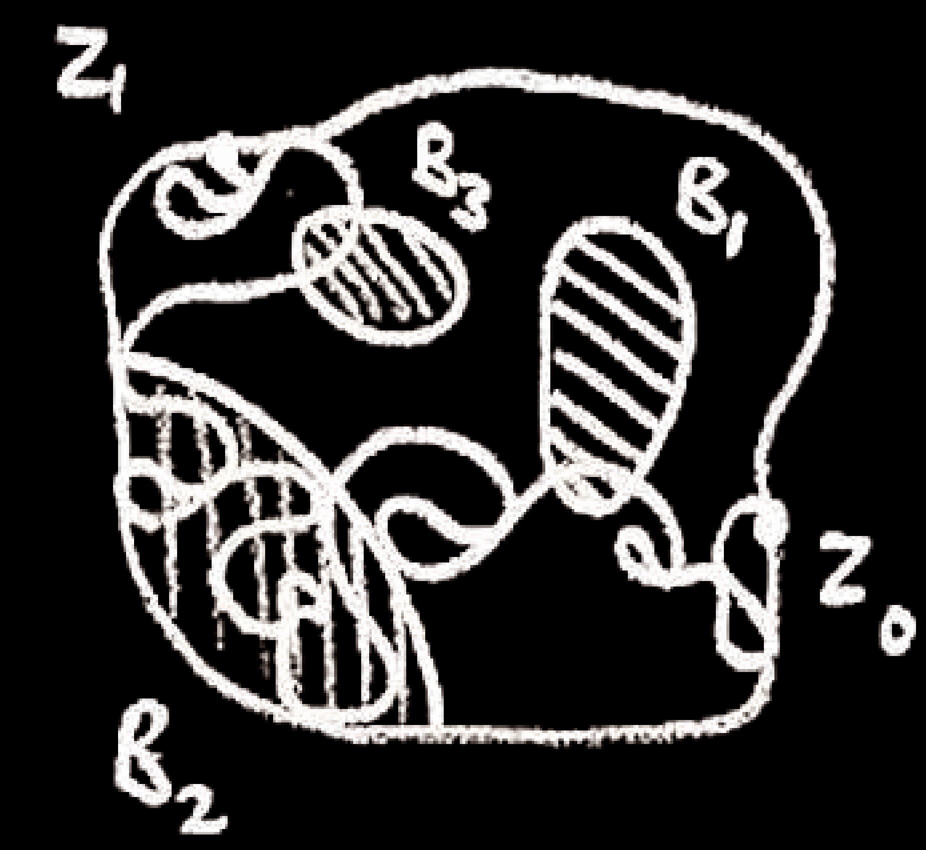
7. SLE and the domain Markov property



$\gamma \in A(B_1, B_2, B_3)$

Recall from §1

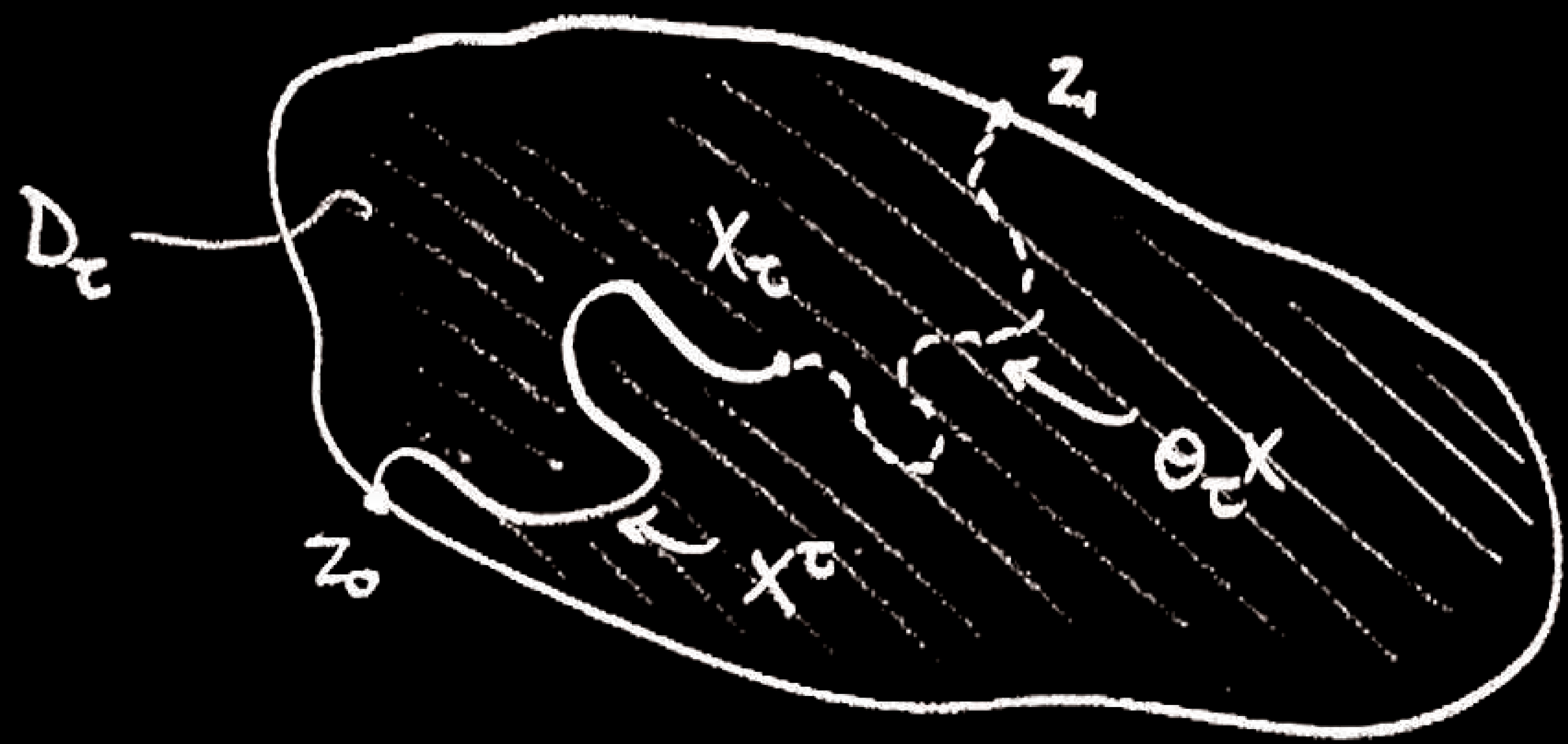
C_D is the set of chords in $D = (U, z_0, z_1)$

$\mathcal{C}_D$ is the σ-algebra on C_D generated by sets $A(B_1, \ldots, B_n)$

Let $(\mu_D : D \in \mathcal{D})$ be a family of probability measures with μ_D on $(C_D, \mathcal{C}_D)$

— also in §8

Say that $(\mu_D : D \in \mathcal{D})$ has the <u>domain Markov property</u> if, for all $D = (U, z_0, z_0) \in \mathcal{D}$, for $X \sim \mu_D$, X does not hit z_1 before time 1, a.s., and, for all parametrization invariant stopping times τ on P_D, conditional on the stopped chord X^τ and on $\tau(X) < 1$, we have $\theta_\tau X \sim \mu_{D_\tau}$



<u>Theorem 7.1</u>

The following statements are equivalent:

(a) the family $(\mu_D : D \in \mathcal{D})$ is conformally invariant and has the domain Markov property,

(b) there exists $\kappa \in [0,\infty)$ such that, for all $D \in \mathcal{D}$ and for any conformal isomorphism $\Phi_D : (\mathbb{H}, 0, \infty) \to D$, μ_D is the law of $[\Phi_D(\gamma)]$, where γ is an $SLE(\kappa)$.

We refer to such μ_D and to $X \sim \mu_D$ as $SLE(\kappa)$ in D.

8. SLE(6) and the locality property

Recall that $(\mu_D : D \in \mathcal{D})$ has the locality property if
for all $D, D' \in \mathcal{D}$, for all initial domains N common to D, D',
if $X \sim \mu_D$, $X' \sim \mu_{D'}$, then $X^N \sim X'^N$,
where X^N is X stopped on leaving N.

Suppose now that $(\mu_D : D \in \mathcal{D})$ is conformally invariant.
Then we can realize $X = \Phi_D(X^\circ)$, $X' = \Phi_{D'}(X^\circ)$
where $\Phi_D, \Phi_{D'}$ are conformal isomorphism to D, D'
from $(\mathbb{H}, 0, \infty)$ and $X^\circ \sim \mu_{(\mathbb{H},0,\infty)}$

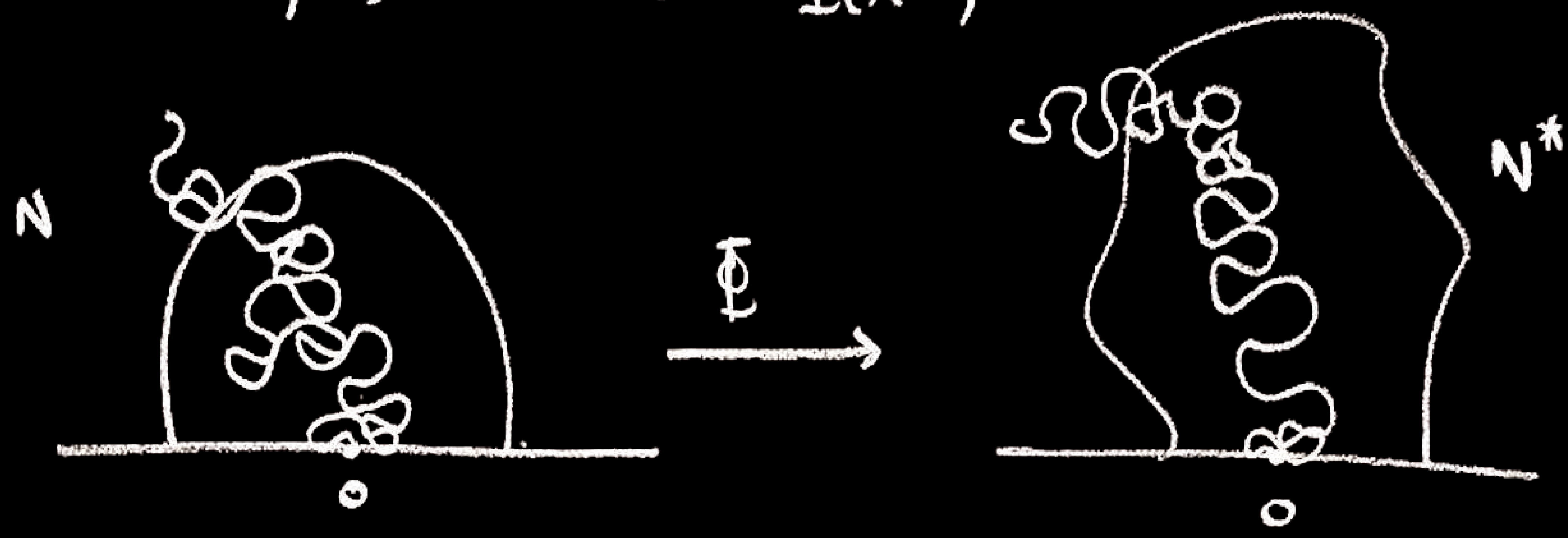
So the locality property is equivalent to the following property of $\mu = \mu_{(\mathbb{H},0,\infty)}$:

let N, N^* be initial domains in $(\mathbb{H}, 0, \infty)$,

let $\Phi : N \longrightarrow N^*$ be a conformal isomorphism, $\Phi(0) = 0$,

with $\Phi(\partial N \cap \mathbb{R}) = \partial N^* \cap \mathbb{R}$;

if $X \sim \mu$, that $X^{N^*} \sim \Phi(X^N)$



Theorem 8.1

SLE(6) has the locality property.

Corollary 8.2

Let U be a simply connected proper domain and let z_0, z_1, z_1' be distinct points in $\hat{U} \setminus U$.

Set $D = (U, z_0, z_1)$, $D' = (U, z_0, z_1')$.

Let X be SLE(6) in D, X' be SLE(6) in D'.

Set $T = \inf\{t \geq 0 : X_t \in [z_1, z_1']\}$,

$\quad T' = \inf\{t \geq 0 : X'_t \in [z_1, z_1']\}$.

Then X^T and $(X')^{T'}$ have the same distribution.