1/I/3H Statistics

State the factorization criterion for sufficient statistics and give its proof in the discrete case.

Let $X_1, \ldots, X_n$ form a random sample from a Poisson distribution for which the value of the mean θ is unknown. Find a one-dimensional sufficient statistic for θ.

1/II/12H Statistics

Suppose we ask 50 men and 150 women whether they are early risers, late risers, or risers with no preference. The data are given in the following table.

	Early risers	*Late risers*	*No preference*	*Totals*
Men	17	22	11	50
Women	43	78	29	150
Totals	60	100	40	200

Derive carefully a (generalized) likelihood ratio test of independence of classification. What is the result of applying this test at the 0.01 level?

[*Distribution* χ^2_1 χ^2_2 χ^2_3 χ^2_5 χ^2_6
99%*percentile* *6.63* *9.21* *11.34* *15.09* *16.81*]

2/I/3H Statistics

Explain what is meant by a uniformly most powerful test, its power function and size.

Let $Y_1, \ldots, Y_n$ be independent identically distributed random variables with common density $\rho e^{-\rho y}$, $y \geq 0$. Obtain the uniformly most powerful test of $\rho = \rho_0$ against alternatives $\rho < \rho_0$ and determine the power function of the test.

2/II/12H **Statistics**

For ten steel ingots from a production process the following measures of hardness were obtained:

73.2, 74.3, 75.4, 73.8, 74.4, 76.7, 76.1, 73.0, 74.6, 74.1.

On the assumption that the variation is well described by a normal density function obtain an estimate of the process mean.

The manufacturer claims that he is supplying steel with mean hardness 75. Derive carefully a (generalized) likelihood ratio test of this claim. Knowing that for the data above

$$S_{XX} = \sum_{j=1}^{n} (X_i - \bar{X})^2 = 12.824,$$

what is the result of the test at the 5% significance level?

[

Distribution	t_9	t_{10}
95% percentile	*1.83*	*1.81*
97.5% percentile	*2.26*	*2.23*

]

4/I/3H **Statistics**

From each of 100 concrete mixes six sample blocks were taken and subjected to strength tests, the number out of the six blocks failing the test being recorded in the following table:

No. x failing strength tests	0	1	2	3	4	5	6
No. of mixes with x failures	53	32	12	2	1	0	0

On the assumption that the probability of failure is the same for each block, obtain an unbiased estimate of this probability and explain how to find a 95% confidence interval for it.

4/II/12H **Statistics**

Explain what is meant by a prior distribution, a posterior distribution, and a Bayes estimator. Relate the Bayes estimator to the posterior distribution for both quadratic and absolute error loss functions.

Suppose $X_1, \ldots, X_n$ are independent identically distributed random variables from a distribution uniform on $(\theta - 1, \theta + 1)$, and that the prior for θ is uniform on $(20, 50)$.

Calculate the posterior distribution for θ, given $\mathbf{x} = (x_1, \ldots, x_n)$, and find the point estimate for θ under both quadratic and absolute error loss function.