing that $Y_i - \widehat{\beta} x_i$ and $\widehat{\beta}$ are independent, or otherwise, determine the joint distribution of $\widehat{\beta}$ and $\widehat{\sigma}^2$.

(iv) Explain carefully how you would test the hypothesis $H_0 : \beta = \beta_0$ against $H_1 : \beta \neq \beta_0$.