NEW FRONTIERS IN RANDOM GEOMETRY (RaG) EP/103372X/1 REPORT 1/9/11 – 1/7/12

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1. Management Process

The Management Committee (MC) comprises the three investigators and the three members of the external Advisory Board (AB), namely Yuval Peres, Stanislav Smirnov, and Wendelin Werner. The local managers have met weekly, and more formally about every two months. The advice of the AB has been sought on a variety of matters including the hiring process. Two members of the AB (Peres and Werner) have spent substantial periods in Cambridge (these visits have been funded by non-EPSRC sources, namely Microsoft Corporation and Cambridge University, respectively).

2. Appointments

This grant was started on 1 September 2011. Two postdoctoral research fellows were appointed following the first advertisement in December 2010.

- Zhongyang Li[1], BSc (Fudan University, 2006), PhD (Brown University, 2011), from 1 September 2011 to 31 August 2014.
- Alan Sola[2], PhD (KTH, Stockholm, 2010), from 1 January 2012 to 31 December 2013.

A further advertisement in December 2011 attracted a number of strong probabilists from around the world. However, these were not considered optimal for the scientific programme, and no appointments were made. We do not anticipate difficulty in making further outstanding appointments. Further positions will be advertised in December 2012.

Cambridge University agreed to the creation of a new tenured post in probability, conditional on the funding by the EPSRC of the RaG programme. The Statistical Laboratory is in the process of advertising and filling this new position.

Date: 7 July 2012.

http://www.statslab.cam.ac.uk/~grg/rag.html.

[1] http://www.statslab.cam.ac.uk/~zl296/

[2] http://www.statslab.cam.ac.uk/~as2221/

3. Research programme

3.1. **Hastings–Levitov conformal aggregation.** Norris, Sola, Turner, and Viklund are studying a regularized version of the HL(α) process. Instead of evaluating the derivative of Φ_{n-1} on the unit circle, one moves out radially to distance $\sigma > 0$ and adjusts particles by the derivative at that point — a natural substitute from the point of view of complex analysis. Simulations of this model reveal that the shape of clusters depends on the size of σ compared with a basic unscaled particle size $d > 0$, and that the clusters cease to be circular if $\alpha > 1$ and σ is of the same order as d or smaller. For large σ, one couples the process with a time-changed HL(0), and uses results of Norris and Turner to prove convergence to a disk. This particular coupling argument fails when $\sigma = d$, but the pictures still suggest convergence to disks when $\alpha < 1$; when $\alpha > 1$, one suspects that a scaling limit, if it exists, is inherently random.

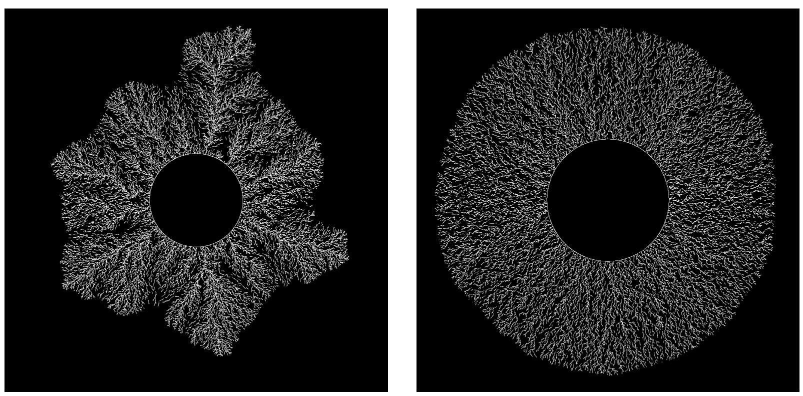

Figure 3.1. HL(1.5) clusters with $\sigma = d$ and $\sigma \gg d$, grown with $d = 0.02$ and $n \asymp d^{-2}$. (Section 3.1)

The larger goal is to broaden the class of particle aggregation models for which the small-particle limit is understood, in particular to models exhibiting macroscopic randomness in the scaling lmit.

3.2. **Self-avoiding walks (SAW).** How does the connective constant for SAWs depend on the choice of graph? Grimmett and Li are studying lower bounds for regular graphs, and strict inequality results for vertex-transitive graphs. Such inequalities are the SAW equivalents of now established theorems for percolation and random-cluster models on general graphs.

3.3. **Planar quantum geometry.** Berestycki and Miermont are working on diffusions in planar quantum geometry and the KPZ formula. They have identified a canonical object which can be thought of as Brownian motion on this "random surface". Conceptually much of the proof is ready. They have identified interesting lines of research for the future: in particular what can be said about the heat kernel?

3.4. **Universality for bond percolation in two dimensions.** In this continuation of work begun before RaG, Grimmett and Manolescu have identified the critical surface of bond percolation on an isoradial graph as the parameter-vector associated with the isoradial embedding. Moreover, the family of such processes form a universality class, in the sense that the values of the critical exponents *at* the critical point are constant across the family.

This leads to several questions of significance, such as: could comparable results hold for random-cluster models; can one prove a 'conditional' Cardy formula for this family.

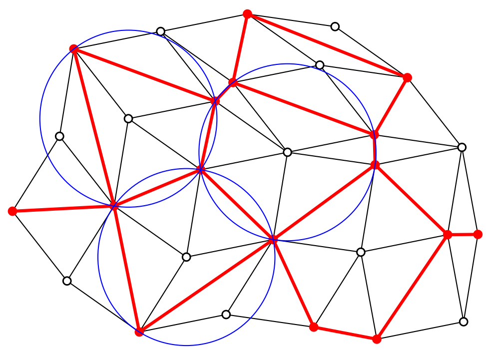

FIGURE 3.2. A rhombic tiling generates an isoradial graph. (Section 3.4)

3.5. **1–2 model.** A 1–2 model is a probability measure on subsets of a hexagonal lattices satisfying the constraint that each vertex has one or two incident edges in the subset. The *homogeneous* cluster is the connected set of vertices in which the present incident edges of each vertex are of the same direction. Li has proved the existence of a phase transition. The remaining problems are to show that the phase transition is sharp, to identify the critical parameters, and to describe the interface separating the homogenous clusters at a critical point.

3.6. **Zeros of random polynomials.** Sola and Pritsker are investigating finer quantitative structure of the geometry of zeros of a random polynomial. Relevant quantities include the discrepancy of zeros, measuring how much the zero-counting measure deviates from arc-length measure in sets such as sectors and annular boxes, and the expected number of zeros in polygons inscribed in the unit disk, and in other subsets with tangencies with the circumference.

3.7. **Coalescing random walks in a random medium.** Berestycki and Barlow are studying coalescing random walks on supercritical percolation clusters. How does the disorder in the geometry affect the density of particles at a large time t? Can the induced fluctuations can be felt in the limit of large time?

3.8. **Speed of a mean-field random walk.** Berestycki, Lubetzky, and Peres have investigated the speed of a random walk on the giant cluster of a random graph. It turns out that there is a critical point at which the speed changes from a positive constant to 0. This gives a lower bound on the mixing time of a random walk on such a cluster, averaged over the starting point of the walk.

4. Activities

4.1. **Output.** The following publications and preprints have been facilitated by funding through RaG. They are available via

http://www.statslab.cam.ac.uk/~grg/rag-pubs.html.

Publications

1. Three theorems in discrete random geometry, G. Grimmett. *Probability Surveys* 8 (2011) 403–441

Preprints

1. A small-time coupling between Lambda-coalescents and branching processes, J. Berestycki, N. Berestycki, V. Limic
2. The genealogy of branching Brownian motion with absorption, J. Berestycki, N. Berestycki, J. Schweinsberg
3. Percolation since Saint-Flour, G. Grimmett, H. Kesten
4. Cycle structure of the interchange process and representation theory, N. Berestycki, G. Kozma
5. Galton–Watson trees with vanishing martingale limit, N. Berestycki, N. Gantert, P. Moerters, N. Sidorova
6. Critical temperature of periodic Ising models, Z. Li
7. Spectral curve of periodic Fisher graphs, Z. Li
8. Bond percolation on isoradial graphs, G. Grimmett, I. Manolescu
9. Asymptotic sampling formulae for Lambda-coalescents, J. Berestycki, N. Berestycki, V. Limic
10. 1–2 model, dimers, and clusters, Z. Li
11. Large scale behaviour of the spatial Lambda–Fleming–Viot process, N. Berestycki, A. M. Etheridge, A. Veber
12. Hastings-Levitov aggregation in the small-particle limit, J. Norris, A. Turner,
13. Weak convergence of the localized disturbance flow to the coalescing Brownian flow, J. Norris, A. Turner, *Communications in Mathematical Physics*
14. Universality for bond percolation in two dimensions, G. Grimmett, I. Manolescu, *Annals of Probability*
15. Inhomogeneous bond percolation on square, triangular, and hexagonal lattices, G. Grimmett, I. Manolescu, *Annals of Probability*

16. Cluster detection in networks using percolation, G. Grimmett, E. Arias-Castro, *Bernoulli*

4.2. **Seminars.** The weekly probability seminar has been lively as always. Details of events may be found at

http://talks.cam.ac.uk/show/archive/9938.

4.3. **Official visitors.** Cambridge Probability has received a number of visitors in 2011-12, for short and longer periods. The following individuals are connected directly to RaG.

- Yuval Peres, November 2011
- Wendelin Werner, January–March 2012
- Hugo Duminil-Copin, February 2012
- Fredrik Johansson Viklund, May 2012

4.4. **Visits by members of RaG.** Members of RaG have made numerous visits to other institutions, and have participated in numerous conferences and workshops. Listed here are visits made by research fellows.

- Jan 2012: *Connections for women: Discrete lattice models in mathematics, physics and computing*, MSRI workshop [Li].
- Jan 2012: *Introductory workshop: Lattice models and combinatorics*, MSRI workshop [Li].
- Feb 2012: *Young European Probabilists 2012 (YEP IX)*, Workshop on two-dimensional statistical mechanics, Eurandom [Li].
- Mar 2012: Columbia University [Sola].
- Mar 2012: *Statistical Mechanics and Conformal Invariance.* MSRI workshop [Sola].
- Mar 2012: *AMS Sectional Meeting, Special Session on Complex Analysis and Probability Theory.* Meeting held in Lawrence, Kansas [Sola].
- Apr 2012: Oklahoma State University, Stillwater OK [Sola].
- May 2012: Lancaster University [Sola].
- May 2012: *Royal Statistical Society.* Applied probability launch meeting, London [Sola].
- Jun 2012: *Conformal Invariance, Discrete Holomorphicity and Integrability*, University of Helsinki workshop [Li].

5. Future activities

Amongst our immediate targets are the following.

- A discussion meeting with scientists from the British Antarctic Survey, scheduled for 19 July 2012.
- The organisation of a one-week Cambridge workshop on probability and random geometry during April 2013.
- The submission of an expression of interest in organising a 6 month programme at the Isaac Newton Institute.

- The search for and appointment of postdoctoral fellows within RaG.

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