9. SLE(8/3) and the restriction property

Recall the notion of filling

- a closed, connected, simply connected subset of $\hat{U}$ containing z_0 and z_1.

S_D = set of fillings of D

$\{S_{\tilde{D}} : \tilde{D} \subseteq D\}$ is a π-system on S_D

(meaning $\tilde{U} \subseteq U$, $\tilde{U} = U$ near z_0, z_1, $\tilde{z}_i = z_i$

$\mathcal{S}_D$ is the σ-algebra of S_D generated by this π-system

$(\mu_D : D \in \mathcal{D})$ family of probability measures

μ_D on $(S_D, \mathcal{S}_D)$

Recall that $(\mu_D : D \in \mathcal{D})$ has the restriction property if

<u>for all</u> $D, D' \in \mathcal{D}$ <u>with</u> $D' \subseteq D$,

<u>for</u> $X \sim \mu_D$, <u>the conditional law of</u> X <u>given</u> $X \subseteq D'$ <u>is</u> $\mu_{D'}$

When $(\mu_D : D \in \mathcal{D})$ is conformally invariant, this is equivalent to the following property of $\mu = \mu_{(\mathbb{H},0,\infty)}$:

<u>for all</u> $D \subseteq (\mathbb{H}, 0, \infty)$, <u>all conformal isomorphisms</u> $\Phi : D \to (\mathbb{H}, 0, \infty)$, <u>for</u> $X \sim \mu$, <u>conditional on</u> $X \subseteq D'$ <u>we have</u> $\Phi(X) \sim \mu$.

Proposition 9.1

Let γ be an SLE($8/3$). Then, for all $D \subseteq (\mathbb{H}, 0, \infty)$,

$$P(\gamma \subseteq D) = \Phi_D'(0)^{5/8}$$

where Φ_D is the unique conformal isomorphism $D \to \mathbb{H}$ with $\Phi_D'(\infty) = 1$.

Theorem 9.2

SLE($8/3$) has the restriction property.