LARGE DEVIATIONS, METASTABILITY, AND STOCHASTIC RESONANCE

Suppose a dynamical system $X'(t) = b(X(t))$ has many stable attractors, say, stable equilibria. Then an invariant measure is concentrated at each of these equilibria. Consider now the diffusion process X^ϵ which is obtained by small, of order $\epsilon << 1$, white noise perturbations of the system. Under mild additional assumptions, the process X^ϵ has a unique normalized invariant measure μ^ϵ. The classical question about the limit of μ^ϵ as $\epsilon \to 0$, under certain assumptions, was answered in the framework of large deviation theory. The invariant measure characterizes the system for large t. But there are two small parameters in the system: $1/t$ and ϵ, and the limit when these parameters tend to zero,in the case of many attractors, depends on how $(1/t, \epsilon)$ approaches zero. This question also can be answered using the large deviation asymptotics. In a generic situation, for each time scale and every initial point there is one attractor where the system spends most of the time. Such an equilibrium is called metastable state. But it turns out that for some important classes of systems, for instance for systems which are close to Hamiltonian, this notion of metastability has essential defects. I will explain that certain distributions on the set of attractors should be considered as the metastable states. This set of distributions is independent of the perturbations. This result is a combination of the averaging principle and large deviation asymptotics.